

Quantum error correction with superconducting circuits

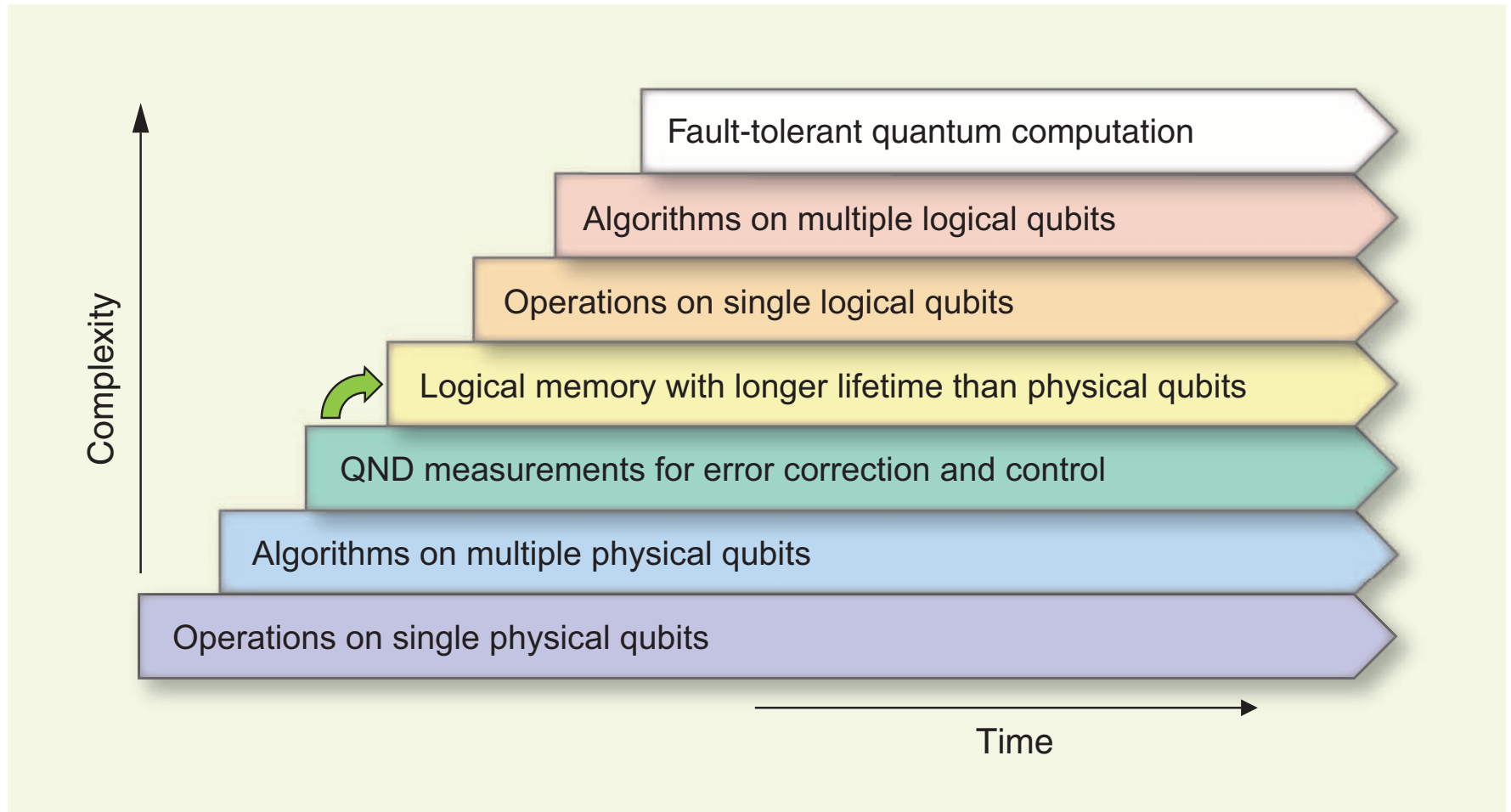
Mazyar Mirrahimi
QUANTIC team (INRIA, LPA, Mines)

QUANTUM INFORMATION PROCESSING

What is next?

- Interesting quantum devices in the next 10 years:
 - = complexity that CANNOT EVER be classically simulated (> 50 qubits or equivalent)
- Outstanding questions:
 - what level of quantum error correction(QEC) needed?
 - how much overhead QEC?
 - what's the best architecture?
 - what are the useful and achievable (on short term) applications?

ROAD-MAP TOWARDS FAULT-TOLERANT QUANTUM COMPUTATION



M.H. Devoret & R.J. Schoelkopf, Science 339, 1169-1174 (2013).

OUTLINE

□ Introduction to quantum error correction

- Classical vs quantum error correction
- Theory of quantum error correction
- Insights on fault-tolerance

□ A continuous-variable alternative

- Cat-qubits for protection against photon-loss
- Nonlinear dissipation paving the way towards fault-tolerance

QUANTUM ERROR CORRECTION

Scheme for reducing decoherence in quantum computer memory

Peter W. Shor*

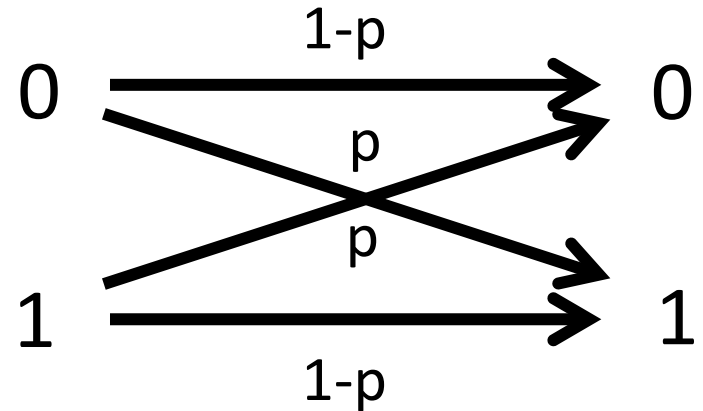
AT&T Bell Laboratories, Room 2D-149, 600 Mountain Avenue, Murray Hill, New Jersey 07974

(Received 17 May 1995)

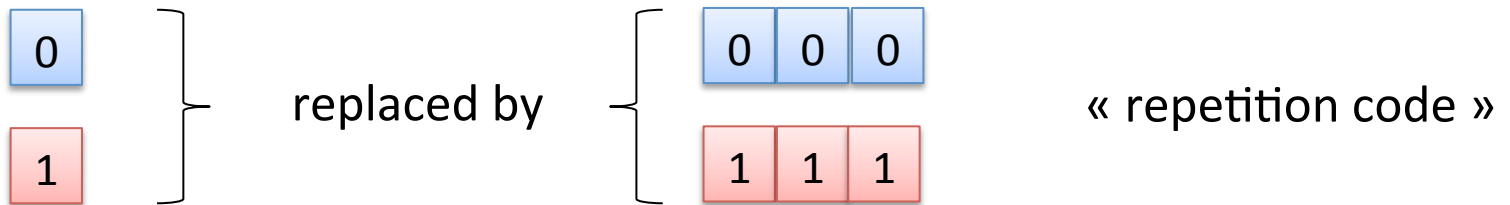
- Decoherence: not a fundamental objection to quantum computation;
- Model continuous decoherence as discrete error channels;
- Redundantly encode quantum information in an entangled state of a multi-qubit system and perform quantum error correction.

CLASSICAL NOISE, CLASSICAL ERROR CORRECTION

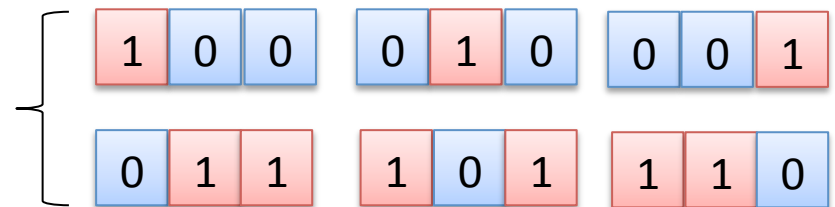
Classical noise: bit-flip errors



Basics of **classical** error correction: redundancy



1-bit errors tractable by **majority vote**:



Probability of intractable 2-bit errors: $3p^2$ (p error probability per unit time)

QUANTUM VS CLASSICAL ERROR CORRECTION

Objective: Protect **any** superposition state $c_0|0\rangle + c_1|1\rangle$ **without** any knowledge of c_0 and c_1 .

Quantum error correction: bit-flip errors

$$c_0 \boxed{0} + c_1 \boxed{1} \quad \longleftrightarrow \quad c_0 \boxed{0} \boxed{0} \boxed{0} + c_1 \boxed{1} \boxed{1} \boxed{1}$$

- **Majority vote erases the information.**
- 1-bit errors tractable by **parity measurement:** Z_1Z_2 and Z_2Z_3
- Four outcomes: (++) No errors, (-+) error on Q1, (+-) error on Q3, (--) error on Q2.

QEC BEYOND BIT-FLIP ERRORS

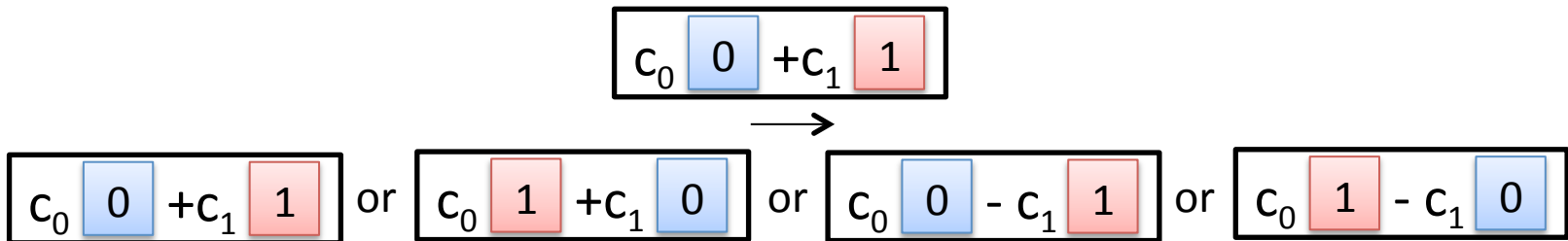
Scheme for reducing decoherence in quantum computer memory

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One needs to correct four possible error channels:
 $I, X, Z, Y=iXZ$



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- Fault-tolerant parity measurements

QEC BEYOND BIT-FLIP ERRORS

Quantum noise: interaction with environment

A general error mechanism:

$$\mathcal{E}(\rho_s) = \text{tr}_{\text{env}} \left[U_\tau (\rho_s \otimes \rho_{\text{env}}) U_\tau^\dagger \right] = \sum_k E_k \rho_s E_k^\dagger$$

with $\sum_k E_k^\dagger E_k = I.$

EXAMPLES

Pure dephasing

$$\mathcal{E}_\varphi(\rho) = E_0 \rho E_0^\dagger + E_1 \rho E_1^\dagger,$$

$$E_0 = \sqrt{1-p}I, \quad E_1 = \sqrt{p}\sigma_z, \quad p = \tau / T_\varphi$$

T1 Relaxation

$$\mathcal{E}_{T1}(\rho) = E_0 \rho E_0^\dagger + E_1 \rho E_1^\dagger,$$

$$E_0 = |0\rangle\langle 0| + \sqrt{1-p}|1\rangle\langle 1|, \quad E_1 = \sqrt{p}|0\rangle\langle 1|, \quad p = \tau / T1$$

QEC BEYOND BIT-FLIP ERRORS

Theory of QEC

Similarly to an error channel, the **error correction (measurement and feedback)** can be modeled by a quantum operation:

$$\rho \rightarrow \mathcal{R}(\rho) = \sum_k \mathbf{R}_k \rho \mathbf{R}_k^\dagger$$

This corrects an error channel $\rho \rightarrow \mathcal{E}(\rho)$ if **for any ρ in the code space**

$$\mathcal{R} \circ \mathcal{E}(\rho) = \rho.$$

QEC BEYOND BIT-FLIP ERRORS

Theorem: discretization of error channels

If the operation \mathcal{R} corrects the error channel \mathcal{E} , it corrects any other error channel \mathcal{F} whose elements F_k are linear combinations of elements E_k with complex coefficients:

$$\mathcal{R} \circ \mathcal{E}(\rho) = \rho \quad \Rightarrow \quad \mathcal{R} \circ \mathcal{F}(\rho) = \rho$$

Corollary: case of qubits

It suffices to correct the operations $\left\{ I, \sigma_x, \sigma_z, \sigma_y = i\sigma_x \sigma_z \right\}$ to correct for any **single-qubit** errors.

FULL QUANTUM ERROR CORRECTION

Four possible error channels for each qubit: I, X, Z, $Y=iXZ$

$$c_0 \begin{matrix} \boxed{0} \\ \boxed{0} \end{matrix} + c_1 \begin{matrix} \boxed{1} \\ \boxed{1} \end{matrix}$$

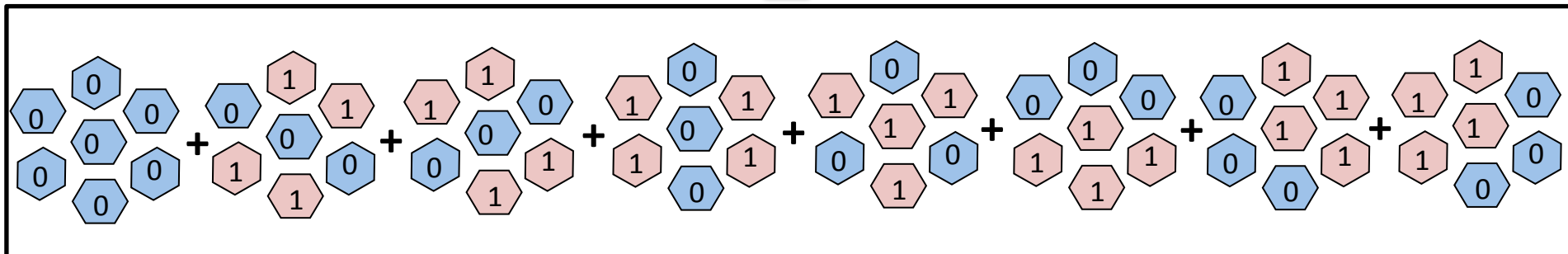


$$c_0 \begin{matrix} \boxed{0} \\ \boxed{0} \end{matrix} + c_1 \begin{matrix} \boxed{1} \\ \boxed{1} \end{matrix} \text{ or } c_0 \begin{matrix} \boxed{1} \\ \boxed{1} \end{matrix} + c_1 \begin{matrix} \boxed{0} \\ \boxed{0} \end{matrix} \text{ or } c_0 \begin{matrix} \boxed{0} \\ \boxed{1} \end{matrix} - c_1 \begin{matrix} \boxed{1} \\ \boxed{0} \end{matrix} \text{ or } c_0 \begin{matrix} \boxed{1} \\ \boxed{0} \end{matrix} - c_1 \begin{matrix} \boxed{0} \\ \boxed{1} \end{matrix}$$

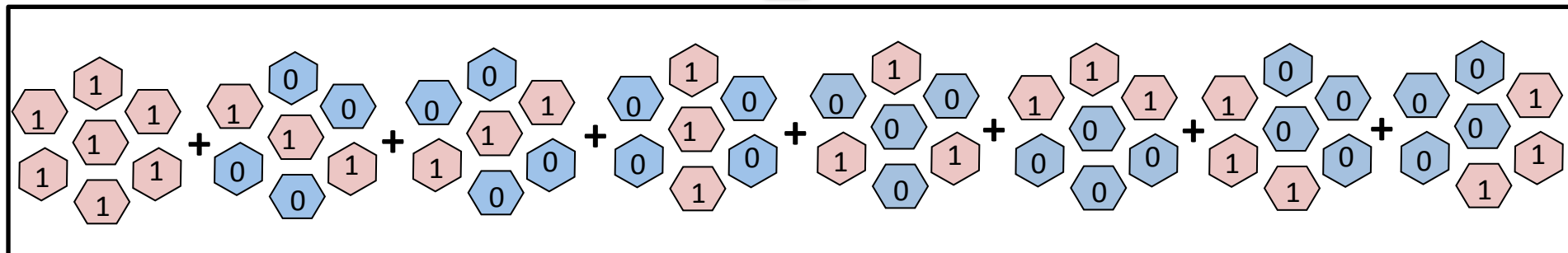
At least five qubits to make all these errors tractable

7-qubit Steane code:

$$0 \longrightarrow$$



$$1 \longrightarrow$$

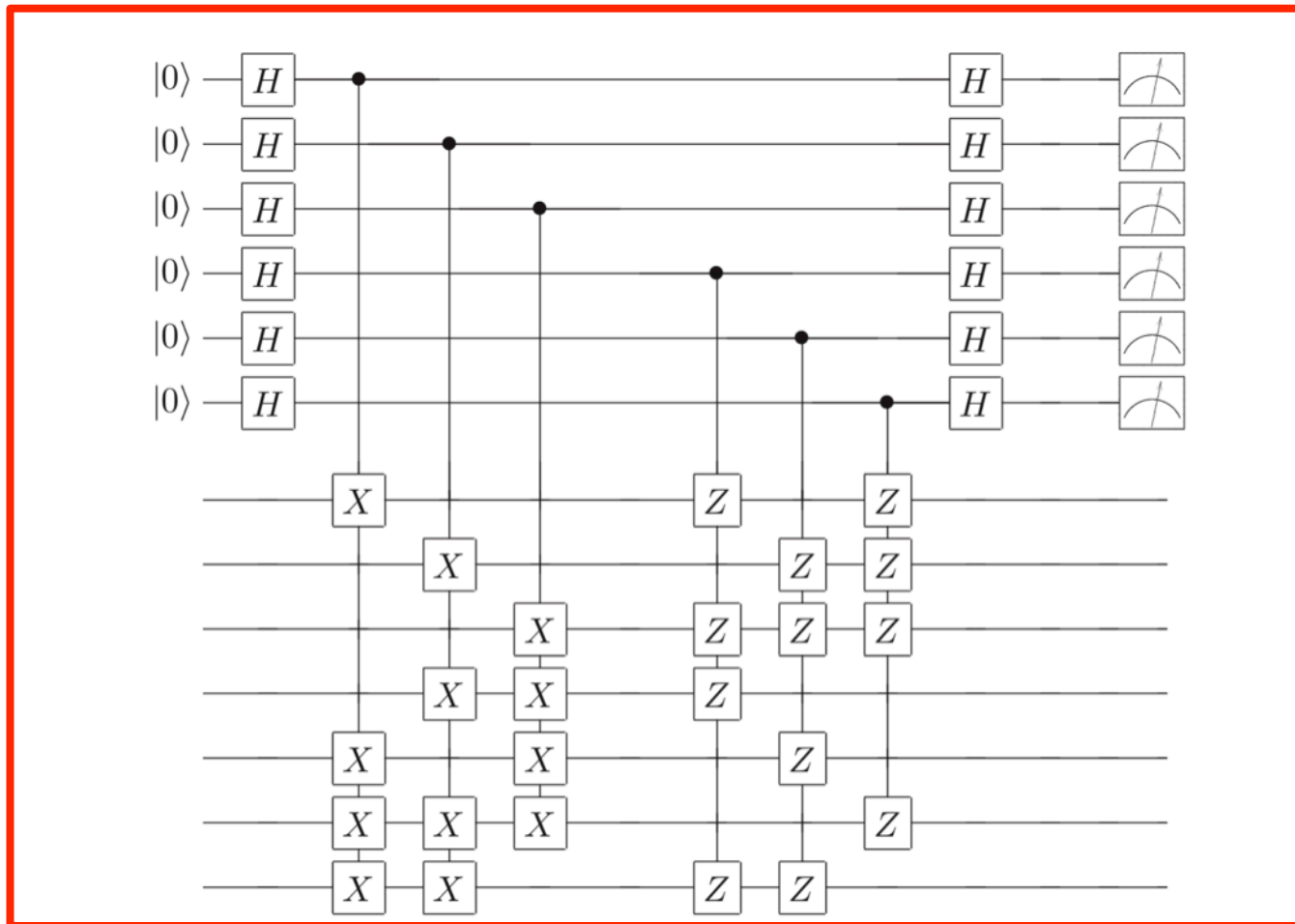


FULL QUANTUM ERROR CORRECTION

$$|0_L\rangle = \frac{1}{\sqrt{8}} \left[|0000000\rangle + |1010101\rangle + |0110011\rangle + |1100110\rangle \right. \\ \left. + |0001111\rangle + |1011010\rangle + |0111100\rangle + |1101001\rangle \right]$$

$$|1_L\rangle = \frac{1}{\sqrt{8}} \left[|1111111\rangle + |0101010\rangle + |1001100\rangle + |0011001\rangle \right. \\ \left. + |1110000\rangle + |0100101\rangle + |1000011\rangle + |0010110\rangle \right]$$

