Non-equilibrium Kondo transport

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Outline

I. Introduction to Kondo model

What, who, why…

II. The Kondo model at equilibrium

Brief and partial survey of the toolbox

- Scaling
- Strong coupling
- Integrability

III. Out-of-equilibrium Kondo model: super Fermi liquid

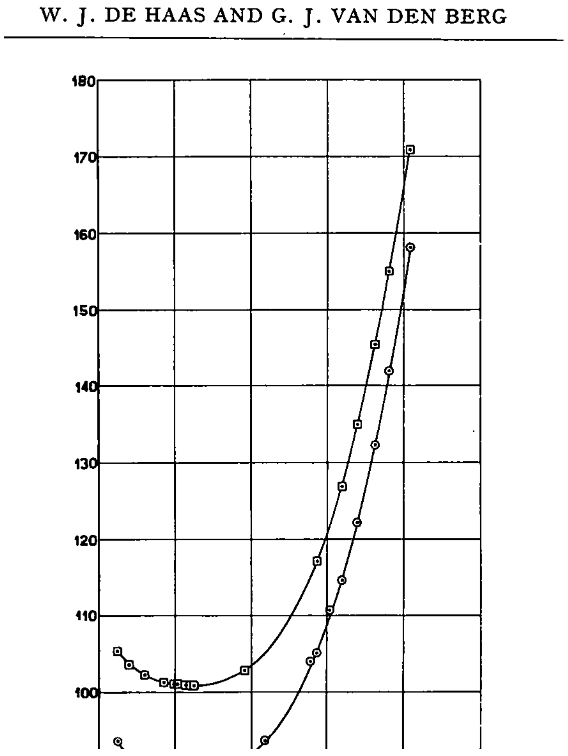
History

Measurements of metal resistivity at low T (1936)

- ➔ Resistivity increases when T is lowered below some characteristic temperature
- ➔ This characteristic temperature decreases when the metal is purer

esistance-temperature curve of gold has been investigated for lifferent wires.

§ 2. *Description of the experiments*. The electrical resistance has een measured with a D i e s s e l h o r s t compensation apparatus y comparing the potential differences between both ends of the



History

Explanation (1964) by J. Kondo:

➔ Magnetic atoms (impurity) cause extra-scattering for conduction electrons

➔ Anti-ferromagnetic Kondo model furnishes a quantitative explanation

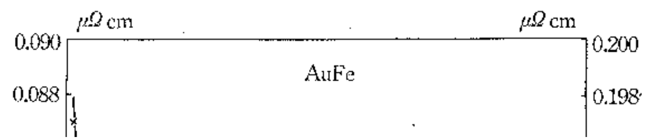
e minimum deduced from these observations ma
d with the existence of localized magnetic mome
ne only case of alloys of Au with the second ser
shows a minimum in resistivity is the alloys wit
se where negative magneto-resistance is found
olved.[2] We also find no resistance minimum in all
ocalized moments is found from the measurements

J. Kondo

nental observations. Now our calculation show

$$-\rho_M (3zJ/\varepsilon_F). \qquad (26)$$

in (23) is the sum of ρ_Δ and ρ_M. Its value i
Ve can expect that both ρ_Δ and ρ_M are of com
bitrarily take $\rho_M = 500\ \mu\Omega$ cm. Then from (26)
2 or $J = -0.15$ ev if $\varepsilon_F = 7$ ev. This magnitude



the Kondo model

Kondo model (J. Kondo,1964)

$$H = \sum_k \sum_{\alpha=\uparrow,\downarrow} \epsilon_k \, c^\dagger_{k,\alpha} c_{k,\alpha} + J \;\; \vec{S}(0) \cdot \vec{S}_{\mathrm{imp}}$$

conduction electrons

coupling

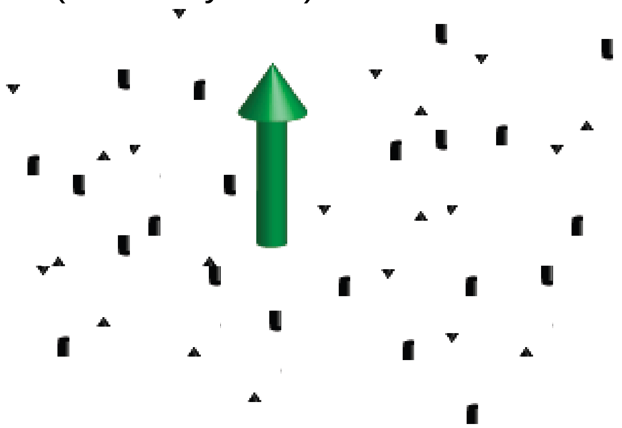
impurity spin (s=1/2)

$$\vec{S}(x) = \sum_{\alpha,\beta} c^\dagger_\alpha(x) \, \frac{\vec{\sigma}_{\alpha,\beta}}{2} \, c_\beta(x)$$

the Kondo model is:

- Archetypical

  Coupling of a single DoF (2-level system) to a Fermi sea



- Universal

  All low energy properties depend on a **single energy scale** T_K

- Non perturbative

  Hard problem !

Kondo models

- Change the impurity spin size : s=1/2, 1 , 3/2 ….