Single round of error correction



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- [Insights on fault-tolerance](#)

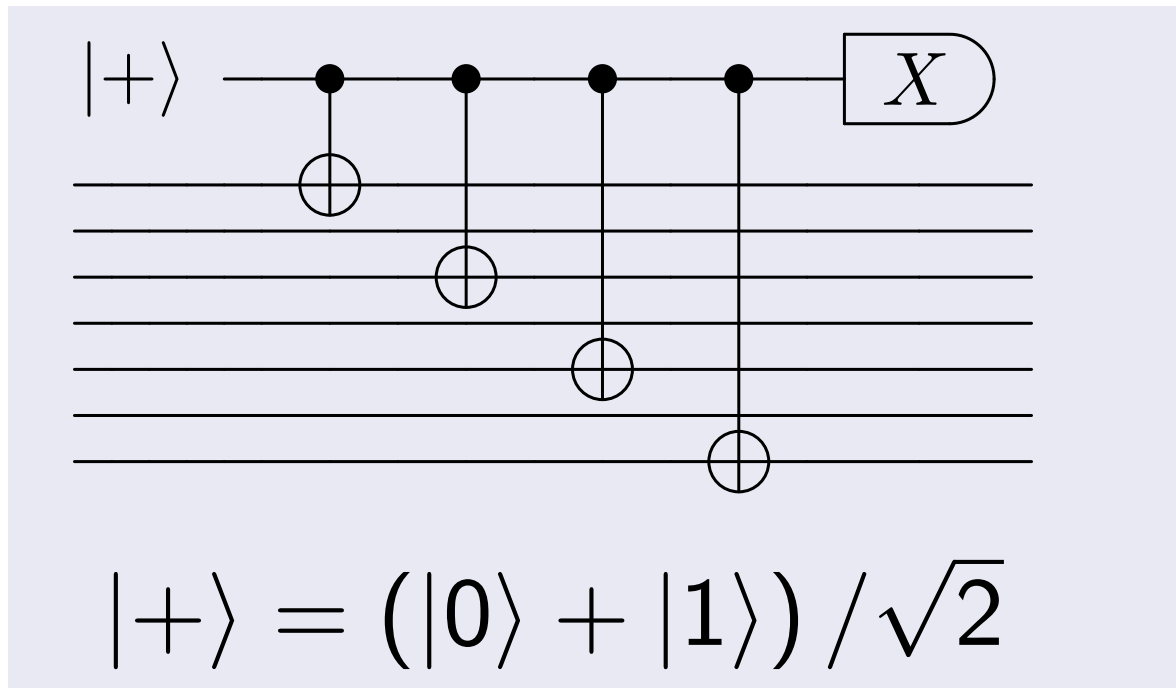
□ A continuous-variable alternative

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FAULT-TOLERANCE

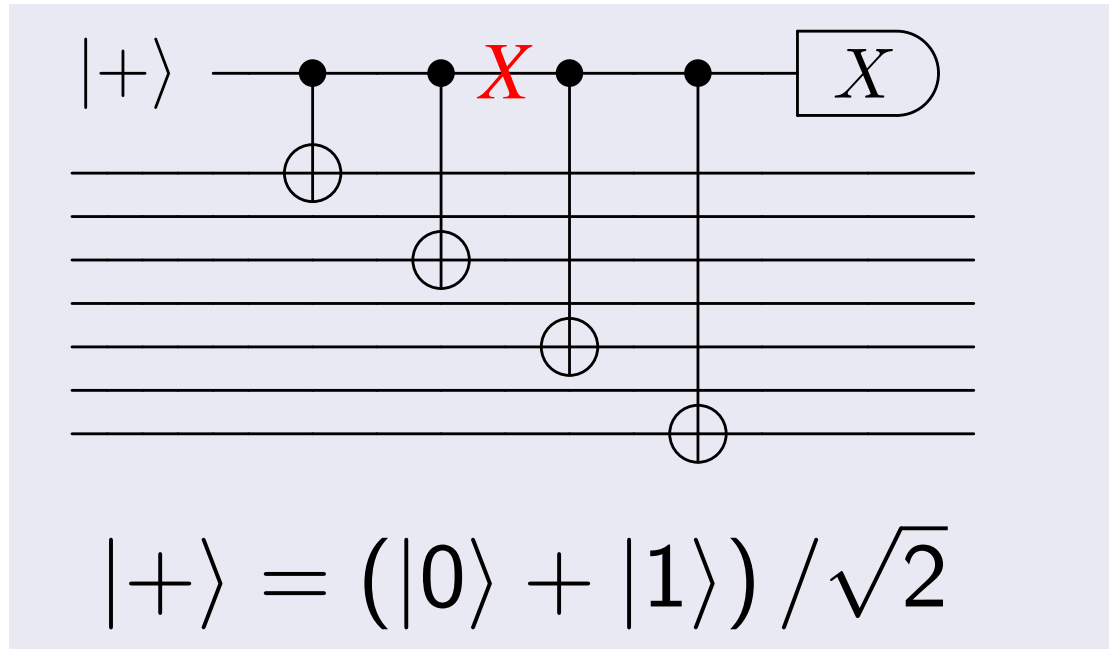
Central idea: through operations, one should not introduce **new error channels** not taken into account by QEC. In particular, one should avoid **propagation/amplification** of errors

Example of parity measurements: simplest circuit to measure the parity $X_1X_3X_5X_7$ for the Steane code.



NOT FAULT-TOLERANT

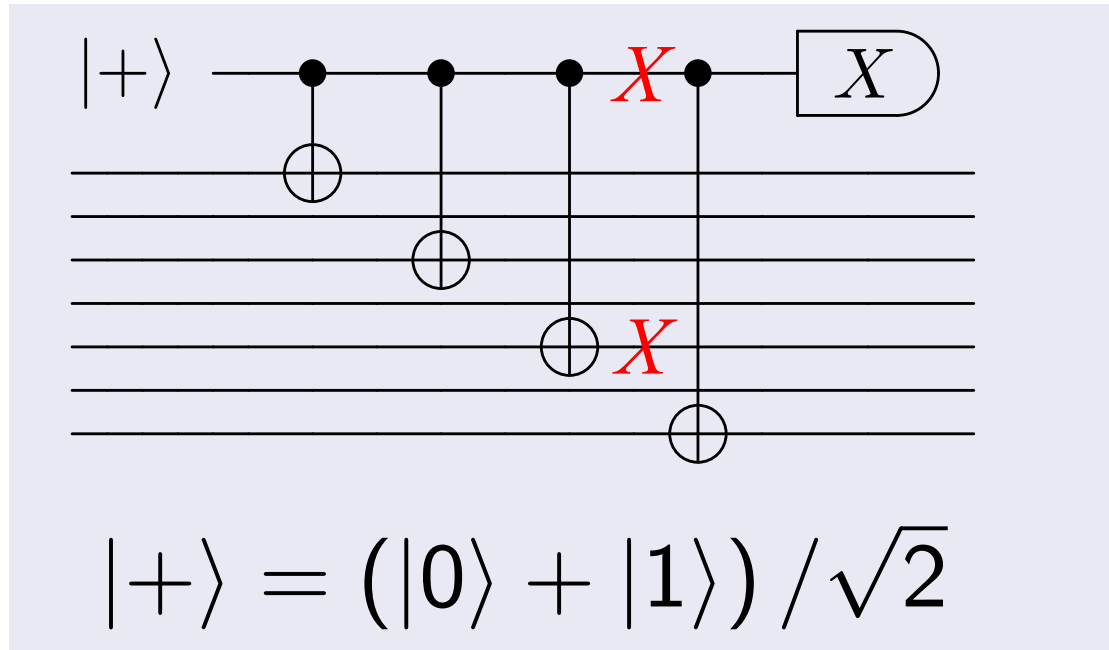
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A bit-flip of the ancilla qubit **propagates to memory qubits.**

NOT FAULT-TOLERANT

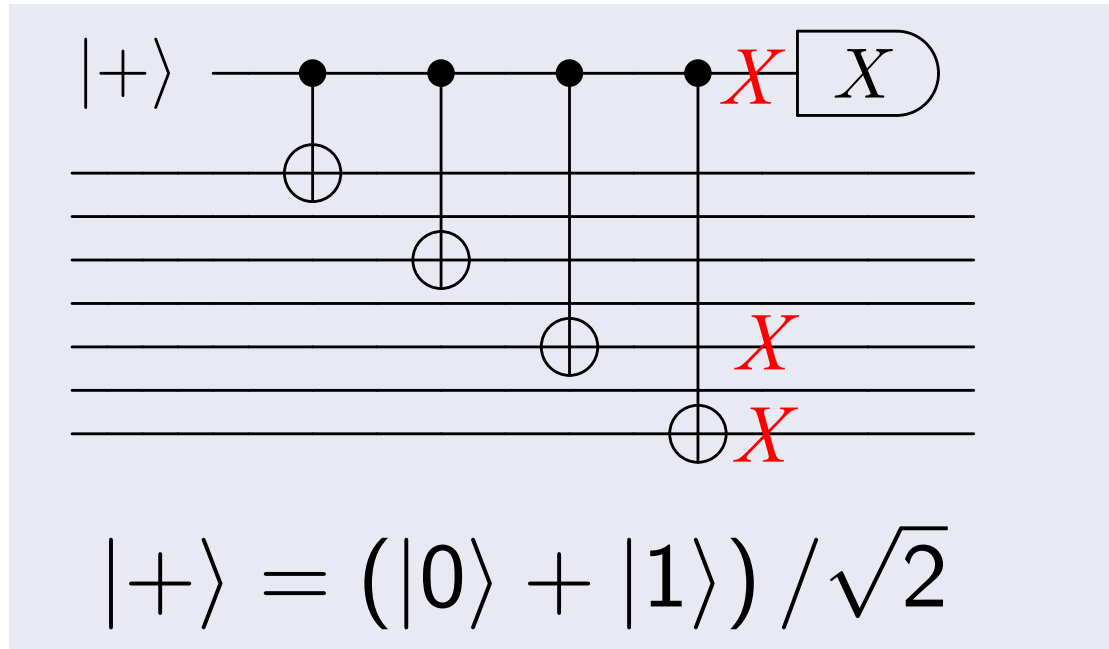
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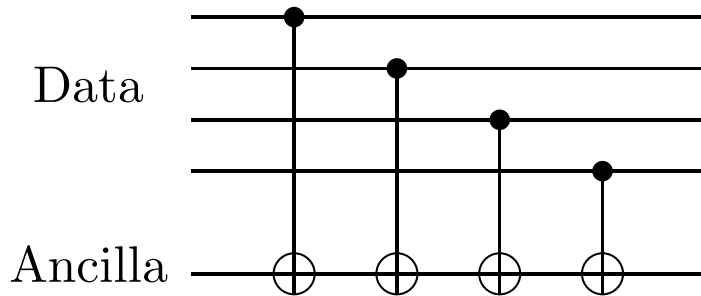
Example of parity measurements: simplest circuit to measure the parity $X_1X_3X_5X_7$ for the Steane code.



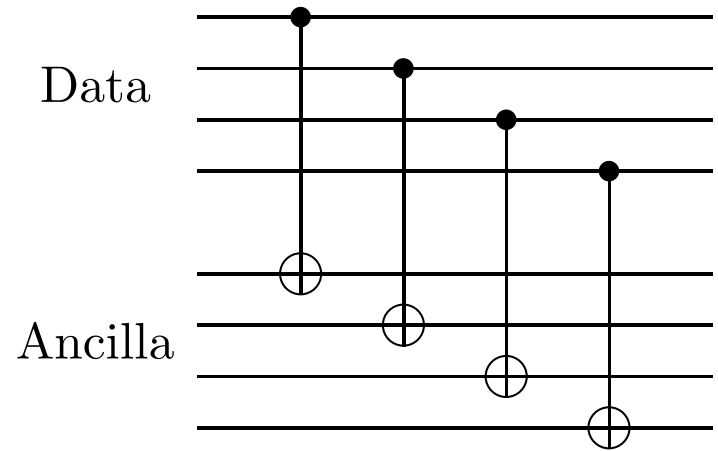
A bit-flip of the ancilla qubit **propagates to memory qubits.**

TOWARDS A SOLUTION

Idea N1: transversal operations



Bad!



Good!

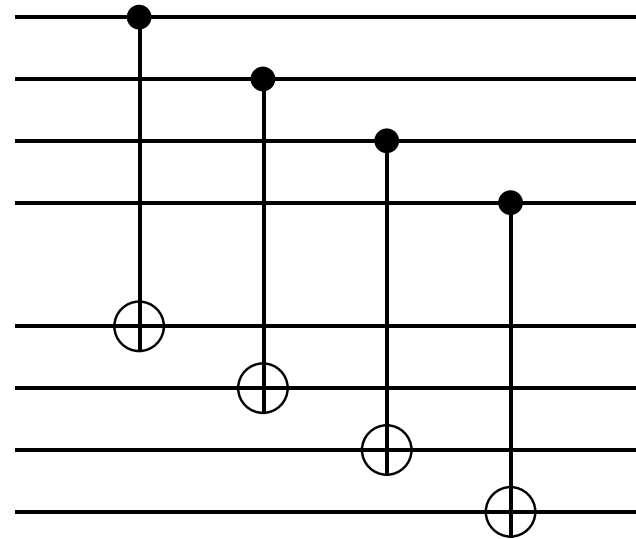
- Each ancilla qubit couples to no more than one memory qubit.
- We readout more than the required information (ancillas get entangled to the codeword).

TOWARDS A SOLUTION

Idea N2: encoding ancillas

$$|\text{Shor}\rangle = \frac{1}{\sqrt{8}} \sum_{\text{even } v} |v\rangle$$

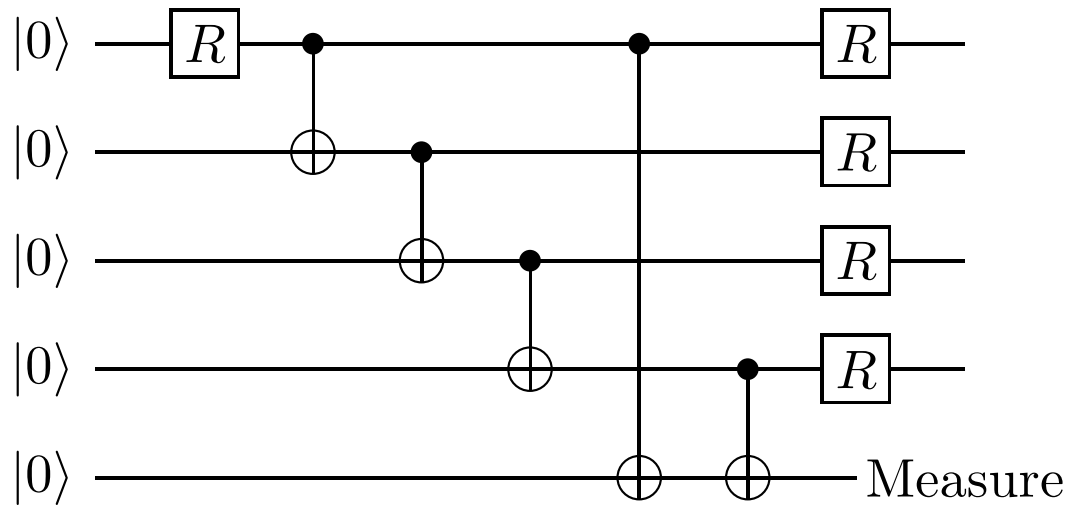
} Ancilla



- The parity of the data qubits is mapped on the parity of the Shor state.
- An error in preparation of the Shor state can propagate.

TOWARDS A SOLUTION

Idea N3: verification of ancillas



$$R = \frac{1}{\sqrt{2}}(X + Z)$$

- Parity measurement is launched if the 5th qubit is measured in 0.
- Otherwise repeat the preparation.

TOWARDS AN ERROR-CORRECTED QUBIT

Three main strategies for implementing a logical qubit:

- A register of physical qubits with full gate operations
- A fabric of physical qubits with nearest neighbor gates
- A superconducting resonator with non-linear drives, non-linear dissipation and photon parity monitoring. These services are provided by Josephson junctions.

Shor (1995)

Steane (1996)

Gottesman, Kitaev, Preskill (2001)

Kitaev (2006)

M.M. et al. (2014)

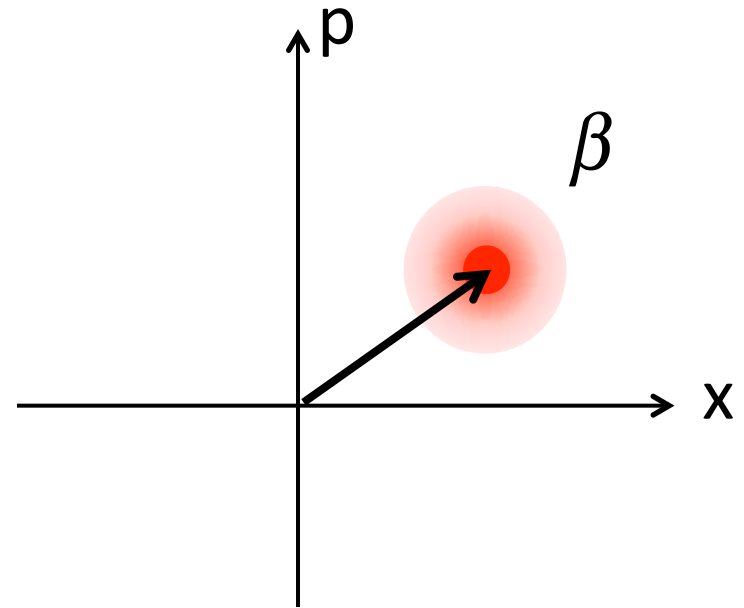
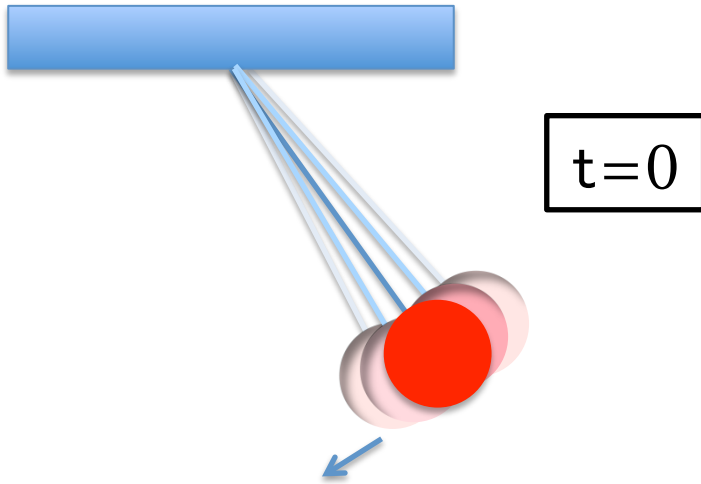
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QUANTUM HARMONIC OSCILLATOR AND COHERENT STATES

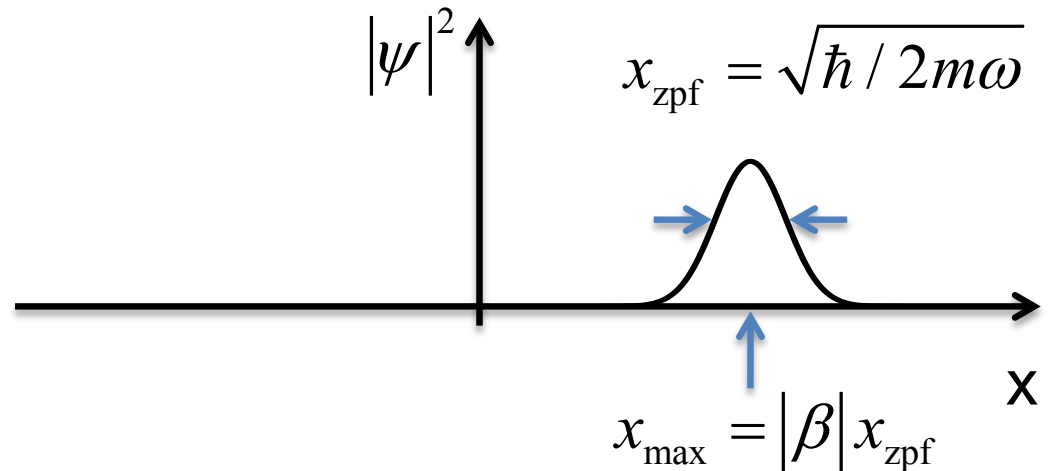
Using classical control (e.g. laser, force), one can only make coherent displacements



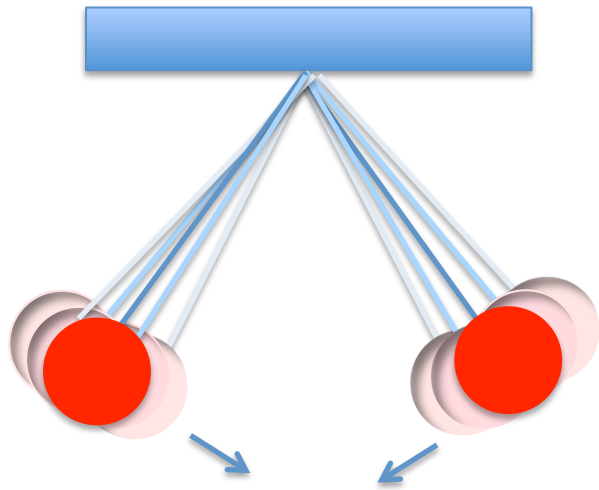
Glauber (coherent) state

$$|\beta\rangle = e^{-\frac{|\beta|^2}{2}} \sum_{n=0}^{\infty} \frac{\beta^n}{\sqrt{n!}} |n\rangle$$

$$\hat{a}|\beta\rangle = \beta|\beta\rangle$$

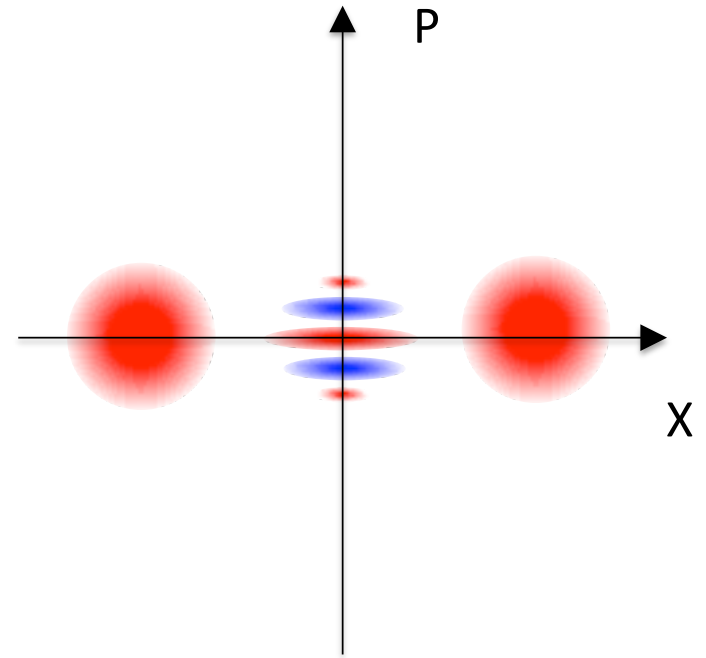


SCHRÖDINGER CAT STATE FOR A HARMONIC OSCILLATOR



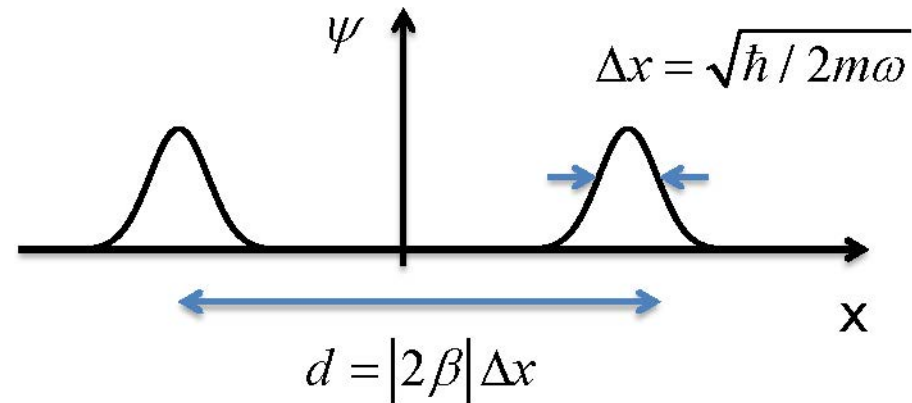
$t=0$

Cat state of an oscillator

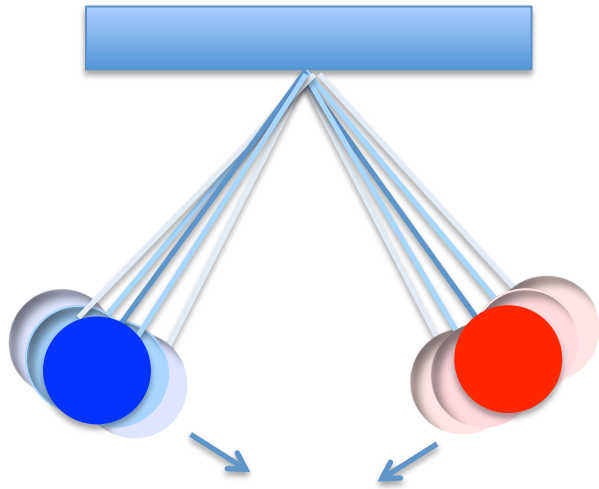


Wigner function $W(\beta)$

$$\begin{aligned}
 |C_{\beta}^{+}\rangle &= \frac{1}{\sqrt{2}} (|\beta\rangle + |-\beta\rangle) \\
 &= \frac{1}{\sqrt{\cosh|\beta|^2}} \sum \frac{\beta^{2n}}{\sqrt{(2n)!}} |2n\rangle
 \end{aligned}$$

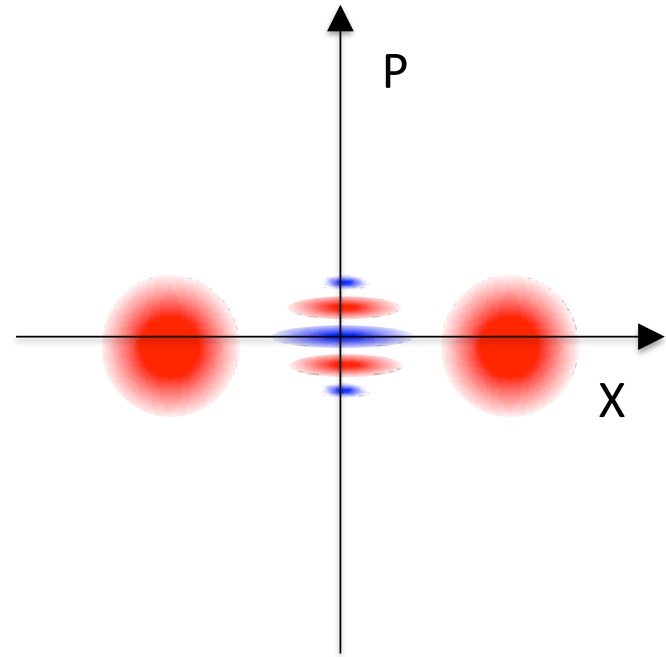


SCHRÖDINGER CAT STATE FOR A HARMONIC OSCILLATOR



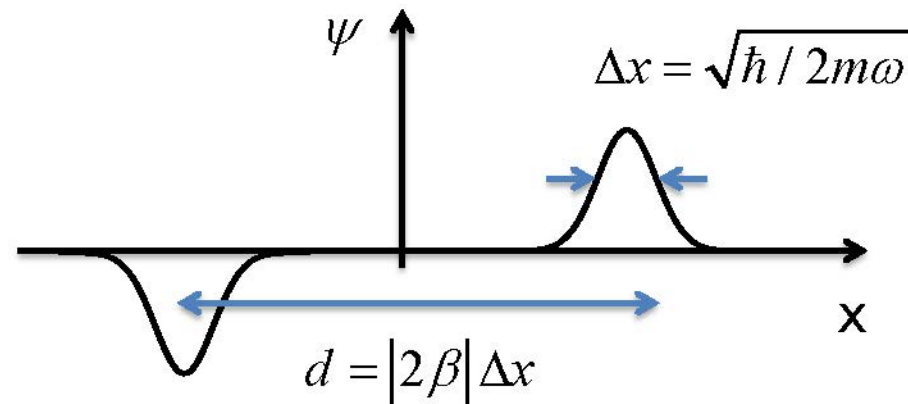
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Cat state of an oscillator

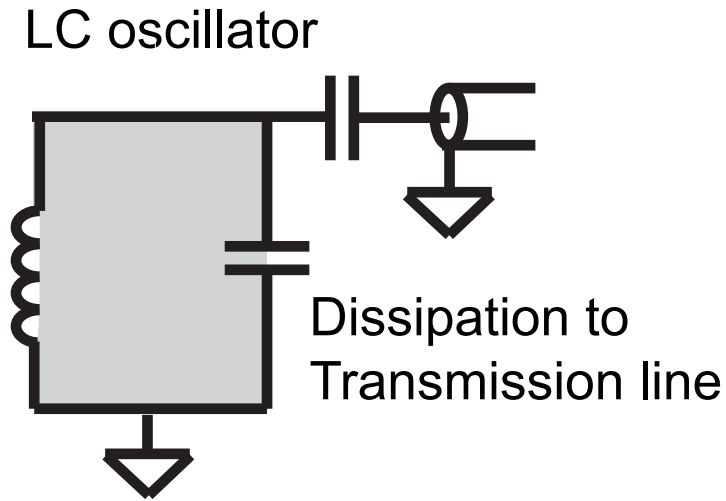


Wigner function $W(\beta)$

$$\begin{aligned}
 |C_{\beta}^{-}\rangle &= \frac{1}{\sqrt{2}} (|\beta\rangle - |-\beta\rangle) \\
 &= \frac{1}{\sqrt{\sinh|\beta|^2}} \sum \frac{\beta^{(2n+1)}}{\sqrt{(2n+1)!}} |2n+1\rangle
 \end{aligned}$$



PHOTON LOSS: MAJOR DECAY CHANNEL OF A H.O.

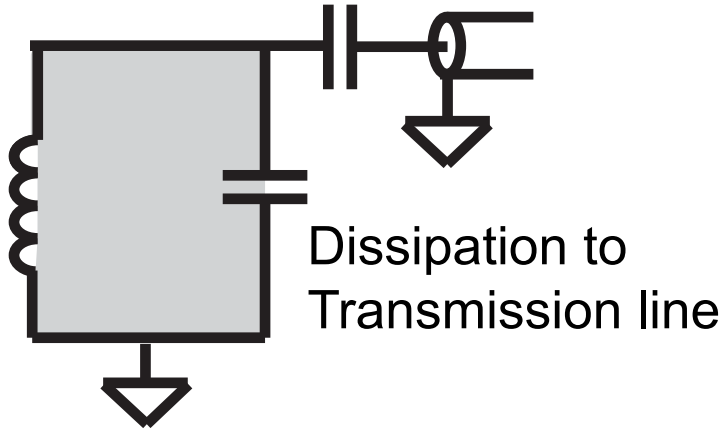


$$\frac{d}{dt}\rho = \kappa D[a]\rho,$$

$$D[a]\rho = a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a.$$

PHOTON LOSS: MAJOR DECAY CHANNEL OF A H.O.

LC oscillator



$$\frac{d}{dt}\rho = \kappa D[\mathbf{a}]\rho,$$

$$D[\mathbf{a}]\rho = \mathbf{a}\rho\mathbf{a}^\dagger - \frac{1}{2}\mathbf{a}^\dagger\mathbf{a}\rho - \frac{1}{2}\rho\mathbf{a}^\dagger\mathbf{a}.$$

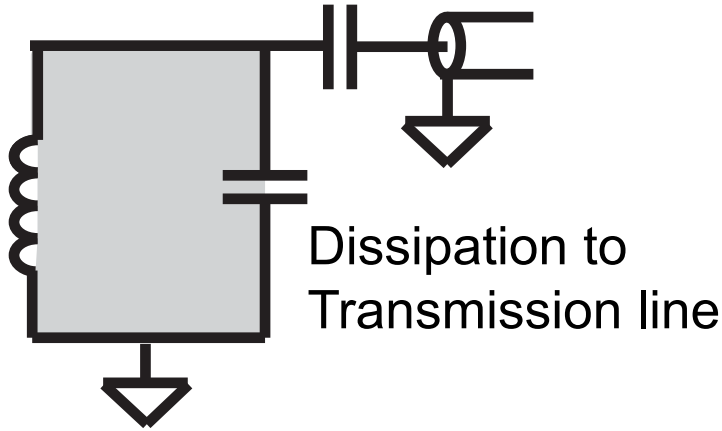
Formulation with error channels:

$$\rho_{\delta t} = \mathcal{E}(\rho_0) = \sum_{l=0}^{\infty} \mathbf{E}_l \rho_0 \mathbf{E}_l^\dagger,$$

$$\mathbf{E}_l = \sqrt{\frac{(1 - e^{-\kappa\delta t})^l}{l!}} e^{-\frac{\kappa\delta t}{2}\mathbf{a}^\dagger\mathbf{a}} \mathbf{a}^l$$

PHOTON LOSS: MAJOR DECAY CHANNEL OF A H.O.

LC oscillator



$$\frac{d}{dt}\rho = \kappa D[\mathbf{a}]\rho,$$

$$D[\mathbf{a}]\rho = \mathbf{a}\rho\mathbf{a}^\dagger - \frac{1}{2}\mathbf{a}^\dagger\mathbf{a}\rho - \frac{1}{2}\rho\mathbf{a}^\dagger\mathbf{a}.$$

Formulation with error channels:

$$\rho_{\delta t} = \mathcal{E}(\rho_0) = \sum_{l=0}^{\infty} \mathbf{E}_l \rho_0 \mathbf{E}_l^\dagger, \quad \mathbf{E}_l = \sqrt{\frac{(1 - e^{-\kappa\delta t})^l}{l!}} e^{-\frac{\kappa\delta t}{2}\mathbf{a}^\dagger\mathbf{a}} \mathbf{a}^l$$

Up to first order in $\kappa\delta t$:

$$\rho_{\delta t} = \mathbf{E}_0 \rho_0 \mathbf{E}_0^\dagger + \mathbf{E}_1 \rho_0 \mathbf{E}_1^\dagger, \quad \mathbf{E}_0 = e^{-\frac{\kappa\delta t}{2}\mathbf{a}^\dagger\mathbf{a}}, \quad \mathbf{E}_1 = \sqrt{\kappa\delta t} e^{-\frac{\kappa\delta t}{2}\mathbf{a}^\dagger\mathbf{a}} \mathbf{a}$$

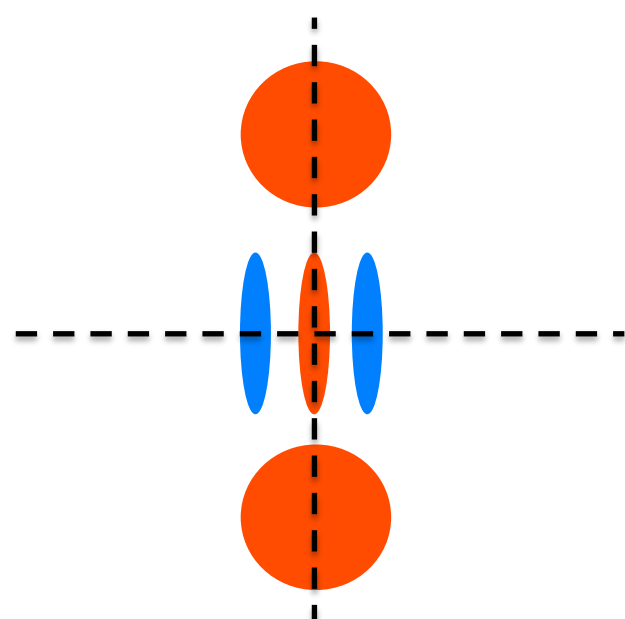
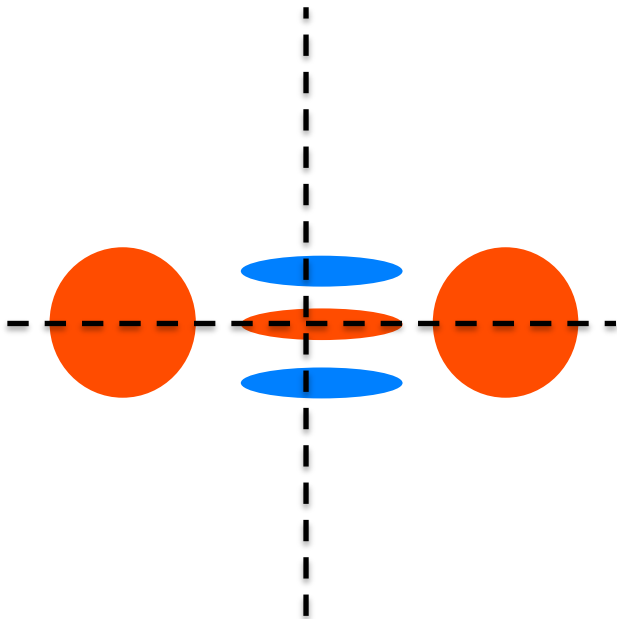
HARDWARE-EFFICIENT QUANTUM ERROR CORRECTION

❑ Encoding and protecting information on a single cavity mode

❑ Minimal QEC hardware : protecting a single high-Q cavity mode (memory), using a single qubit (providing non-linearity), one low-Q mode (entropy evacuation).

Idea:

$$|0_L\rangle = |C_\alpha^+\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + |-\alpha\rangle) \quad |1_L\rangle = |C_{i\alpha}^+\rangle = \frac{1}{\sqrt{2}}(|i\alpha\rangle + |-i\alpha\rangle)$$

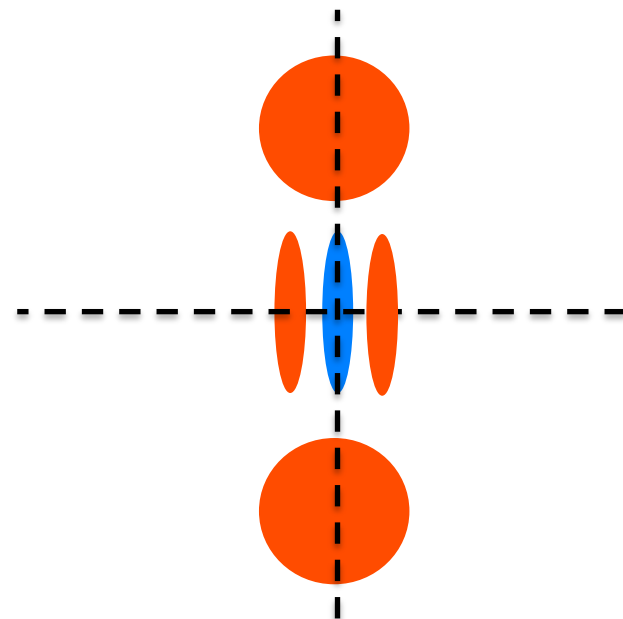
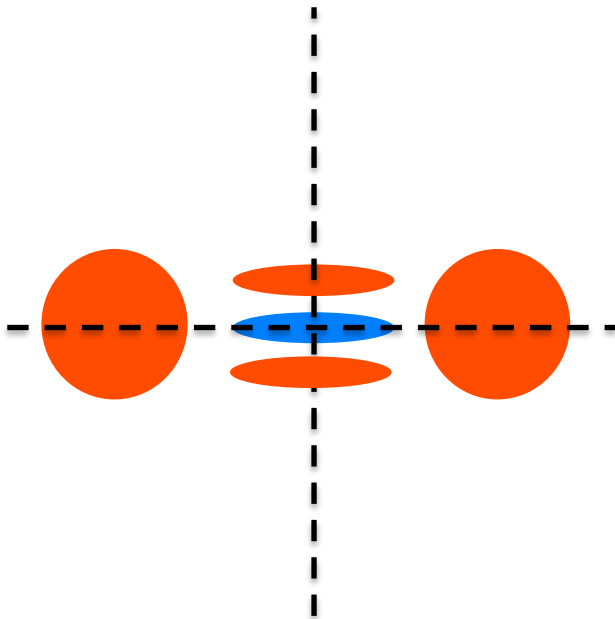


HARDWARE-EFFICIENT QUANTUM ERROR CORRECTION

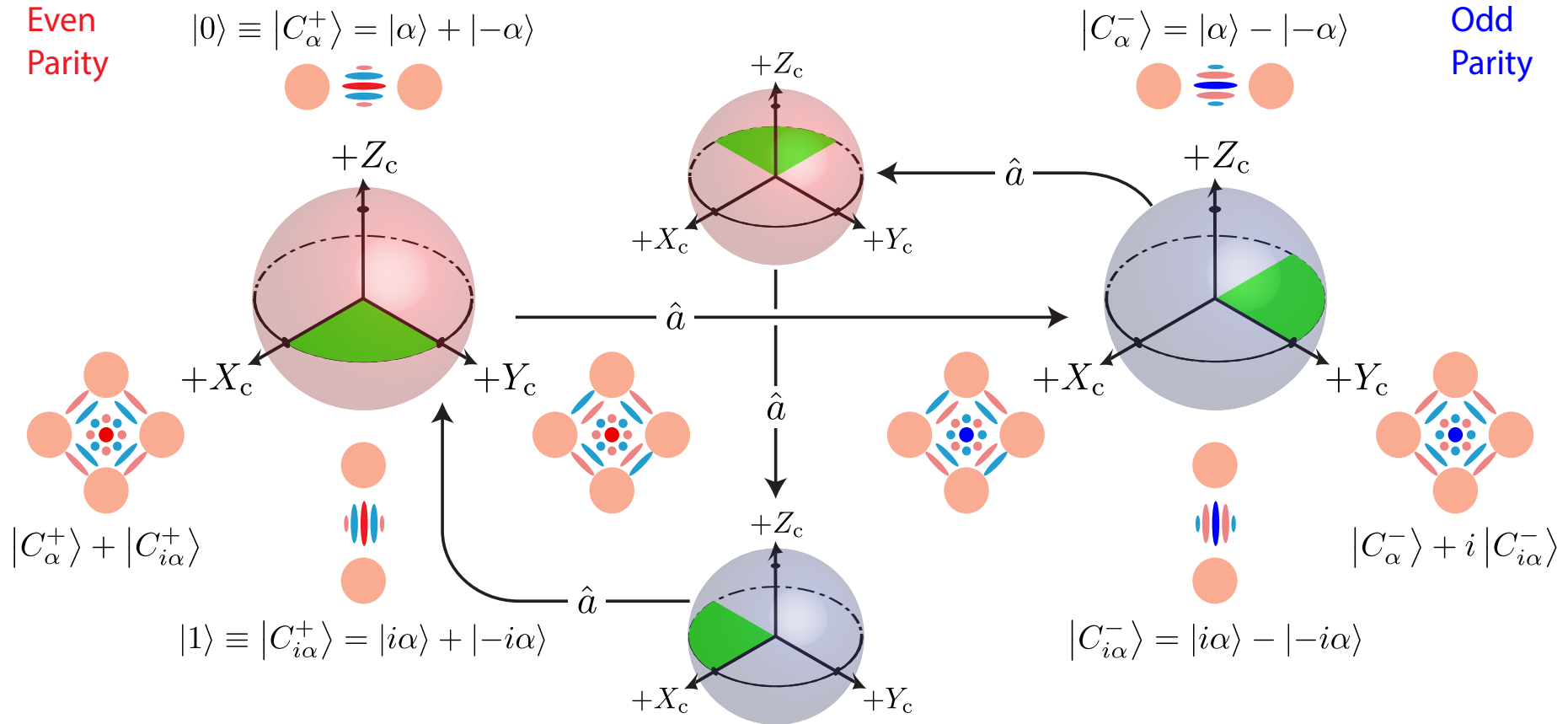
- ❑ Encoding and protecting information on a single cavity mode
- ❑ Minimal QEC hardware : protecting a single high-Q cavity mode (memory), using a single qubit (providing non-linearity), one low-Q mode (entropy evacuation).