- Add channel degeneracy (several Fermi seas): k=1,2,3…

- Enlarge the symmetry group

  SU(2) → SU(N)

  N=4 : realized in Quantum Dots

  N→∞ : as theoretical trick (heavy fermions)

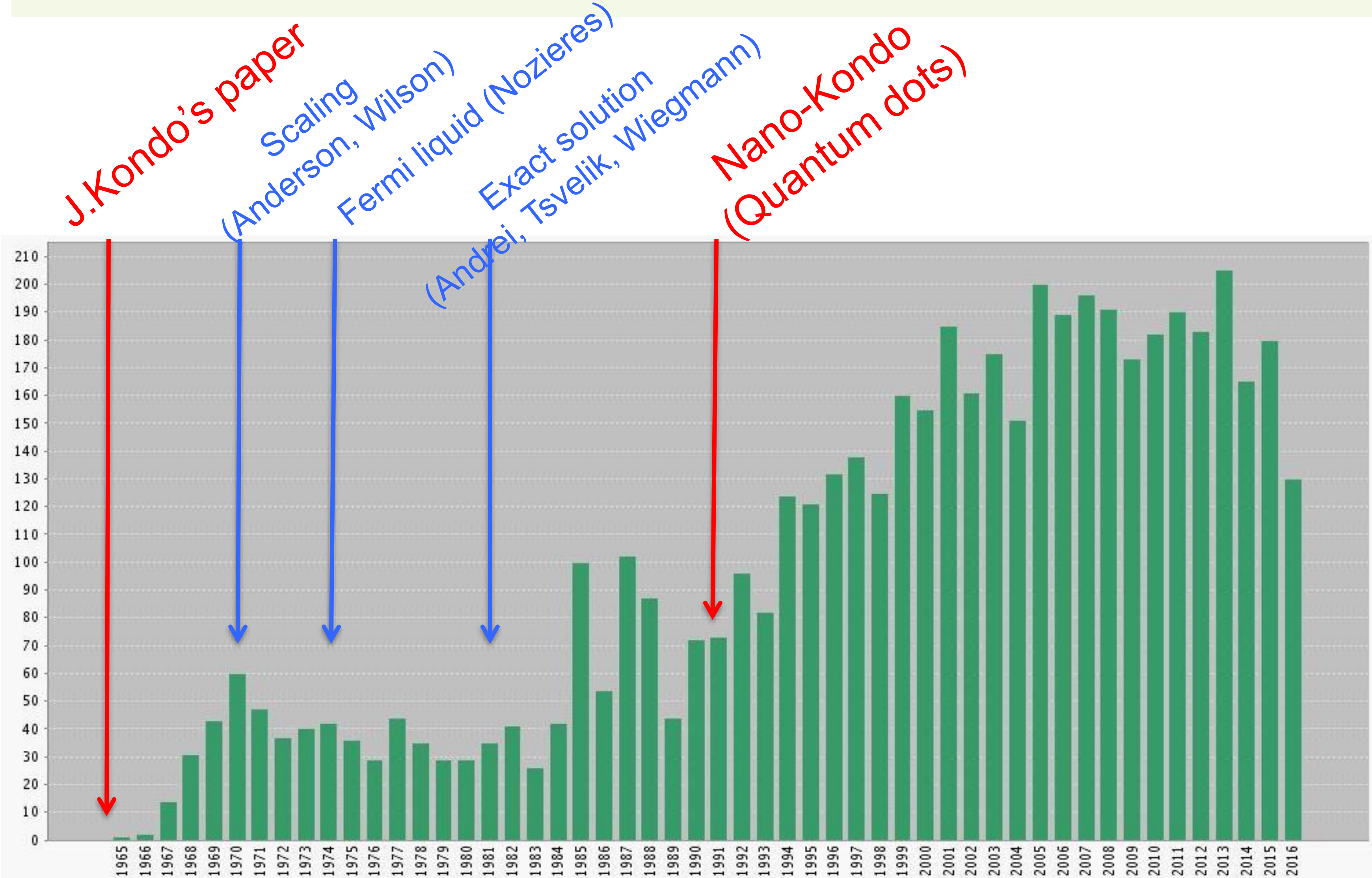
SU(2) → SO(N)

exotic Kondo effect

In this talk, only the original Kondo model will be discussed

- k=1 (one channel)
- s=1/2

Kondo model popularity



II. Equilibrium Kondo

Some tools for describing Kondo effect

- Scaling and strong coupling
- Fermi liquid picture
- Bethe Ansatz

Scaling - 1

- Kondo's result : big problem at low T ! Perturbation theory is ill-defined

$$R = R_0 \left[1 - 4J\rho_F \ln\left(\frac{k_B T}{D}\right) + \ldots \right]$$

Density of states at Fermi level

Electronic bandwidth

Higher powers of $\ln(k_B T/D)$

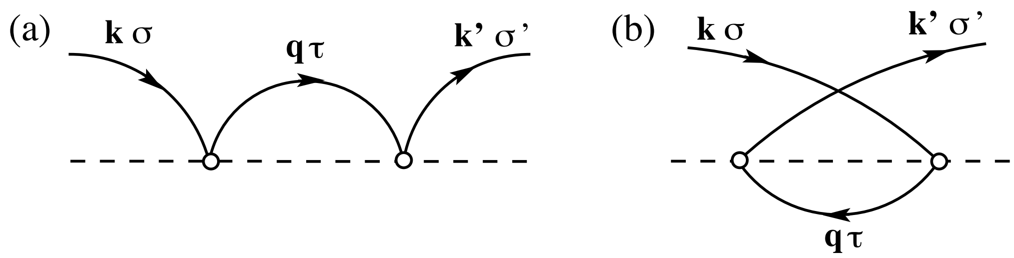
- Resummation of most divergent terms (Abrikosov 1965):

$$R = \frac{R_0}{\left[1 + 2J\rho_F \ln\left(\frac{k_B T}{D}\right)\right]^2} \quad \rightarrow \quad k_B T_K \sim D \exp\left(-\frac{1}{2J\rho_F}\right)$$

- What happens when T approaches T_K ? How to describe it ?