Another possibility:

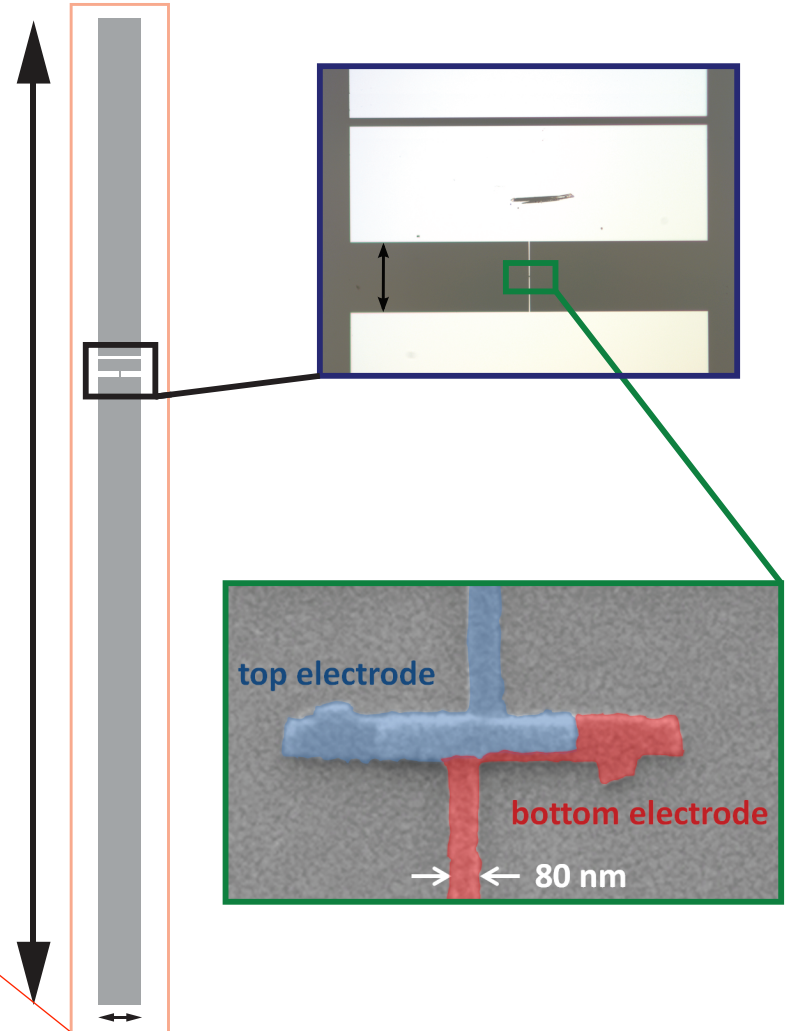
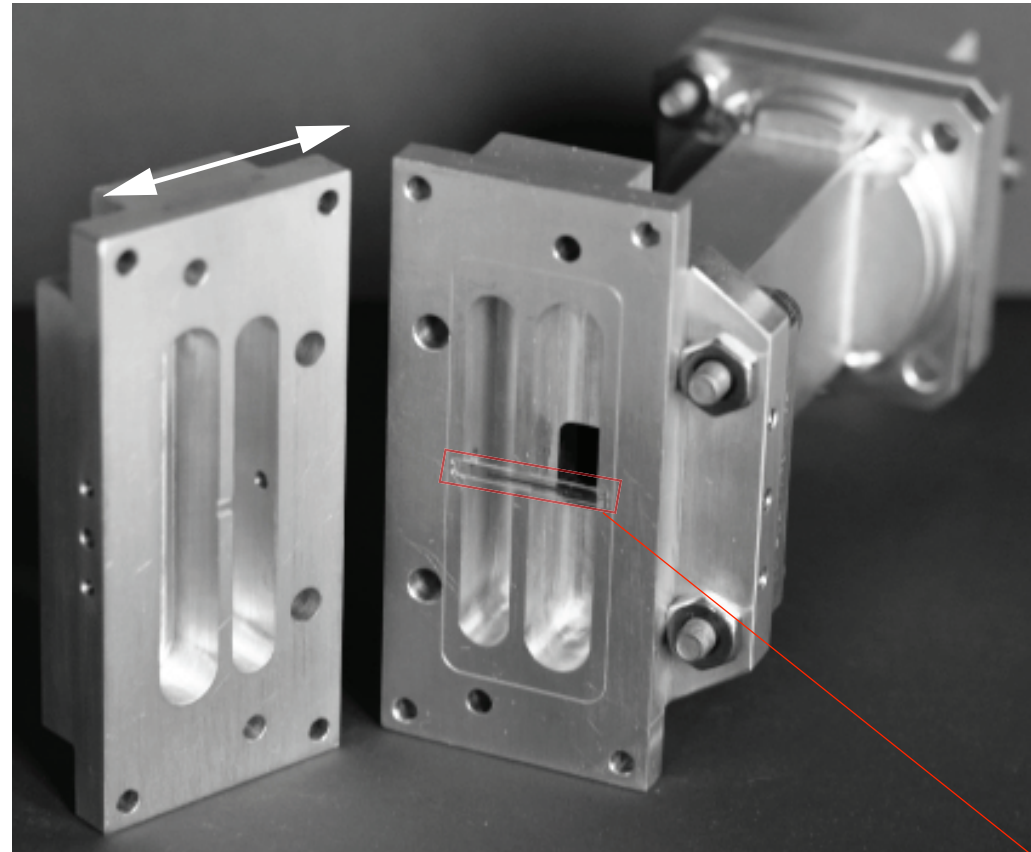
$$|0_L\rangle = |C_\alpha^-\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle) \quad |1_L\rangle = |C_{i\alpha}^-\rangle = \frac{1}{\sqrt{2}}(|i\alpha\rangle - |-i\alpha\rangle)$$



TO LIVE AND DIE IN A CAVITY

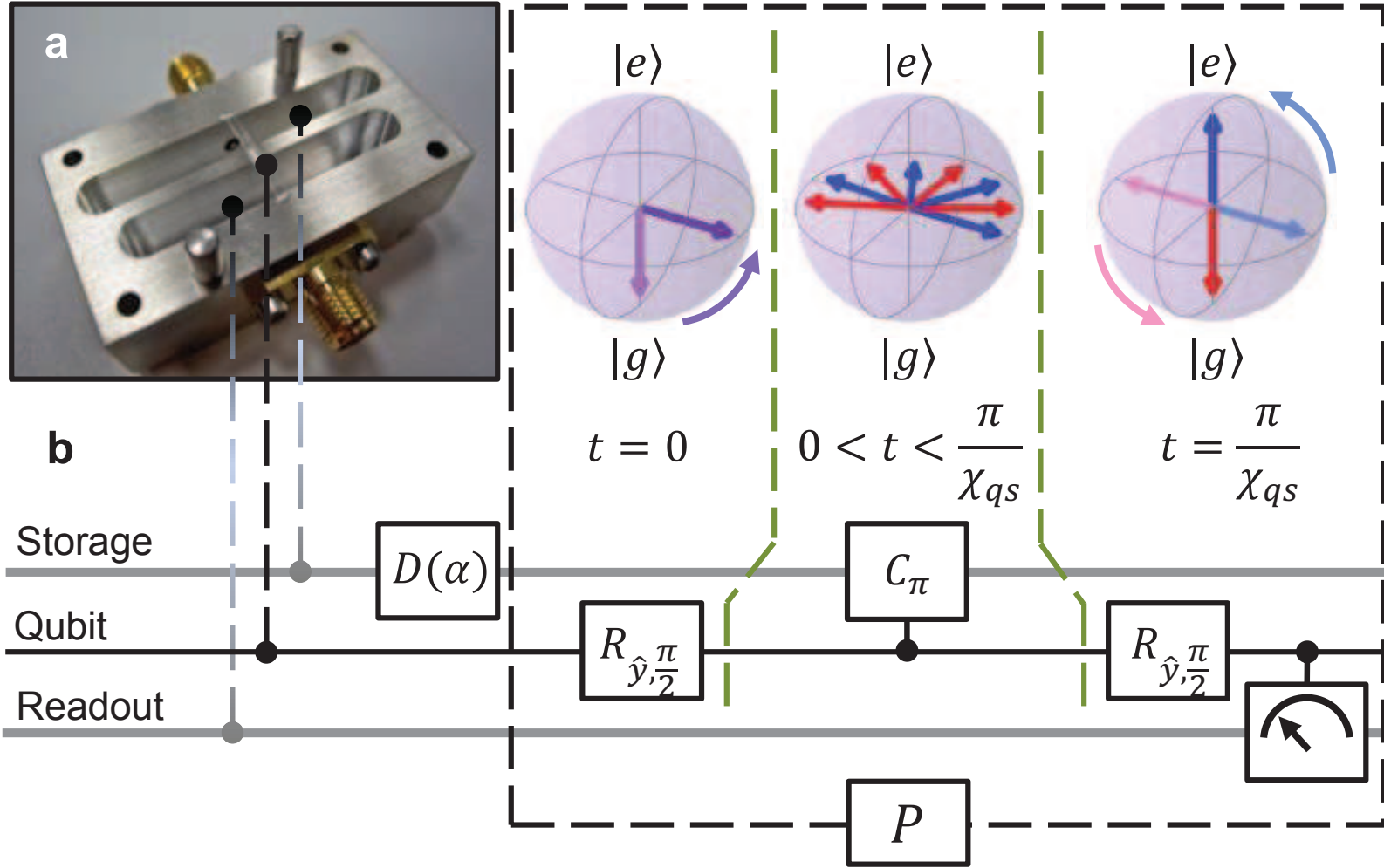


DEVICE



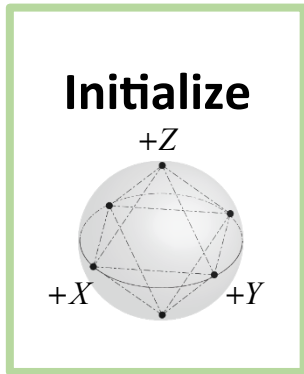
H. Paik *et. al.* PRL (2011),
G. Kirchmair *et. al.* Nature (2013)

MEASURING PHOTON-NUMBER PARITY



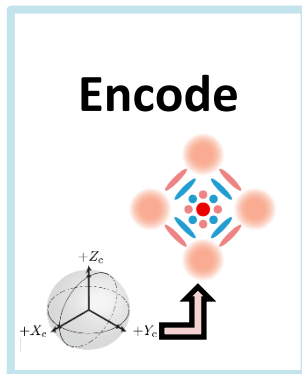
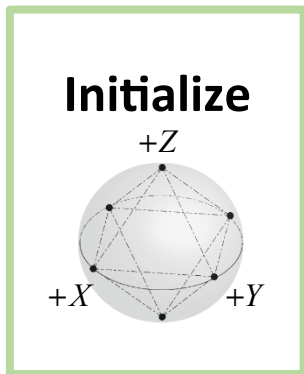
Bertet et al. PRL 89, 200402 (2002)
Sun et al. Nature 511, 444-448 (2014)

ERROR CORRECTION START-TO-FINISH



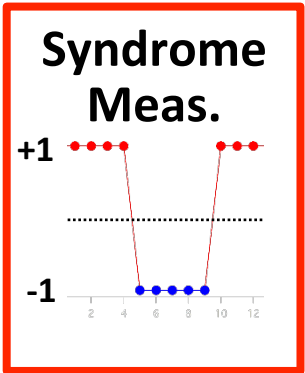
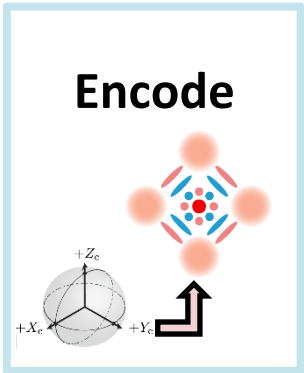
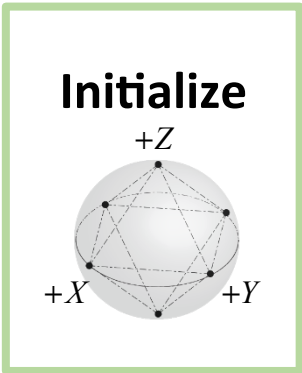
$$|g\rangle|0\rangle \rightarrow |\psi\rangle = (c_g |g\rangle + c_e |e\rangle)|0\rangle$$

ERROR CORRECTION START-TO-FINISH



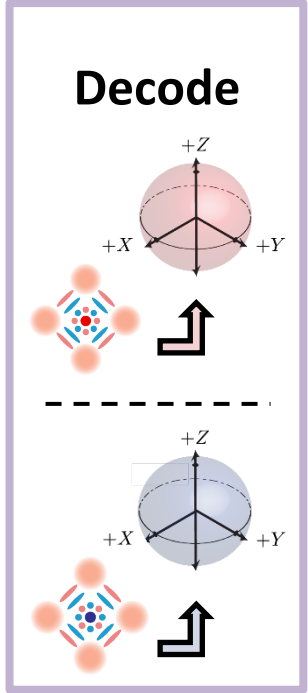
$$|\psi_L\rangle = (c_g |C_\alpha^+\rangle + c_e |C_{i\alpha}^+\rangle) |g\rangle$$

ERROR CORRECTION START-TO-FINISH

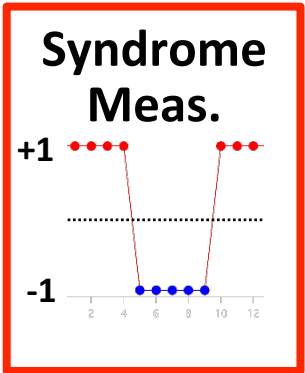
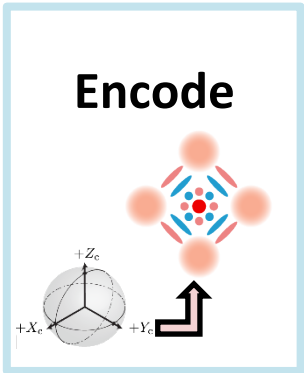
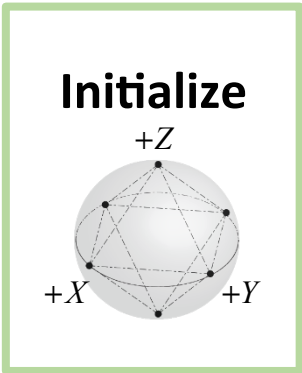


Even jump #

Odd jump #

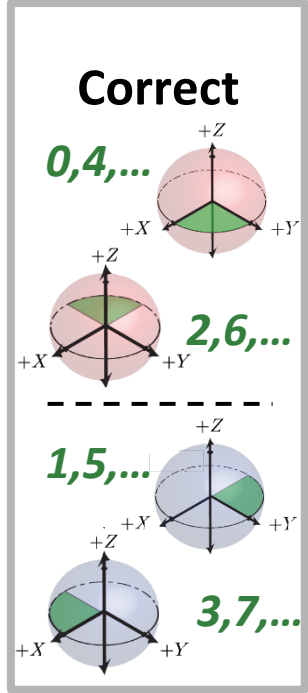
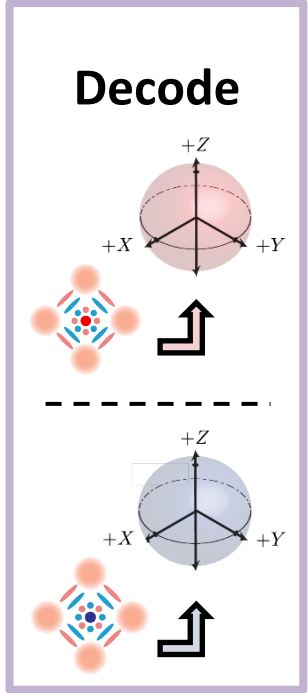


ERROR CORRECTION START-TO-FINISH

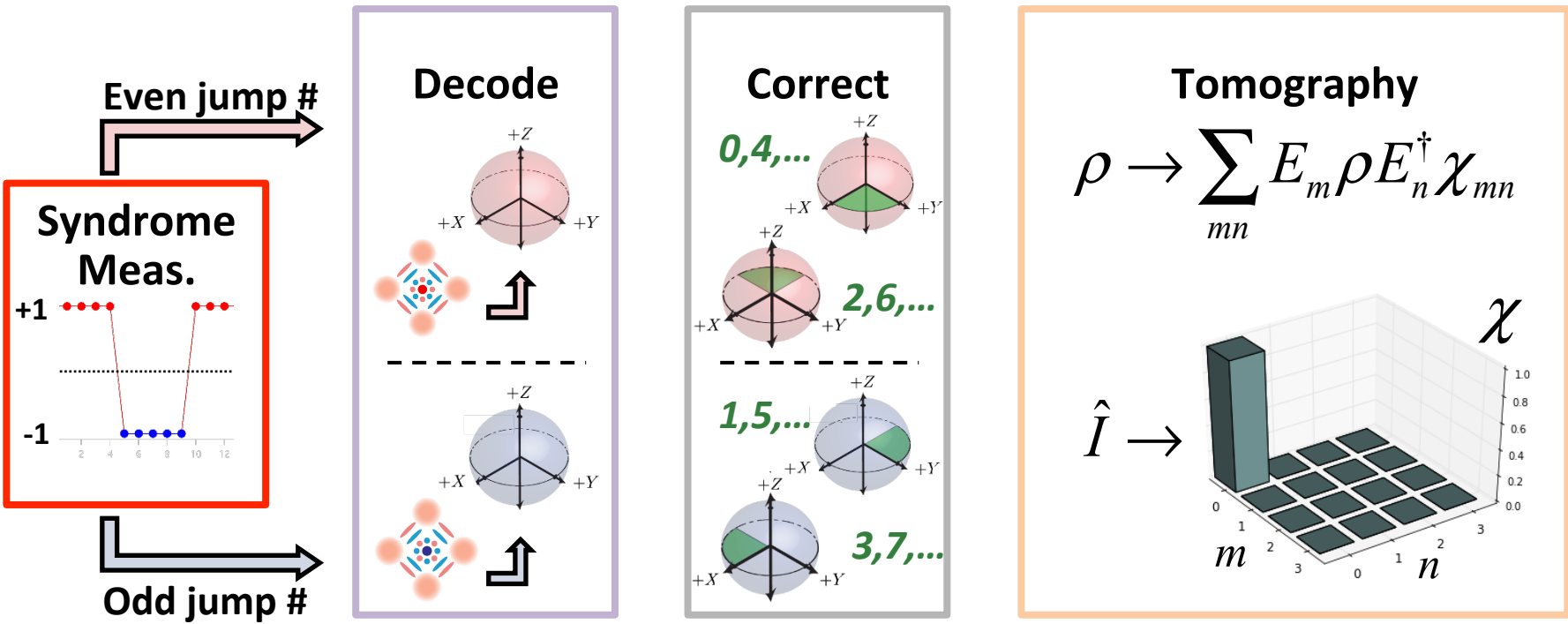


Even jump #

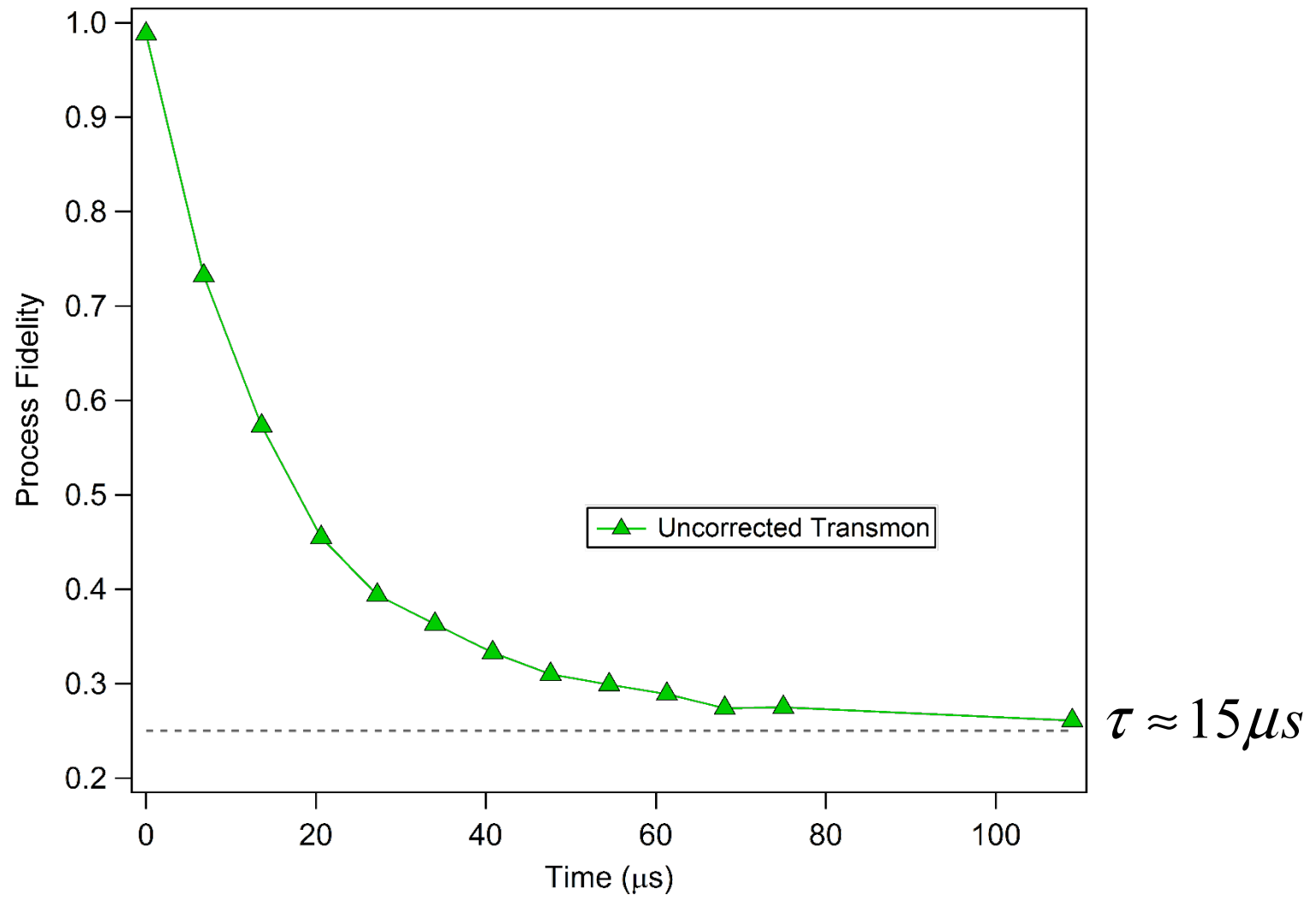
Odd jump #



ERROR CORRECTION START-TO-FINISH



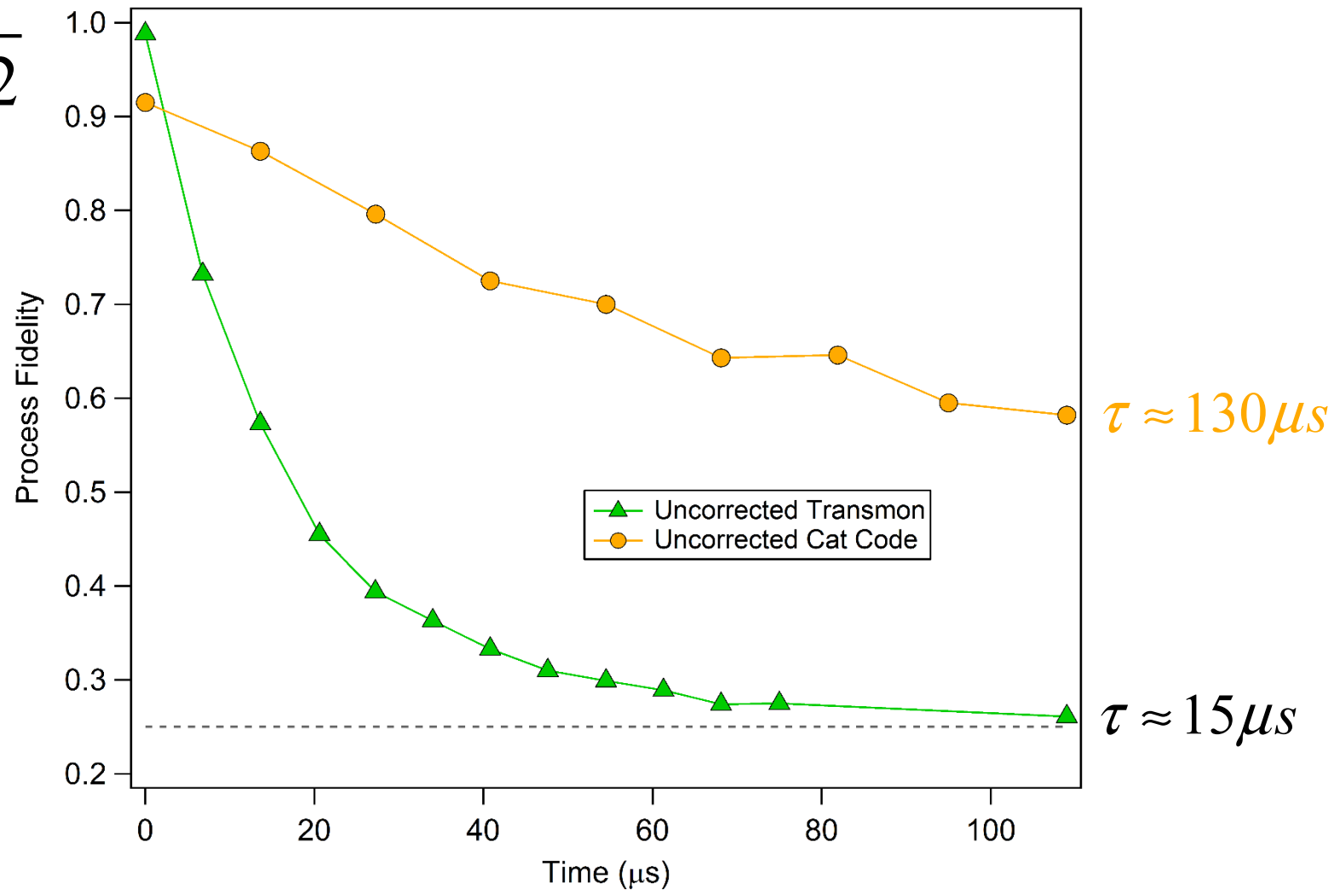
ERROR CORRECTION START-TO-FINISH



*Ofek et al., Nature 536, 441-445, 2016.

ERROR CORRECTION START-TO-FINISH

$$\alpha = \sqrt{2}$$



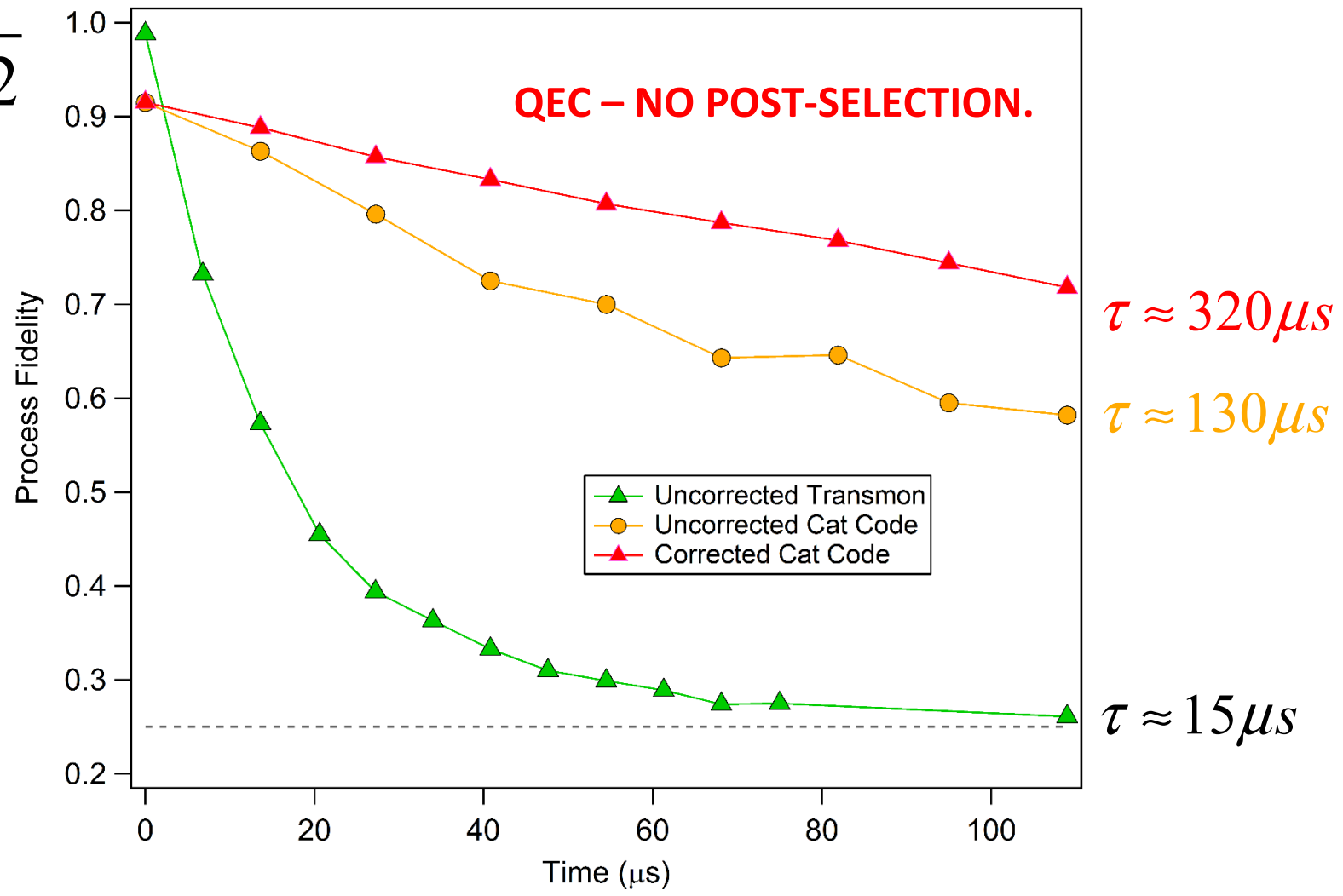
$\tau \approx 130 \mu s$

$\tau \approx 15 \mu s$

*Ofek et al., Nature 536, 441-445, 2016.

ERROR CORRECTION START-TO-FINISH

$$\alpha = \sqrt{2}$$

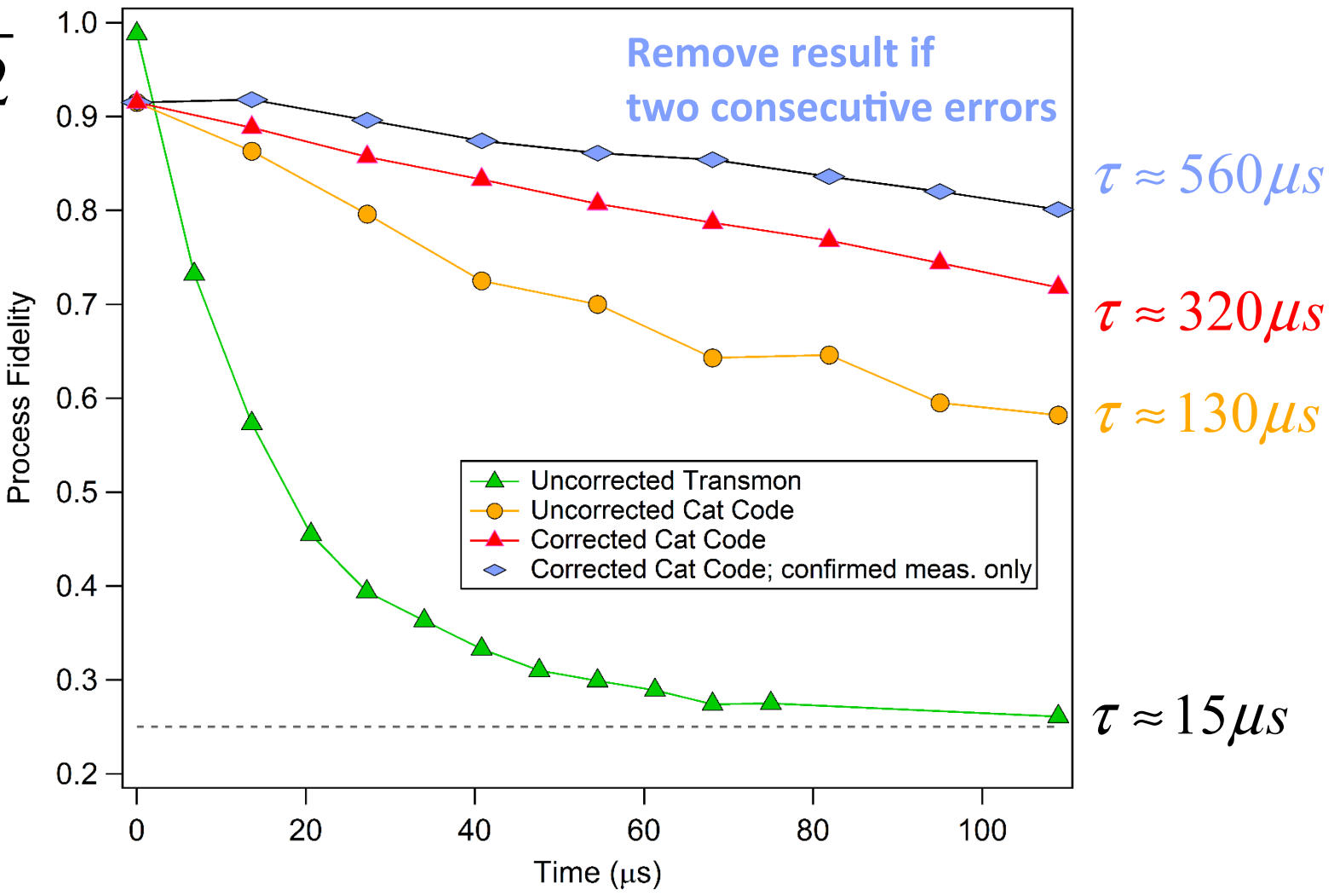


*Ofek et al., Nature 536, 441-445, 2016.

ALLOW POSTSELECTION?

Throwing out ~ 20% of data which are probably msmt. errors...

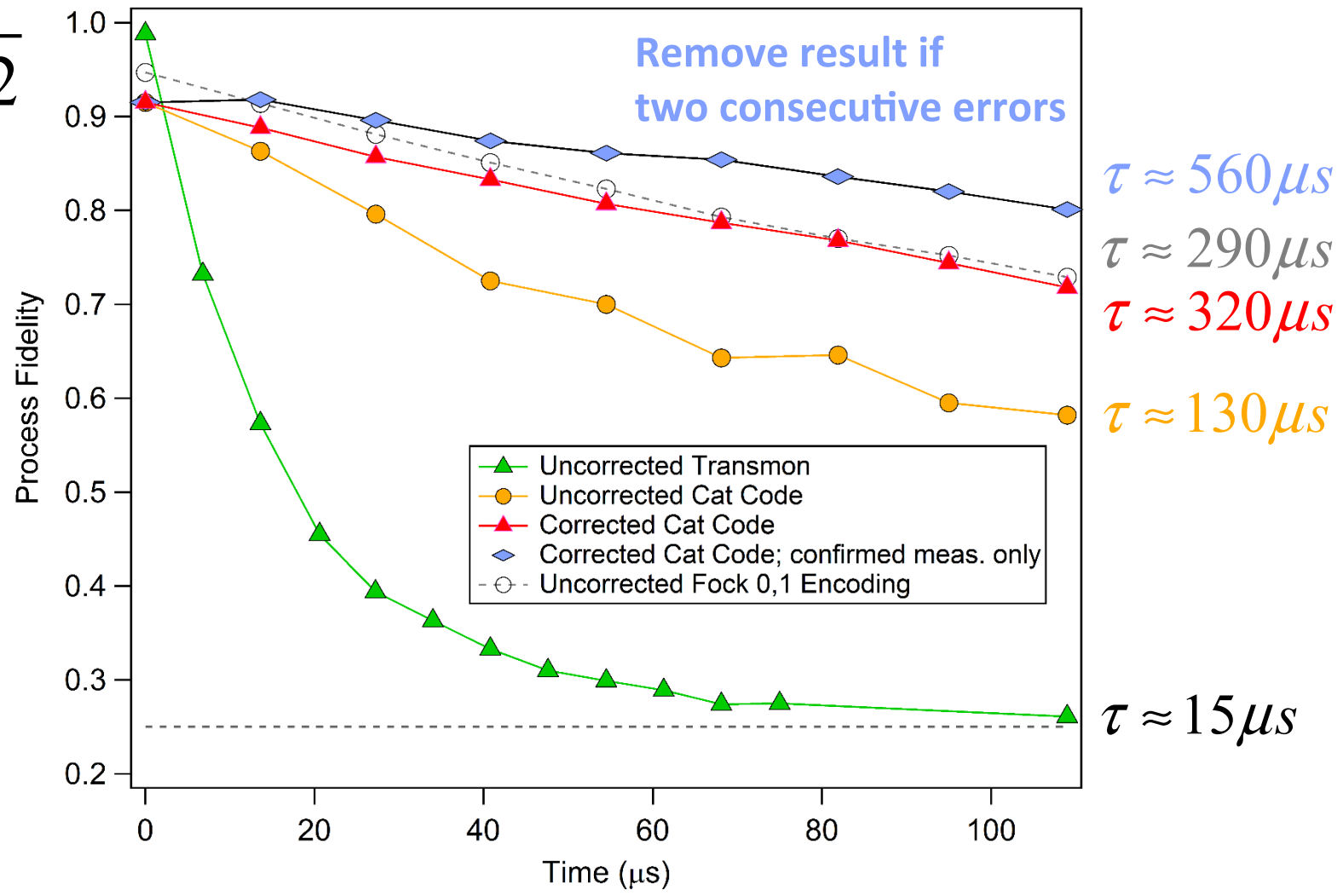
$$\alpha = \sqrt{2}$$



*Ofek et al., Nature 536, 441-445, 2016.

COMPARING TO SINGLE PHOTON ENCODING

$$\alpha = \sqrt{2}$$



*Ofek et al., Nature 536, 441-445, 2016.

TOWARDS FAULT-TOLERANT QC

Most important limitations:

- Energy decay.
- Uncorrected errors: dephasing due to combination of Kerr and decay.
- Non fault-tolerance: propagation of errors.