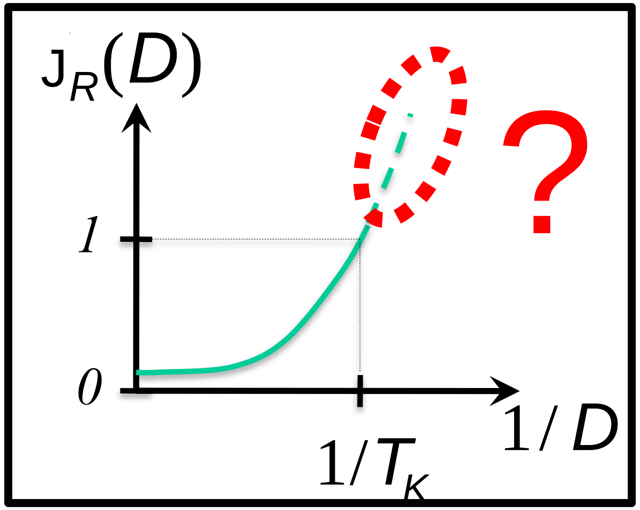
Scaling - 2

- Renormalization Group: Anderson (1970) confirmed numerically by Wilson (1974)
- Idea : reduce the high energy cutoff (bandwidth D) and define an effective coupling (« running coupling constant ») J(D)



- Result:

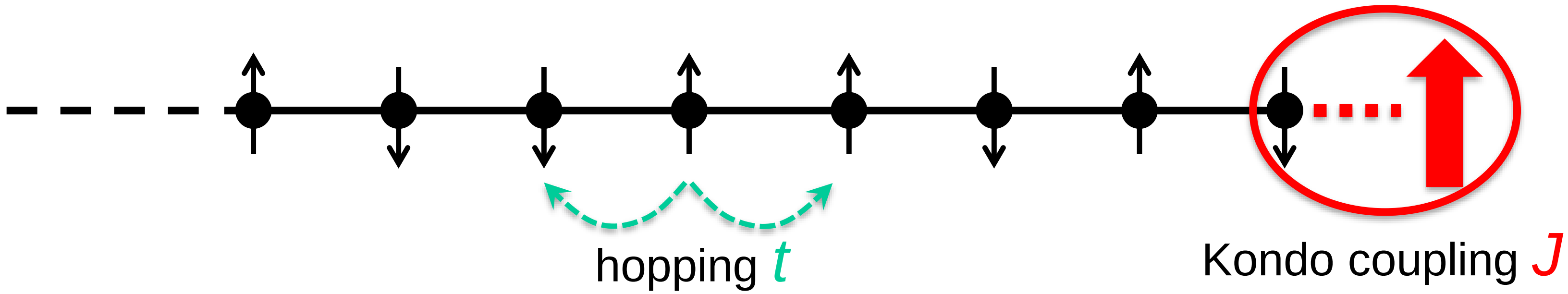
$$\frac{dJ(D)}{d\ln D} = -2\rho_{\mathrm{F}}\big(J(D)\big)^2$$



Strong coupling picture - 1

Ingredients: - Lattice model: semi infinite chain

- Infinite Kondo coupling $J/t = \infty$



- Formation of (infinitely bound) singlet
  ➔ the impurity spin is **screened:**
  Groundstate of the Kondo model
- Last site effectively decouples
  ➔ electrons experience phase shift $\pi / 2$

Strong coupling picture - 2

Expansion at large J/*t* (Nozieres 1974):

- Assume finite J/t adiabatically deforms the eigenstates
- Assume the scattering matrix on the Kondo singlet is **analytical**

LOCAL FERMI LIQUID → physics depends on 2 phenomenological parameters

ends only on the spin and energy   Note that (3) is an *exact* de

Equivalent to an effective theory:

$$H = H_{\mathrm{free}} + \frac{1}{T_K}\mathcal{O}_2(x=0)$$

free electrons (with phase shift $\pi/2$)

effect of the singlet fluctuations

Integrability

The kondo model possesses an infinity
of converved quantities

→ Integrable (Andrei, Tsvelik, Wiegmann 1983)

→ Eigenstates can be obtained exactly:

- → They can be described as **quasiparticles**
- → These quasiparticles scatter nicely on the impurity (factorization)