Rest of this talk: parametric methods to achieve **fault-tolerance**.

Idea: driven-dissipative mechanism to restrict the dynamics to 2- or 4-dimensional manifolds $\text{Span}\{|\pm\alpha\rangle\}$ and $\text{Span}\{|\pm\alpha\rangle, |\pm i\alpha\rangle\}$.

OUTLINE

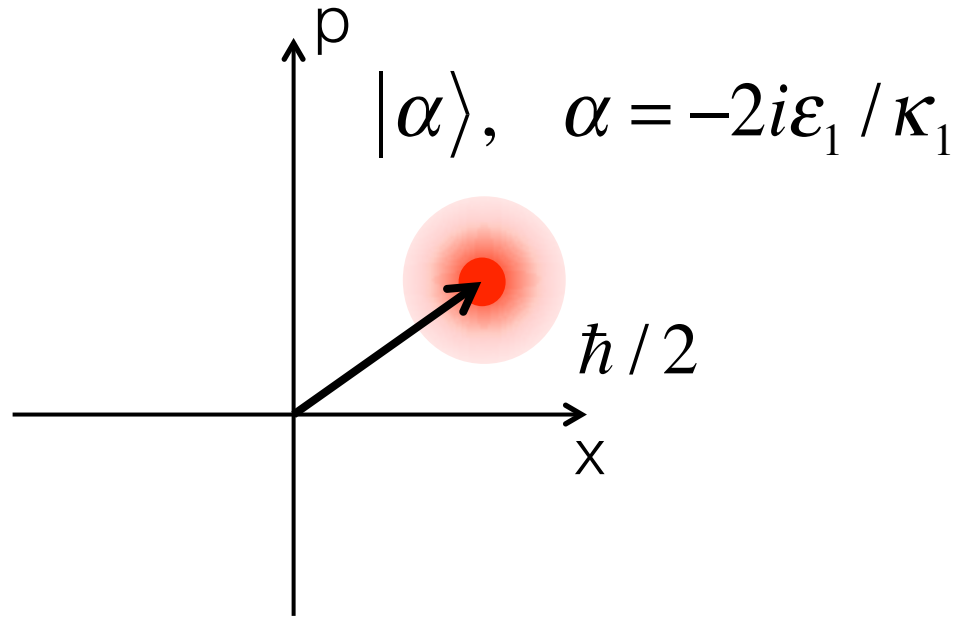
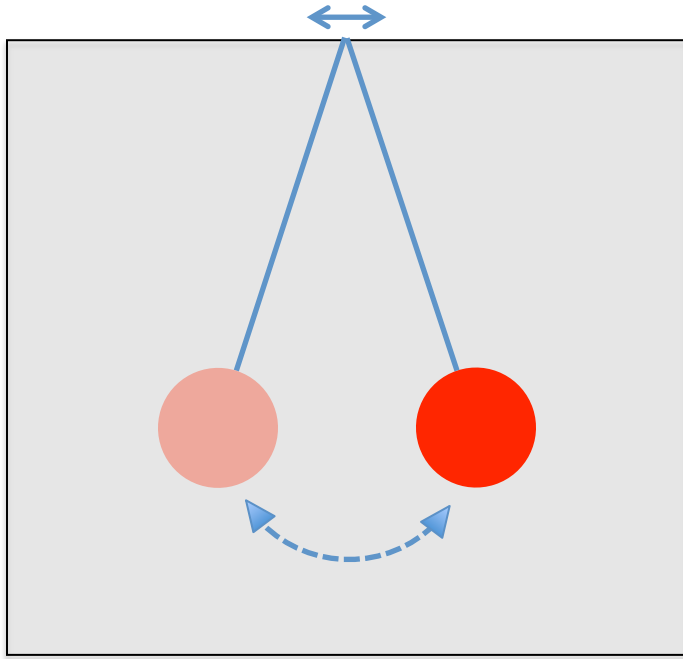
□ Introduction to quantum error correction

- Classical vs quantum error correction
- Theory of quantum error correction
- Insights on fault-tolerance

□ A continuous-variable alternative

- Cat-qubits for protection against photon-loss
- Nonlinear dissipation paving the way towards fault-tolerance

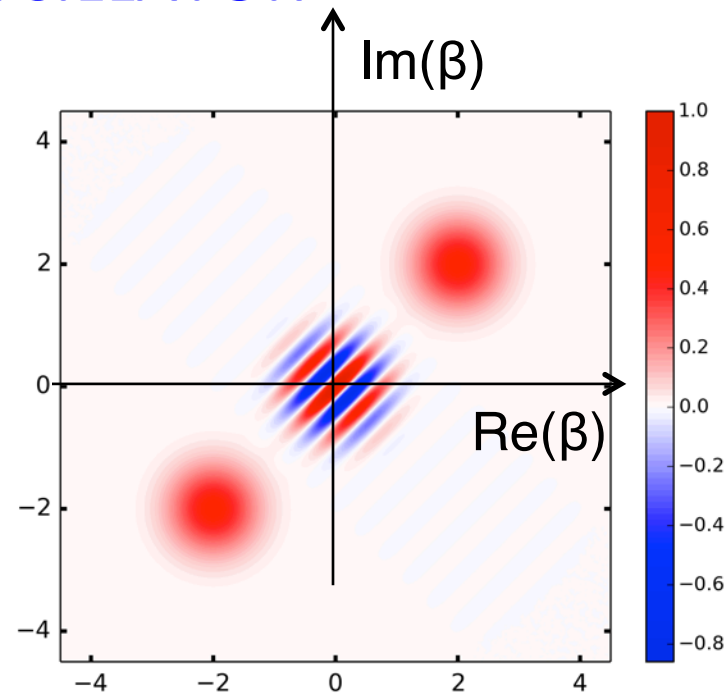
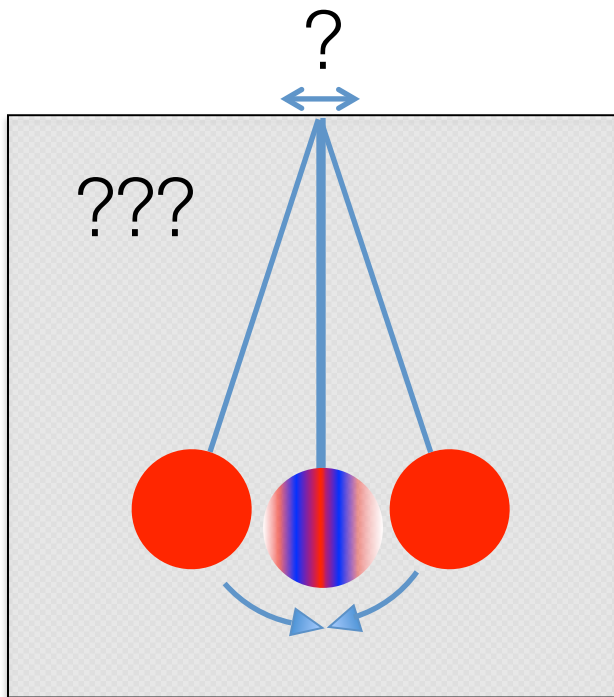
DRIVEN-DAMPED HARMONIC OSCILLATOR



Hamiltonian : $H = \varepsilon_1^* \hat{a} + \varepsilon_1 \hat{a}^\dagger$

loss operator : $\sqrt{\kappa_1} \hat{a}$

A SPECIAL DRIVEN-DAMPED OSCILLATOR



Wigner function $W(\beta)$

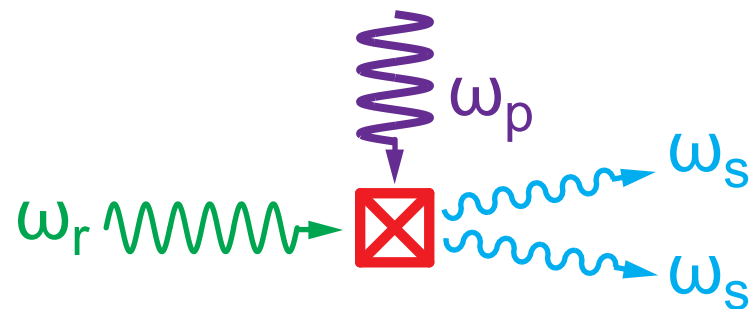
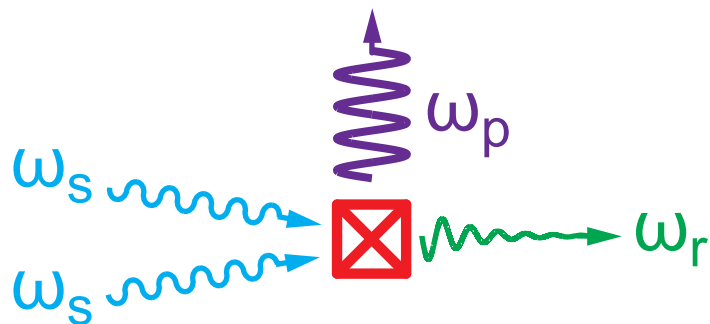
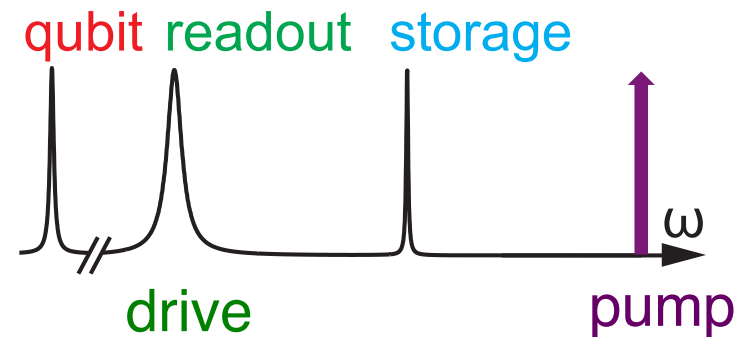
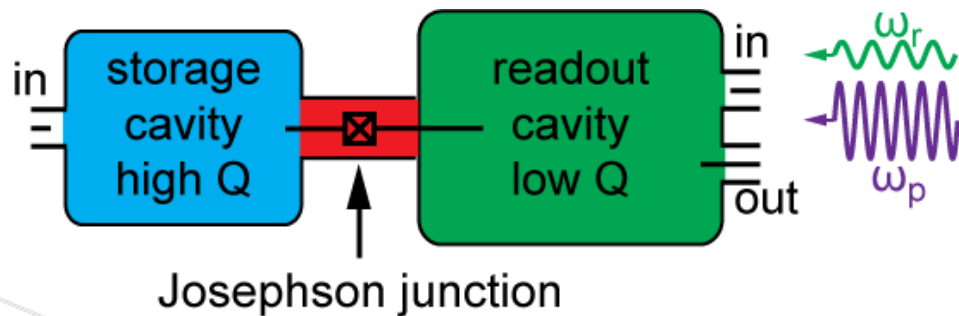
Hamiltonian : $H = \varepsilon_2^* \hat{a}^2 + \varepsilon_2 (\hat{a}^\dagger)^2$

loss operator : $\sqrt{\kappa_2} \hat{a}^2$

$$\alpha = \pm \sqrt{-2i\varepsilon_2 / \kappa_2}$$

TWO-PHOTON EXCHANGE

$$\omega_p = 2\omega_s - \omega_r$$



$$H_{sr} = (g_2^* \hat{a}_s^2 \hat{a}_r^\dagger + \text{h.c.})$$

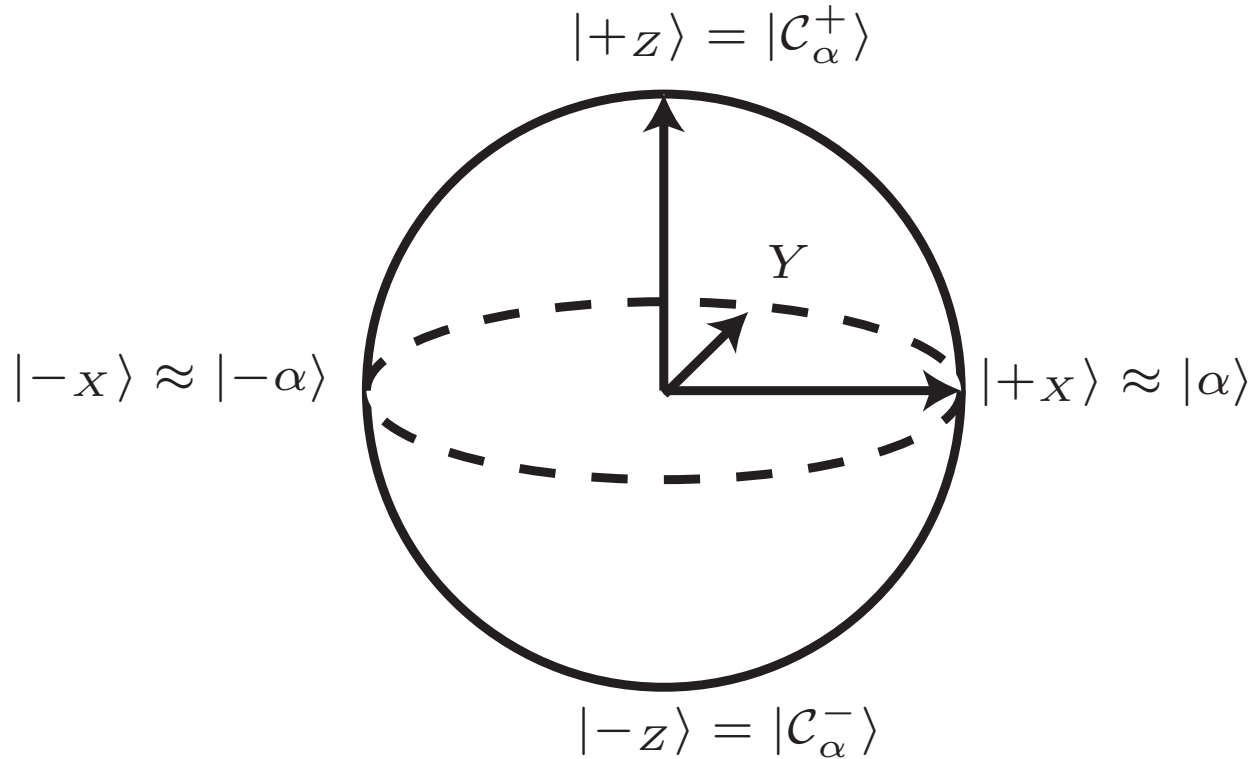
+

$$\varepsilon \hat{a}_r^\dagger + \text{h.c.}$$

effective loss operator: $\sqrt{\kappa_2} \hat{a}_s^2$

effective drive: $\varepsilon_2 (\hat{a}_s^\dagger)^2 + \text{h.c.}$

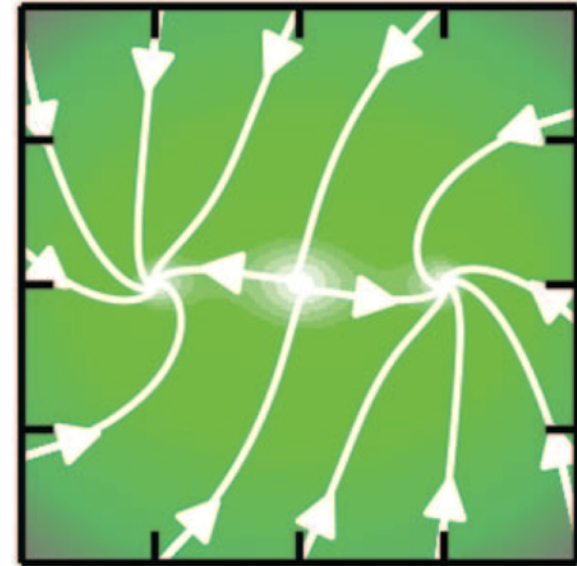
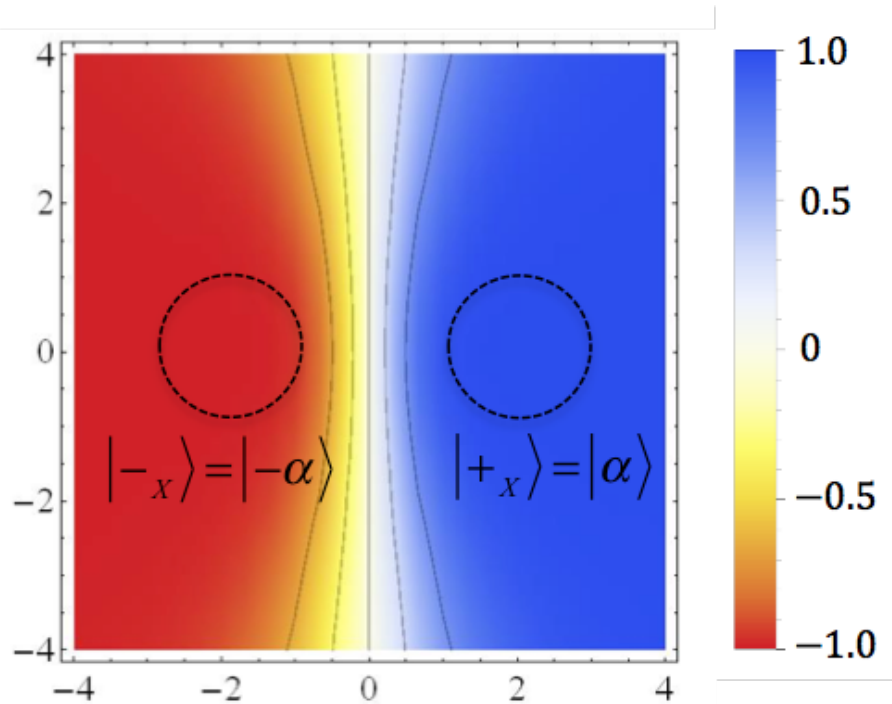
CHOICE OF QUBIT BASIS



$$|+_z\rangle = |C_\alpha^+\rangle = N_+ (|\alpha\rangle + |-\alpha\rangle) = \sum c_{2n} |2n\rangle$$

$$|-_z\rangle = |C_\alpha^-\rangle = N_- (|\alpha\rangle - |-\alpha\rangle) = \sum c_{2n+1} |2n+1\rangle$$