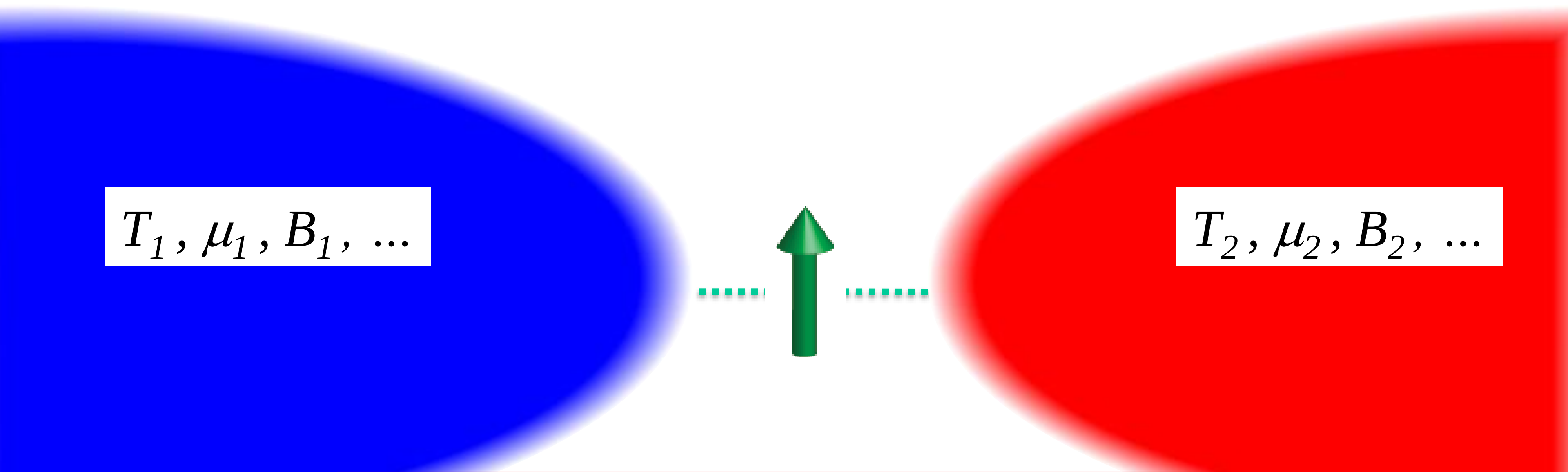
→ Thermodynamics can be obtained at arbitrary energy:

susceptibility $\chi(T/T_K)$

specific heat $C(T/T_K)$

III. Out-of-equilibrium Kondo

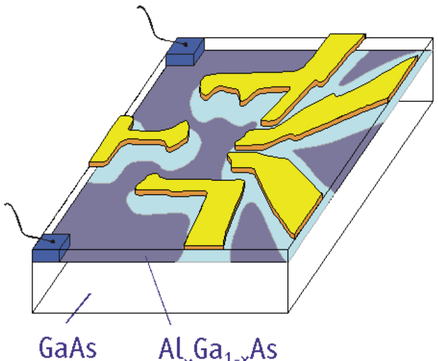
- Several baths (macroscopic, at equilibrium)
- Out-of-equilibrium forcing
- Flow (of charge, spin, energy, …) through impurity

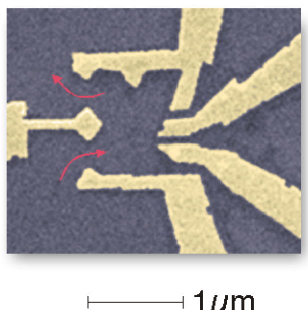


Questions: currents $I(\mu_1,\mu_2,\ldots)$?
fluctuations $\Delta I(\mu_1,\mu_2,\ldots)$?
state of the system ?

« Nano-impurities »

- Quantum dots:
  - ➔ 2D electron gas

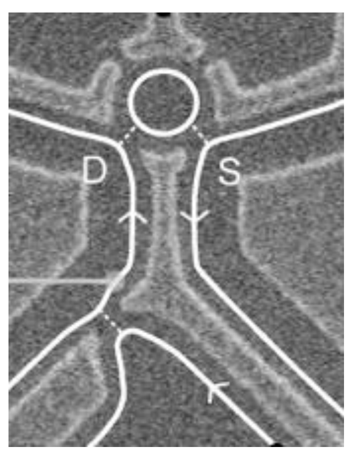




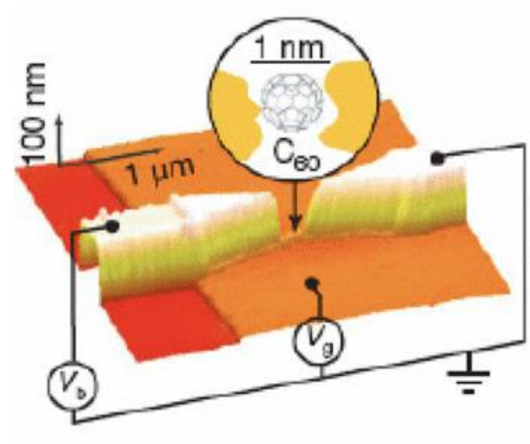
(C. Marcus, Harvard)

  - ➔ Quantum Hall edge states

(F. Pierre, LPN)



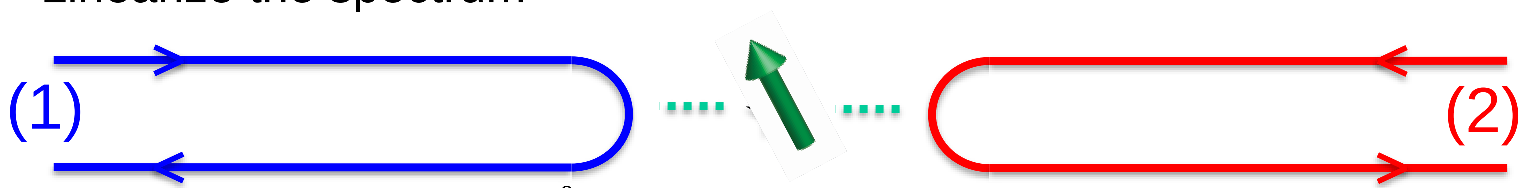
- Molecules: metallic electrodes
  - ➔ break junctions
  - ➔ electromigration