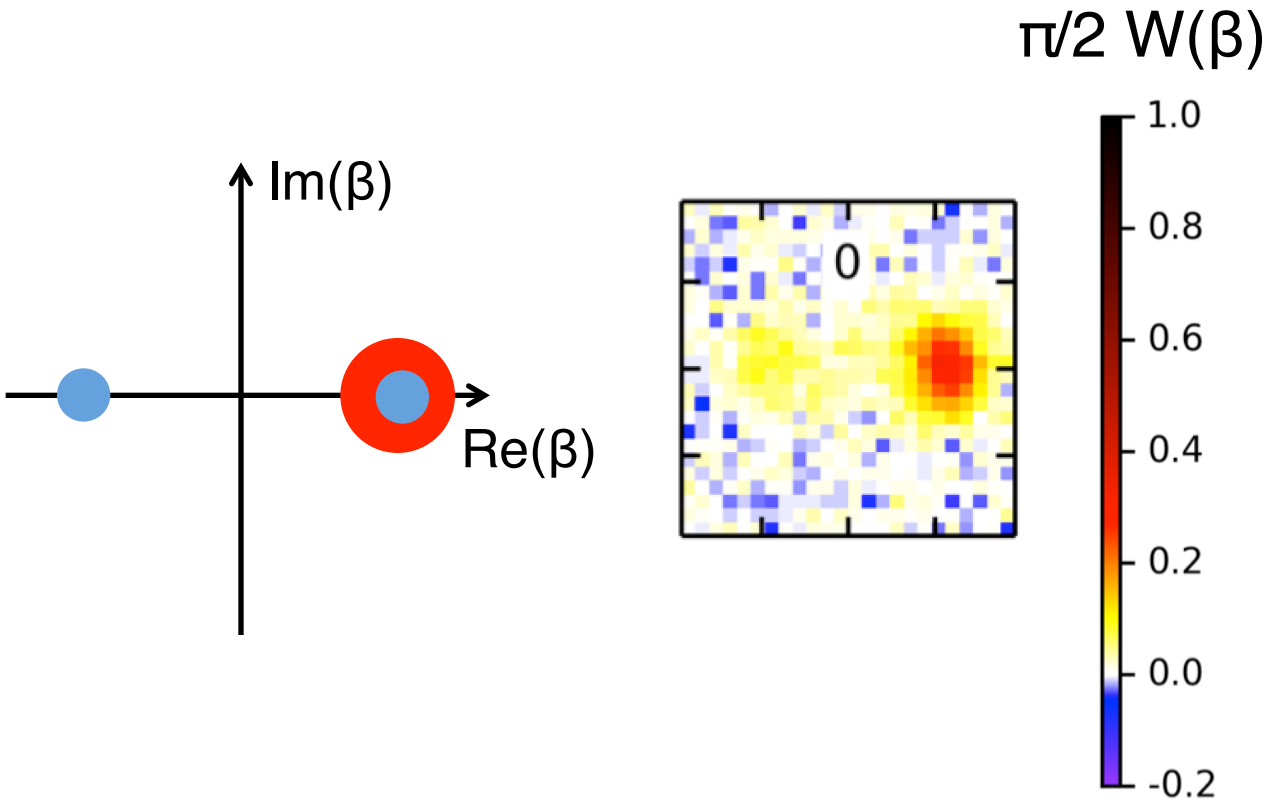
A QUBIT WITHOUT PHASE-FLIPS



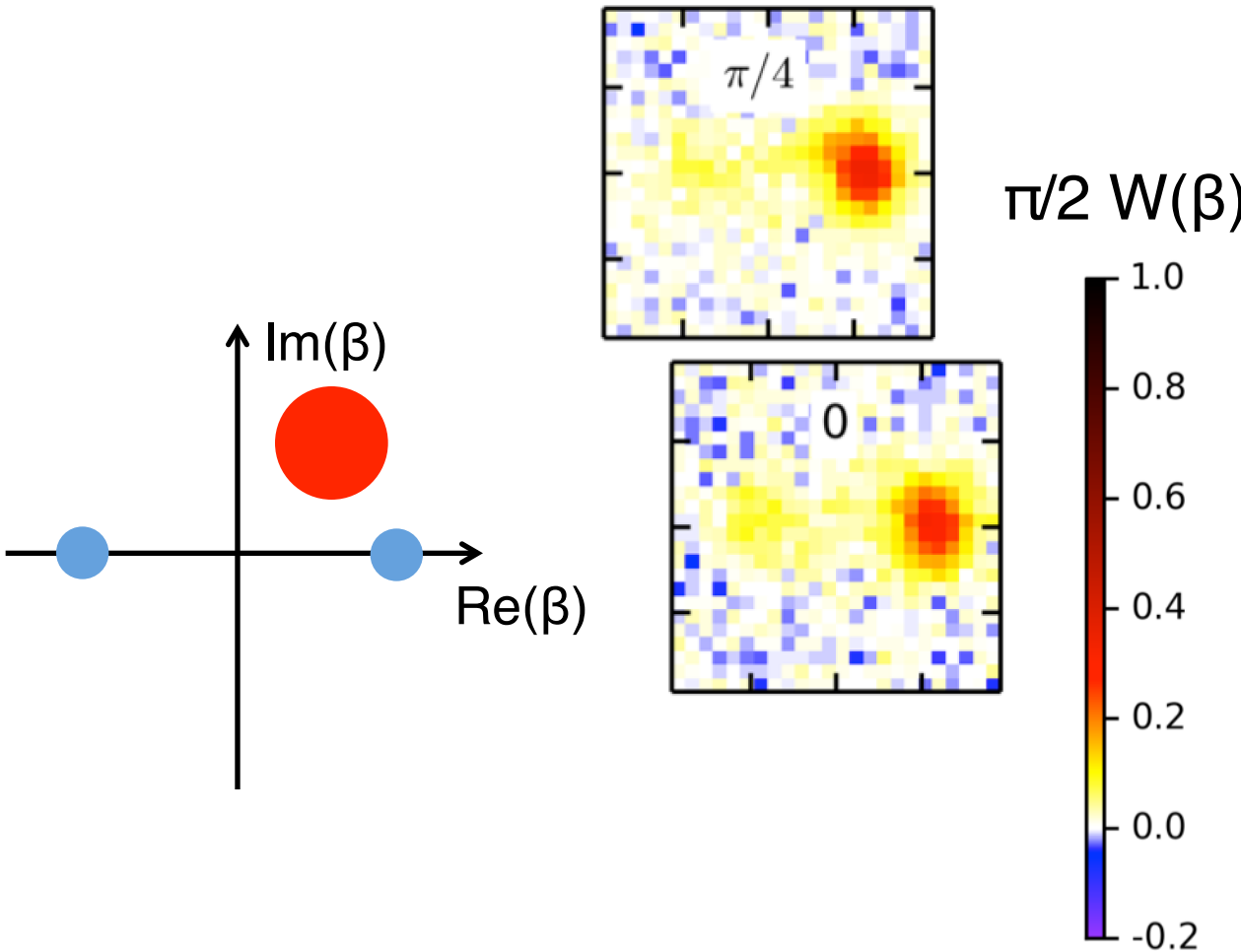
$$\text{Tr}(\sigma_X^L \rho_\infty) \text{ for } \rho_0 = |\beta\rangle\langle\beta|$$

Phase-flip errors induced by reasonable (local in the phase space) errors are suppressed exponentially in $|\alpha|^2$.

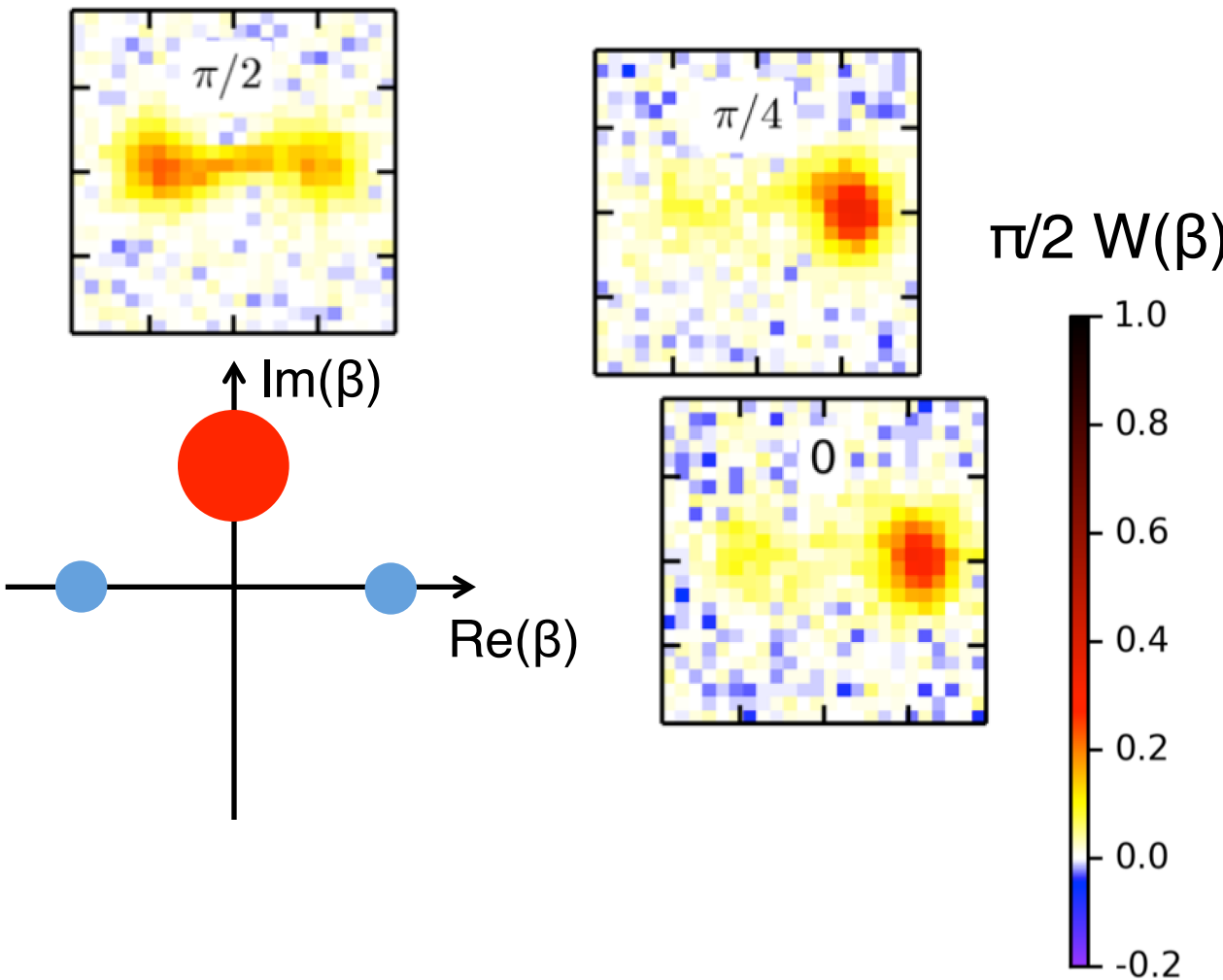
BI-STABLE BEHAVIOR (SEMI-CLASSICAL)



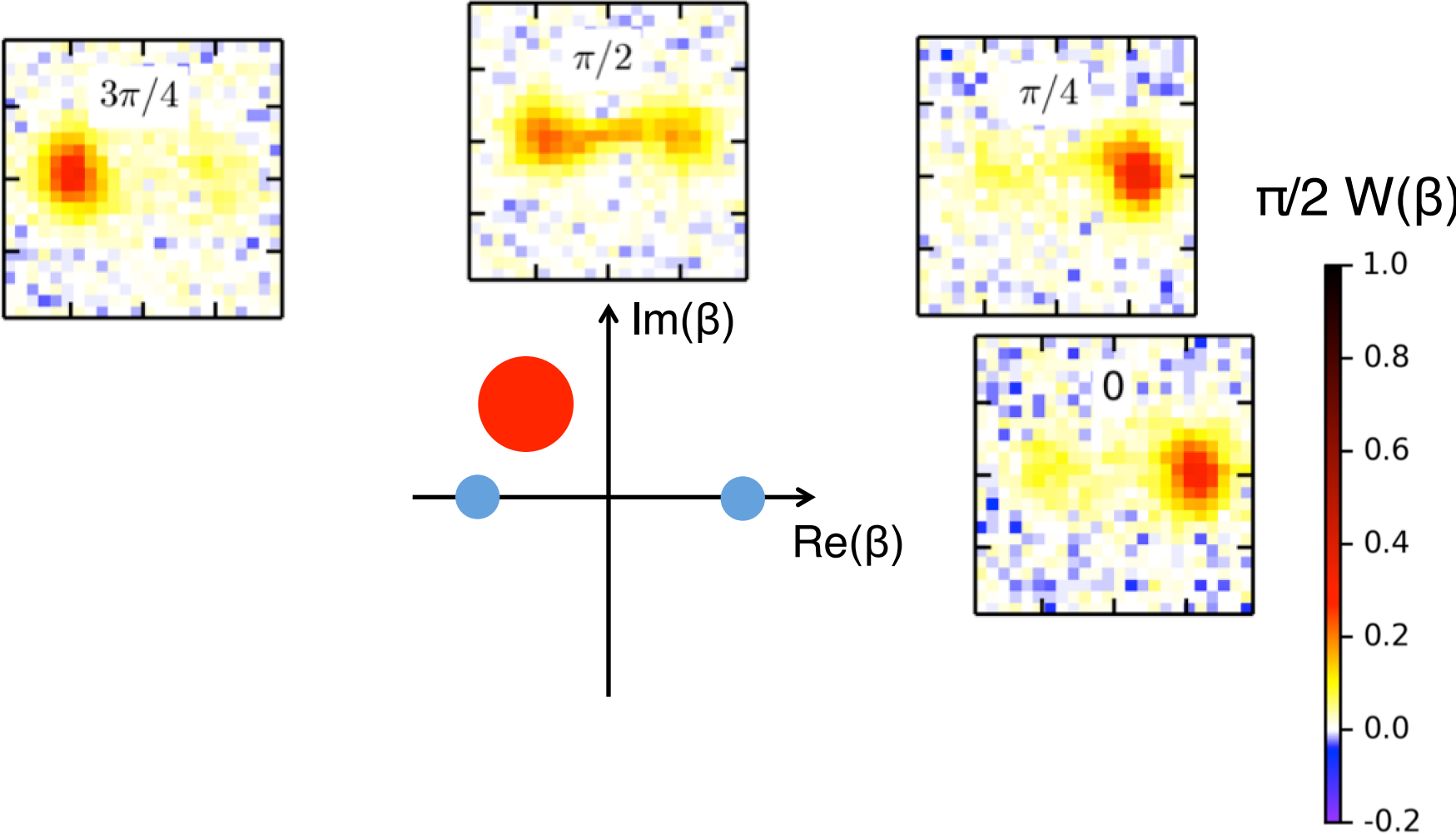
BI-STABLE BEHAVIOR (SEMI-CLASSICAL)



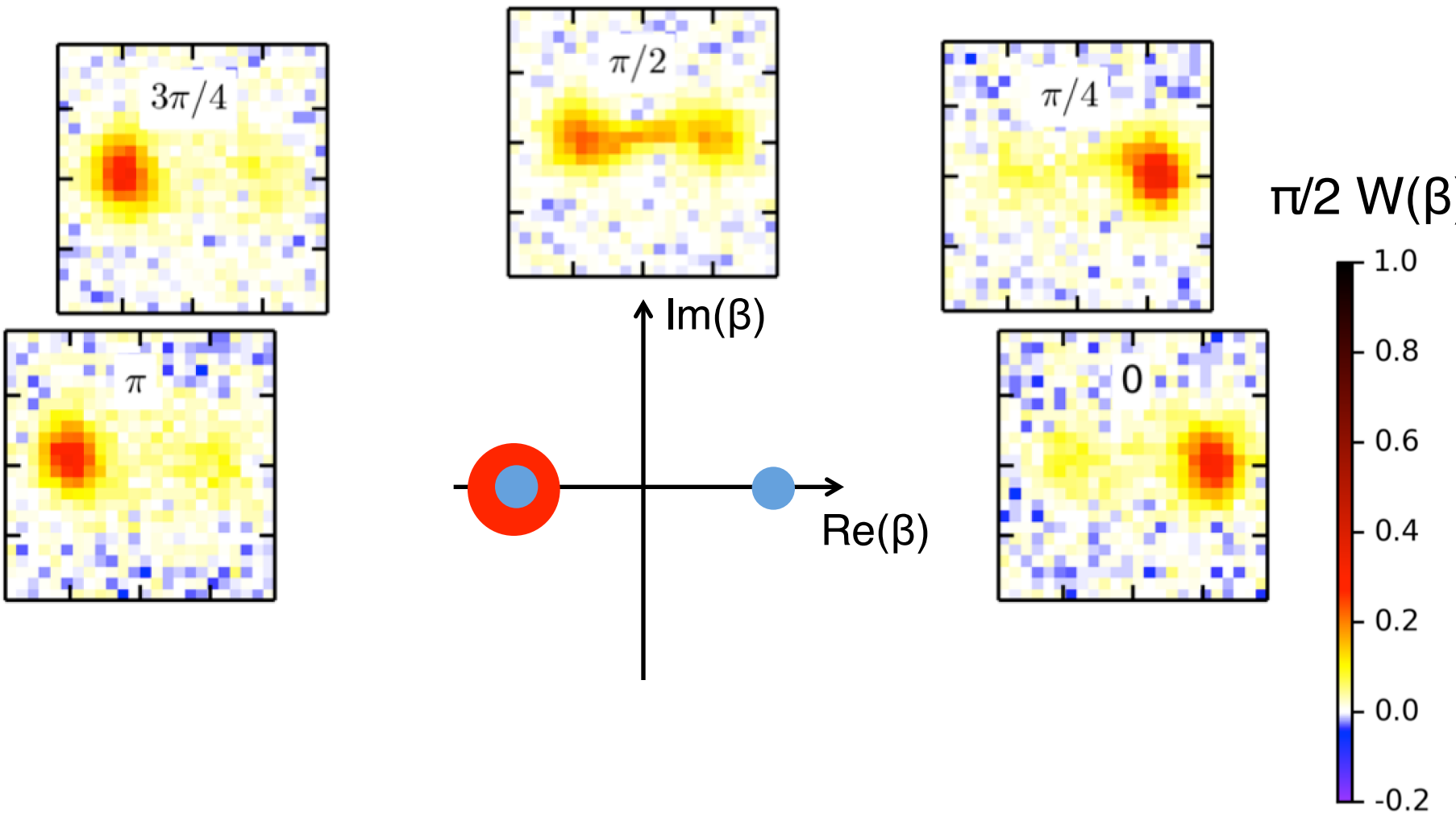
BI-STABLE BEHAVIOR (SEMI-CLASSICAL)



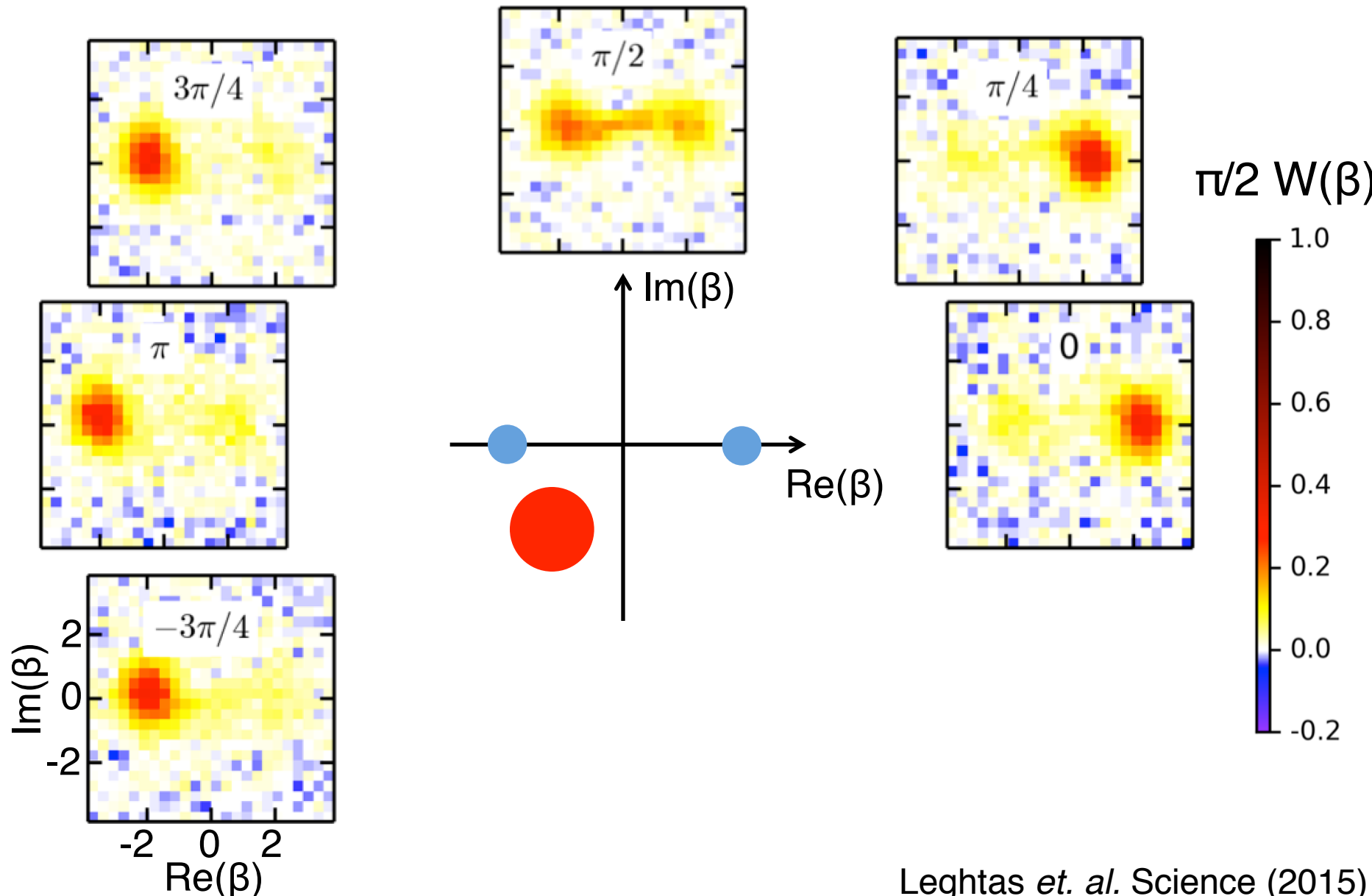
BI-STABLE BEHAVIOR (SEMI-CLASSICAL)



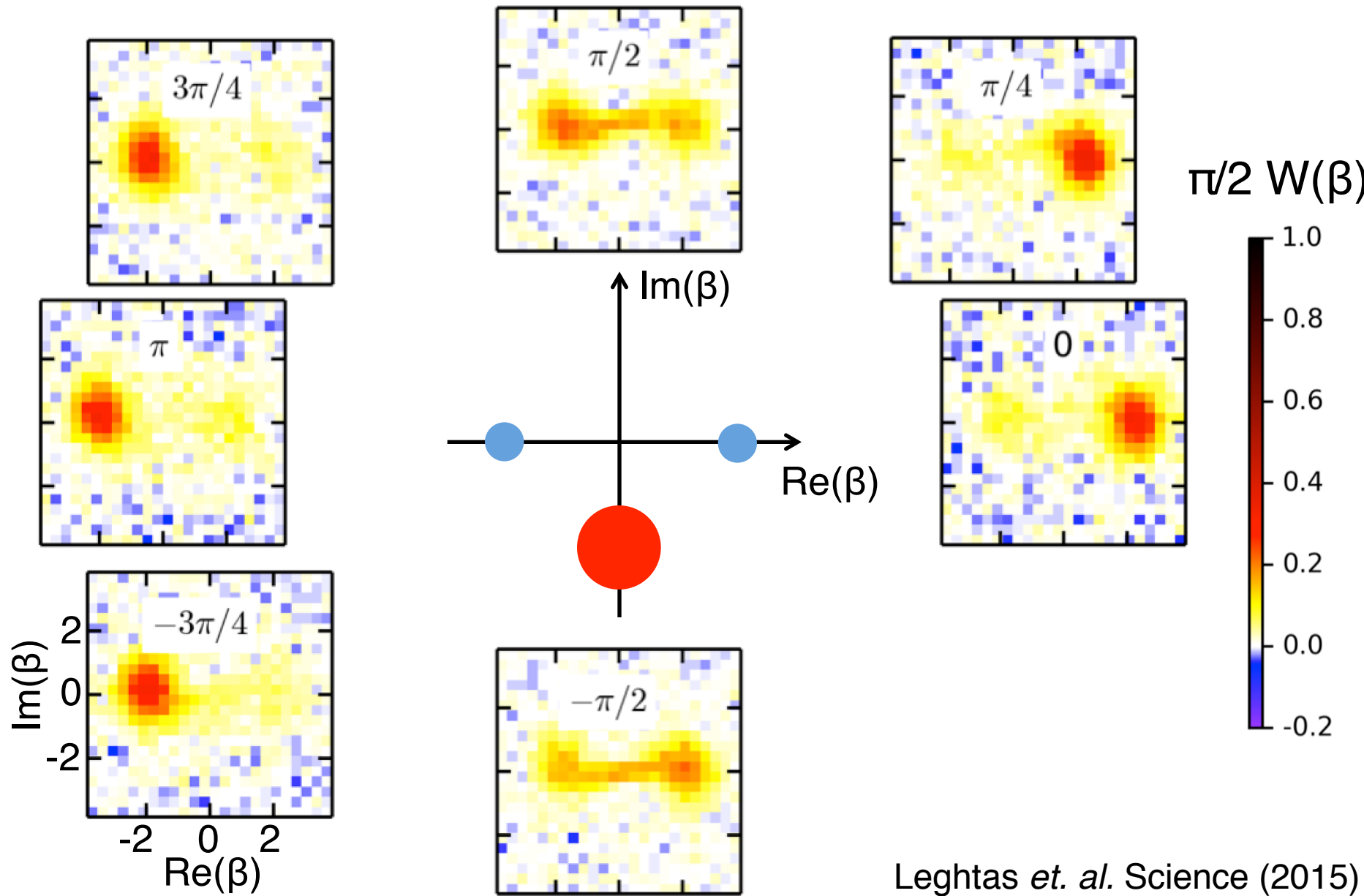
BI-STABLE BEHAVIOR (SEMI-CLASSICAL)



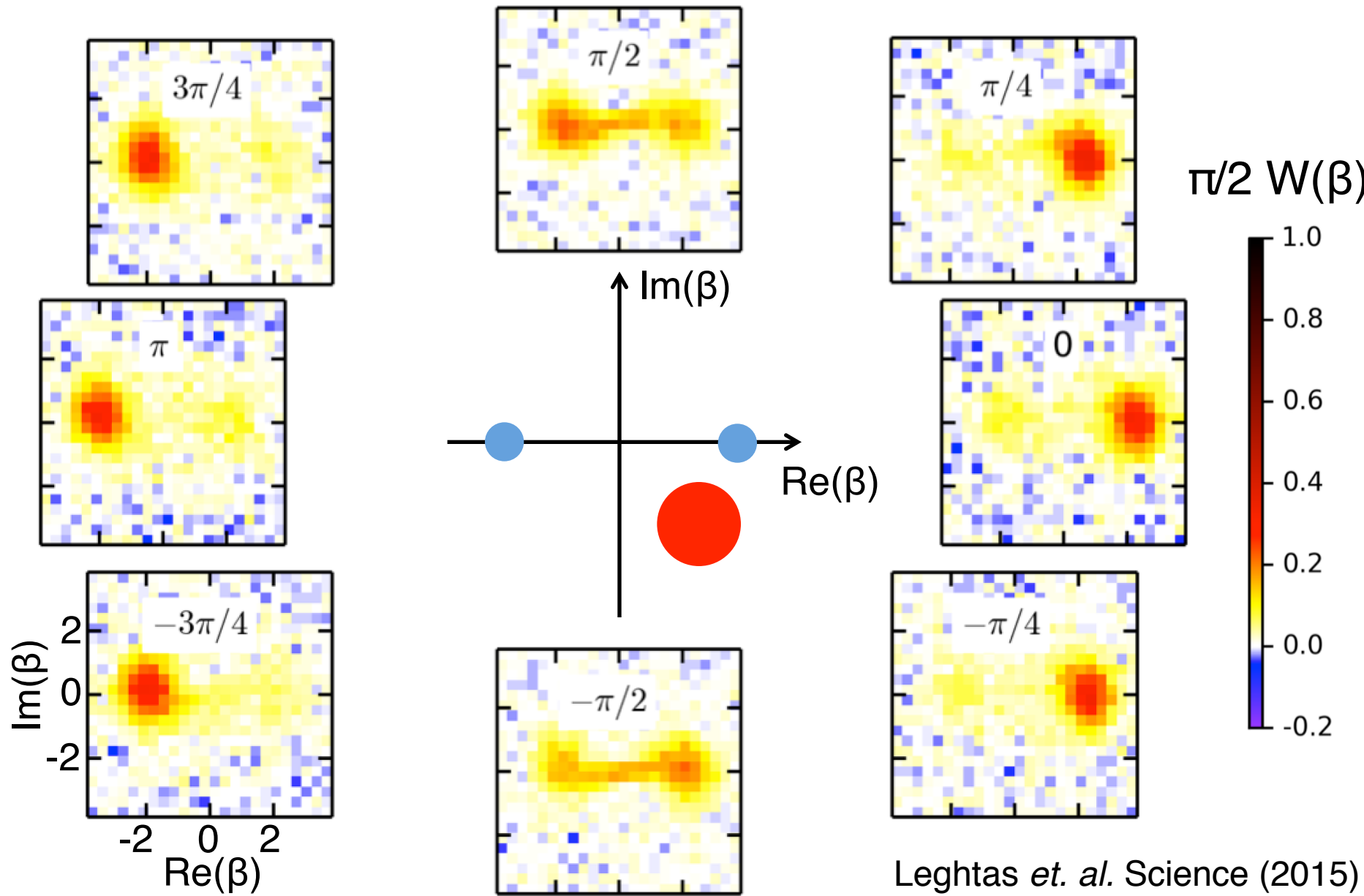
BI-STABLE BEHAVIOR (SEMI-CLASSICAL)



BI-STABLE BEHAVIOR (SEMI-CLASSICAL)

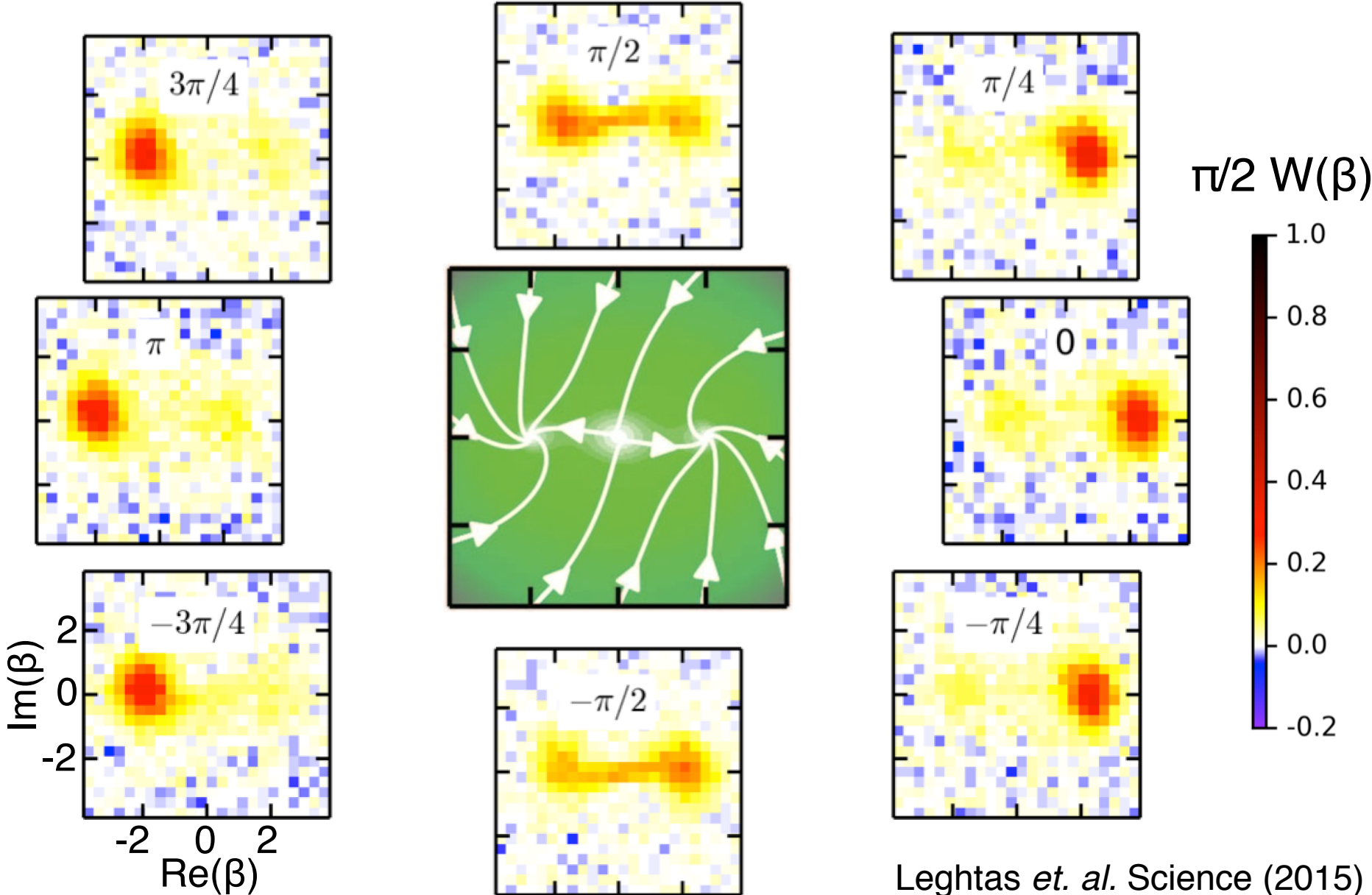


BI-STABLE BEHAVIOR (SEMI-CLASSICAL)



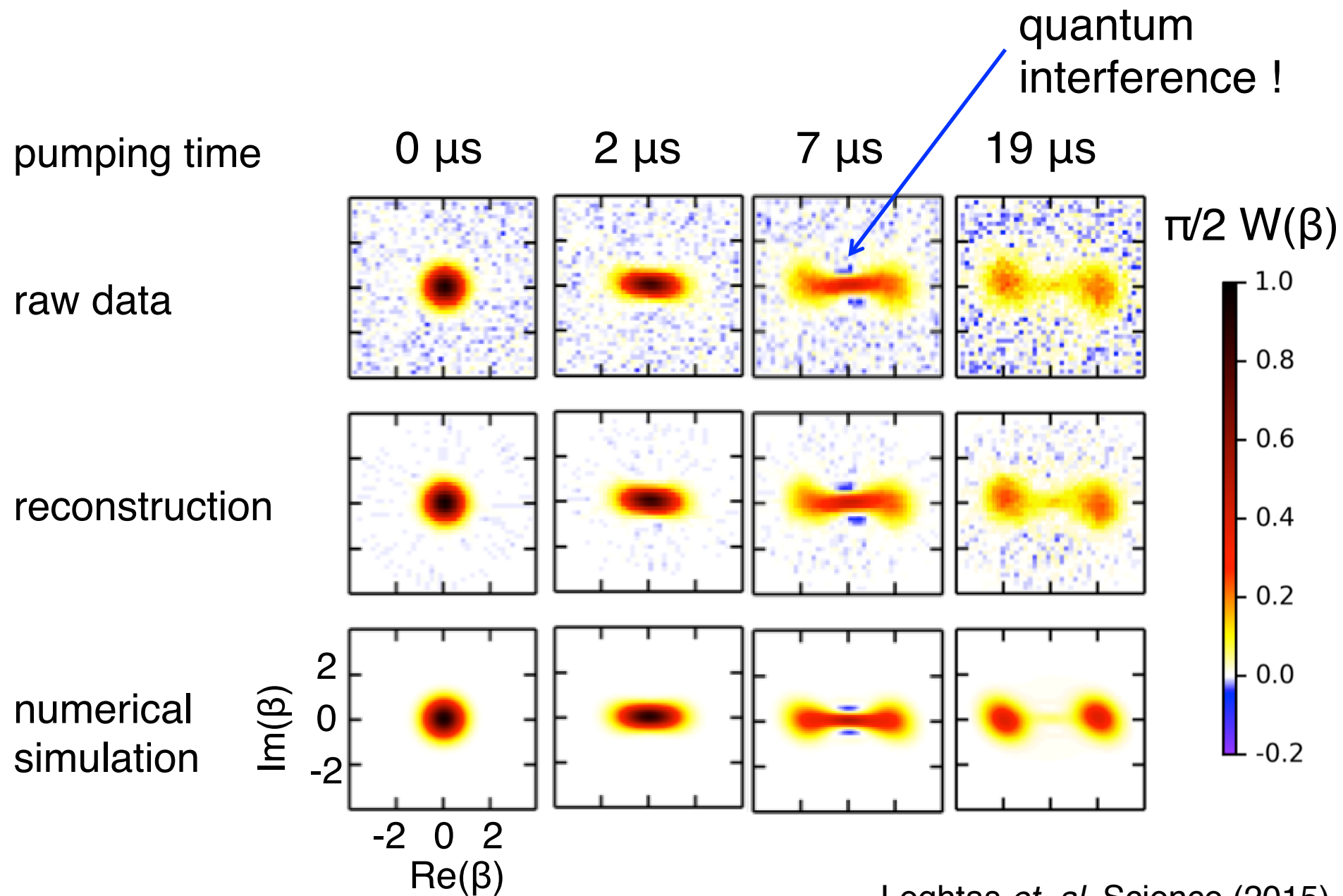
Leghtas *et. al.* Science (2015)

BI-STABLE BEHAVIOR (SEMI-CLASSICAL)

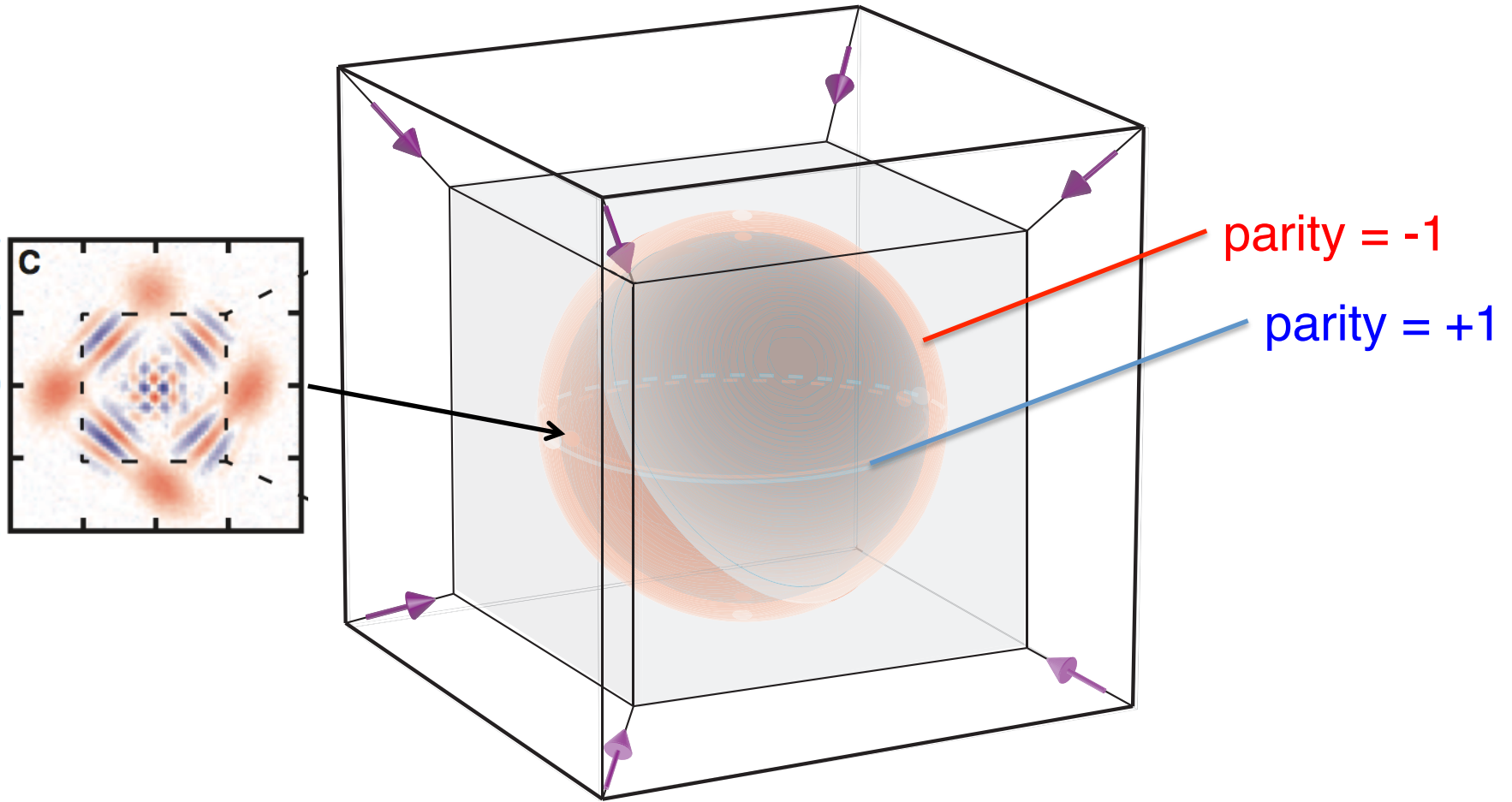


Leghtas *et. al.* Science (2015)

CAT SQUEEZES OUT OF VACUUM



FUTURE: FULL QUANTUM ERROR CORRECTION

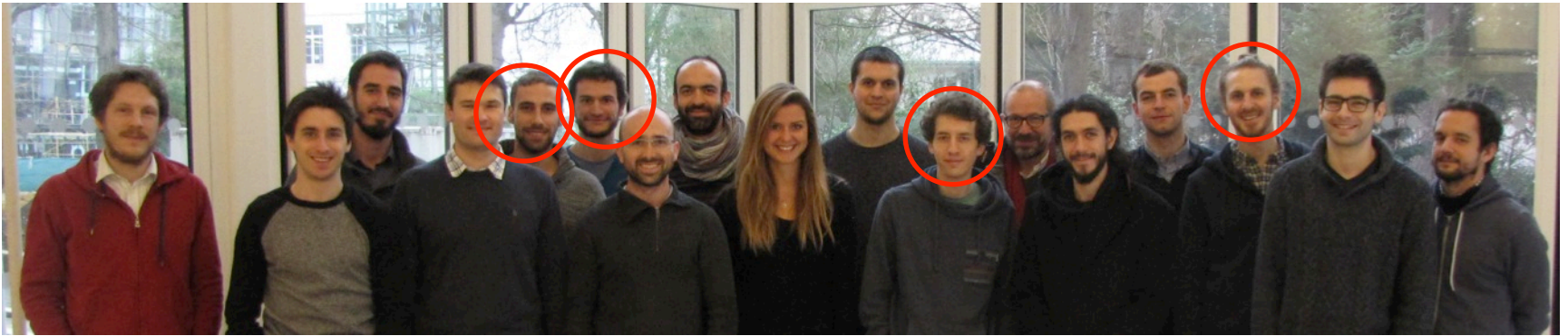


loss operator $\sqrt{\kappa_4} \hat{a}^4$

$$H = \varepsilon_4 (\hat{a}^\dagger)^4 + \varepsilon_4^* \hat{a}^4$$

M.M. *et. al.* NJP (2014)
S. Mundhada *et. al.* in prepatation.

COLLABORATORS



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Schoelkopf's group, (Nissim Ofek, Andrei Petrenko)