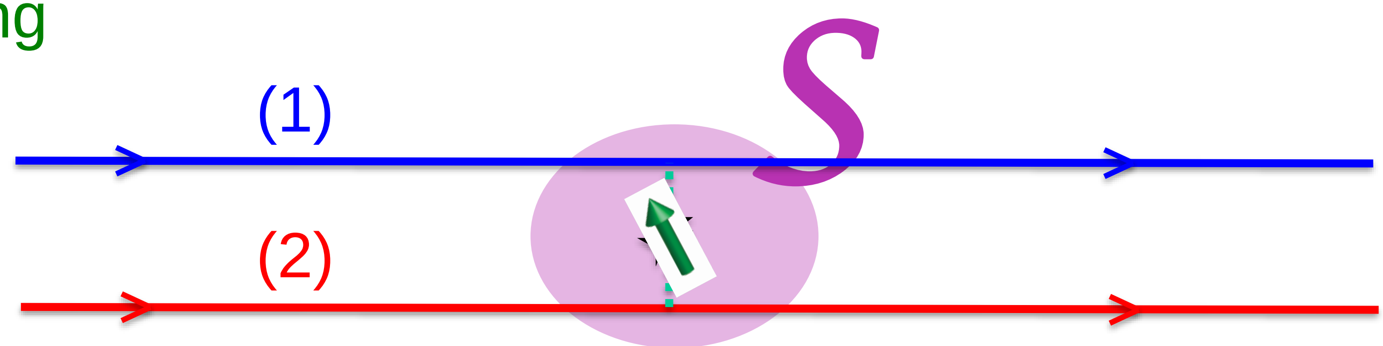
(W. Wernsdorfer, Institut Néel)

Modeling the baths

- Modes that couple to the impurity are 1D (conduction channel)
- Linearize the spectrum



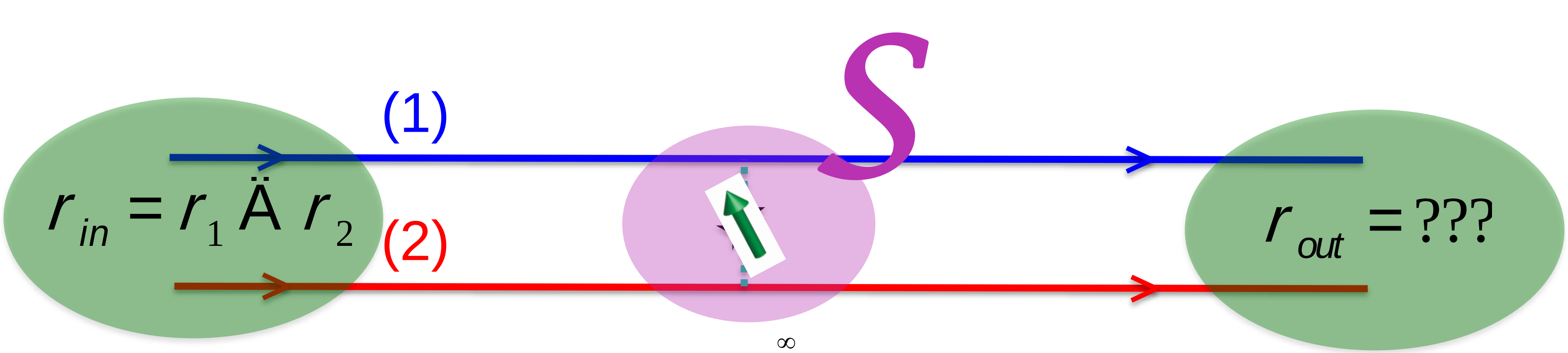
$$H_0[\Psi] = \sum_{a=1,2} -iv_F \int_{-\infty}^{0} dx \left[\Psi^\dagger_{aR}(x)\partial_x \Psi_{aR}(x) - \Psi^\dagger_{aL}(x)\partial_x \Psi_{aL}(x) \right]$$



$$H_0[\Psi] = \sum_{a=1,2} -iv_F \int_{-\infty}^{\infty} dx \; \Psi^\dagger_{a}(x)\partial_x \Psi_{a}(x)$$

Chiral theory involving only right-moving fields: **scattering problem**

Modeling the baths



$$H_0[\Psi] = \sum_{a=1,2} -iv_F \int_{-\infty}^{\infty} dx \;\; \Psi_a^{\dagger}(x)\partial_x \Psi_a(x)$$

Chiral theory involving only right-moving fields: **scattering problem**

Scattering matrix S

- Without interactions:
  - S factorizes in one-body scattering matrices $S^{(1)}$
  - Landauer Buttiker formalism works

$$I = \int dE \left(f_1(E) - f_2(E) \right) |S_{12}^{(1)}|^2(E)$$

- Generically, **with** interactions:
  - S is genuinely **many-body:** **particle production**
  - Landauer Buttiker fails

- Integrable model
  - Existence of quasi-particles with factorized scattering: GOOD
  - Write the (electronic) incoming Fermi sea's in terms of integrable quasiparticles: **VERY HARD IN GENERAL: BAD**
  - Some exceptions…

Integrability + non-equilibrium

a few available solutions !

- Dressed TBA
  - Quantum Hall edge states tunneling (P. Fendley, A. Ludwig, H. Saleur 1995)
  - Self-dual Interacting Resonant Level Model (E.B., P. Schmitteckert, H. Saleur 2008)
- Map to equilibrium problem
  - Boundary sine Gordon model (V. Bazhanov, S. Lukyanov, A.B. Zamolodchikov 1999)
- Effectively non-interacting systems (map to free fermions)
  - 1-ch Kondo (A. Schiller, U. Hershfield 1998) } Toulouse point
  - Luttinger Liquid (A. Komnik, O. Gogolin 2003) } Toulouse point
  - 2-ch Kondo (E. Sela, I. Affleck 2009) QCP & vicinity

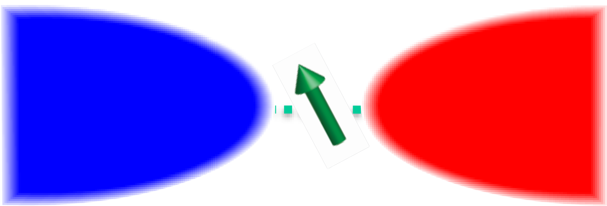
The game is not over

- Integrable theories have nevertheless a rich structure:
  - Infinite number of conserved quantities
  - Renormalization group flow is controlled non-perturbatively

One can use this rich structure to develop a controlled expansion out of equilibrium, in the strong coupling regime (at least in some cases)

Integrable Strong Coupling Expansion $V, T, w, \ldots$ £ T_K

Weak - Strong Coupling



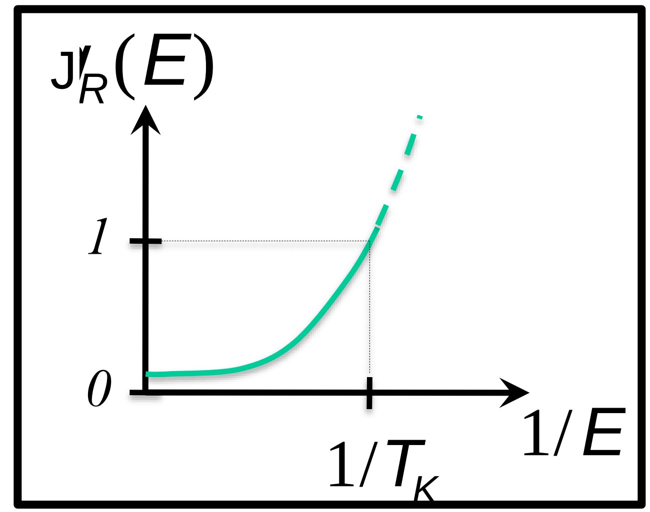
$$H = H_0 + \mathrm{J}\, H_K$$

baths

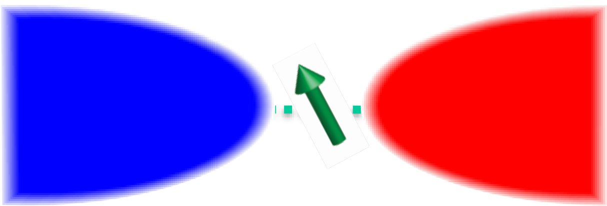
impurity/bath coupling

$$H_K = \vec{S}_{imp} \cdot \vec{S}_{bath}(0)$$

Spin exchange



Weak - Strong Coupling

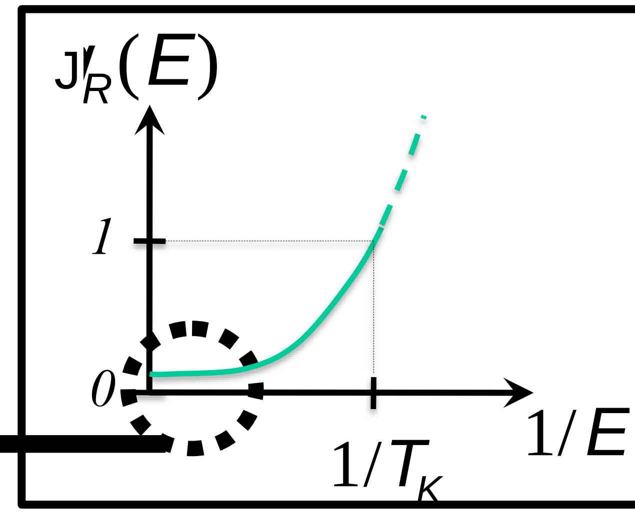


$$H = H_0 + J\, H_K$$

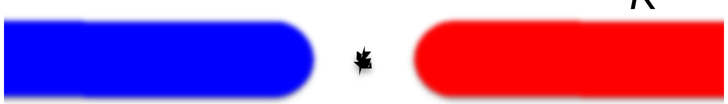
baths → H_0; impurity/bath coupling → H_K

$$H_K = \vec{S}_{imp} \cdot \vec{S}_{bath}(0)$$

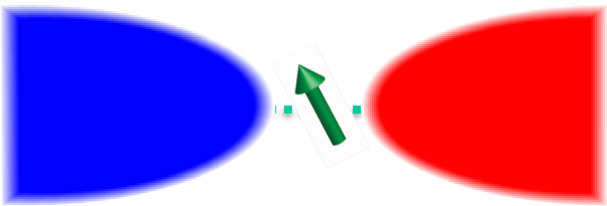
Spin exchange → $\cdot$



Weak coupling limit, $E >> T_K$



Weak - Strong Coupling

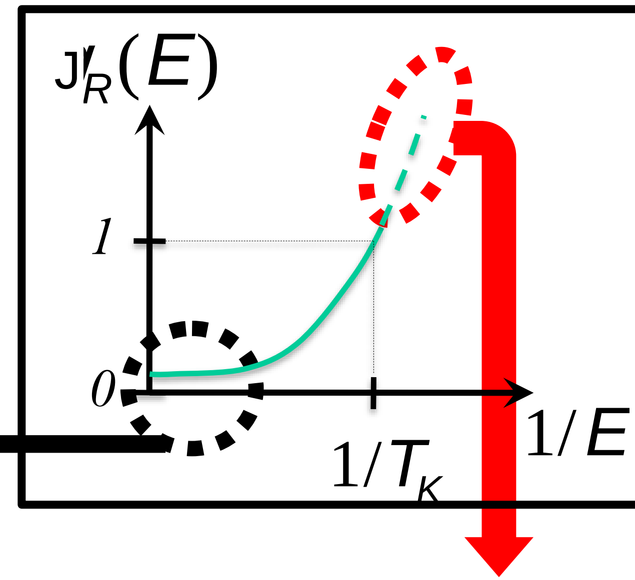


$$H = H_0 + J\, H_K$$

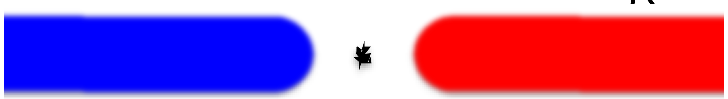
baths — impurity/bath coupling

$$H_K = \vec{S}_{imp} \cdot \vec{S}_{bath}(0)$$

Spin exchange



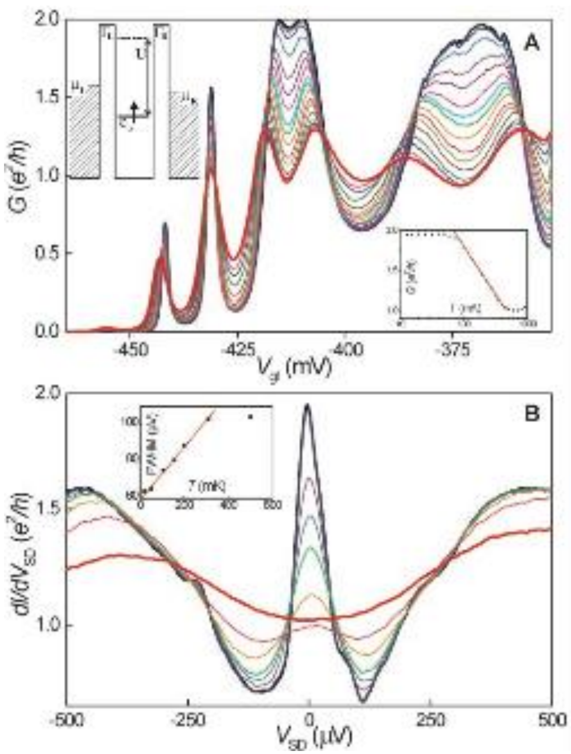
Weak coupling limit, $E >> T_K$



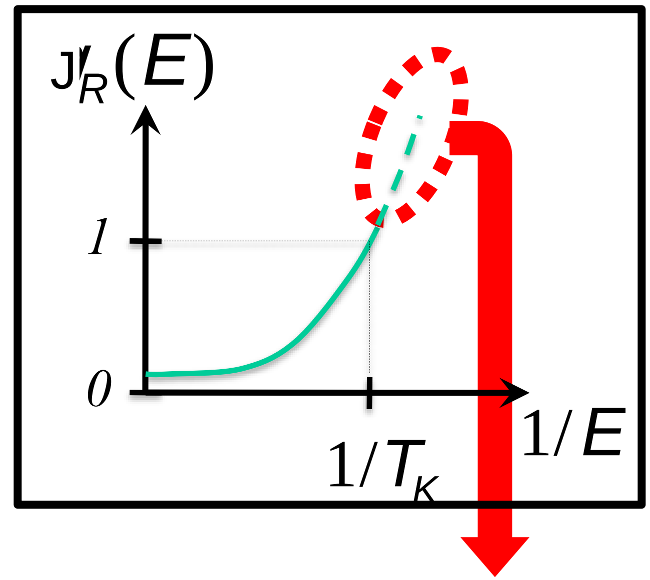
Strong coupling limit, $E << T_K$

At low energy :
strong coupling regime
“Physics is non perturbative”

Weak - Strong Coupling



« Kondo resonance is a strong coupling phenomenon »



Strong coupling limit, $E \ll T_K$

At low energy :
strong coupling regime
"Physics is non perturbative"

‘Standard’ perturbation theory

- Keldysh method:
  - allows for a formal expression of the out-of-equilibrium density matrix

$$\hat{\rho}(t) = \mathcal{U}(0,t)\,\hat{\rho}(0)\,\mathcal{U}(0,t)^{-1} \qquad\qquad \mathcal{U}(0,t) = \mathcal{P}\, e^{-i\gamma\int_0^t dt'\, H_B(t')}$$

  - but how to evaluate/resum the perturbative expansion?

    Fails in the strong coupling regime

- Need for a non-perturbative approach: Integrability !

Strategy

Run the RG backwards

Want to describe the strong coupling regime $T, V, w \ldots$ £ T_K

1. Incorporate out-of-equilibrium forcing AT the strong coupling fixed point "$T_K =$ ¥"

2. Use Integrability + Keldysh to build the (many-body) scattering matrix

3. Expand in inverse powers of T_K

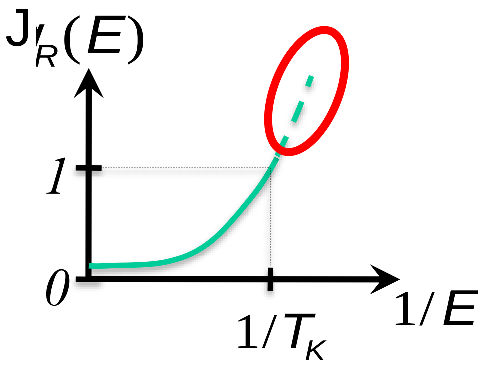
Net result:

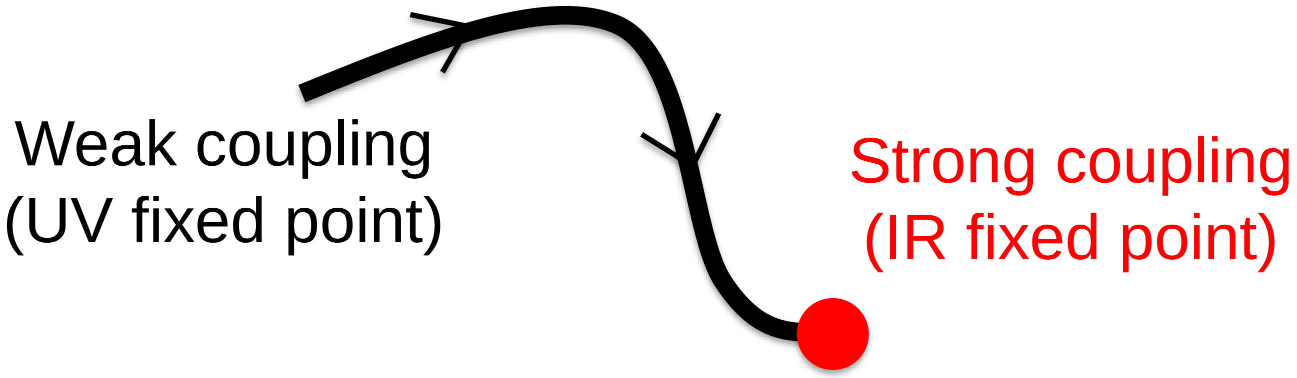
→ Exact effective operator encoding
out-of-equilibrium + many-body scattering

→ Taylor expansion of the universal scaling functions
for local observables, at arbitrary order in principle

Strong coupling fixed point

- Perturbation is **relevant**
- Strong coupling fixed point described by BCFT





- Step 1: Out-of-equilibrium SC fixed point $(T_{\mathrm{K}} = ¥)$

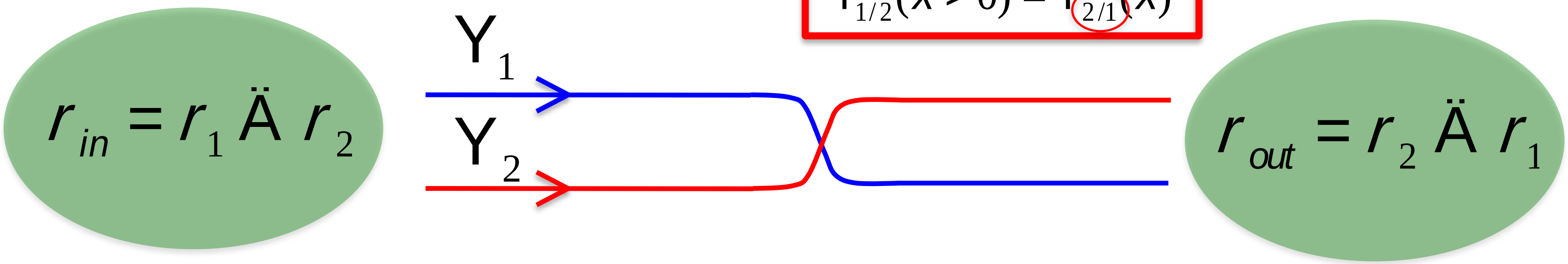
Strong coupling fixed point

- Boundary conditions: $\Phi(x=0^-) = \mathcal{B}\cdot\Phi(x=0^+)$
- **“Transparent fields”**: $\tilde{\Phi}(x<0) = \Phi(x)\ ;\ \tilde{\Phi}(x>0) = \mathcal{B}\cdot\Phi(x)$

  They don’t see the impurity!

  BC for fermions: $\pi/2$ phase shift

$$\tilde{\Psi}_{1/2}(x<0) = \Psi_{1/2}(x)$$
$$\tilde{\Psi}_{1/2}(x>0) = \Psi_{2/1}(x)$$



- Forcing out-of-equilibrium easily represented!
- Amounts to a gauge transformation $\mathcal{U}_{N.Equ}(z)$ for the transparent fields

$$\rho_{in} \propto e^{-\frac{H_0[\Psi_1] - \mu_1 Q_1}{T_1}} \otimes e^{-\frac{H_0[\Psi_2] - \mu_2 Q_2}{T_2}} \Rightarrow \langle I \rangle = \left(2e^2/h\right)\left(\mu_1(t) - \mu_2(t)\right)$$

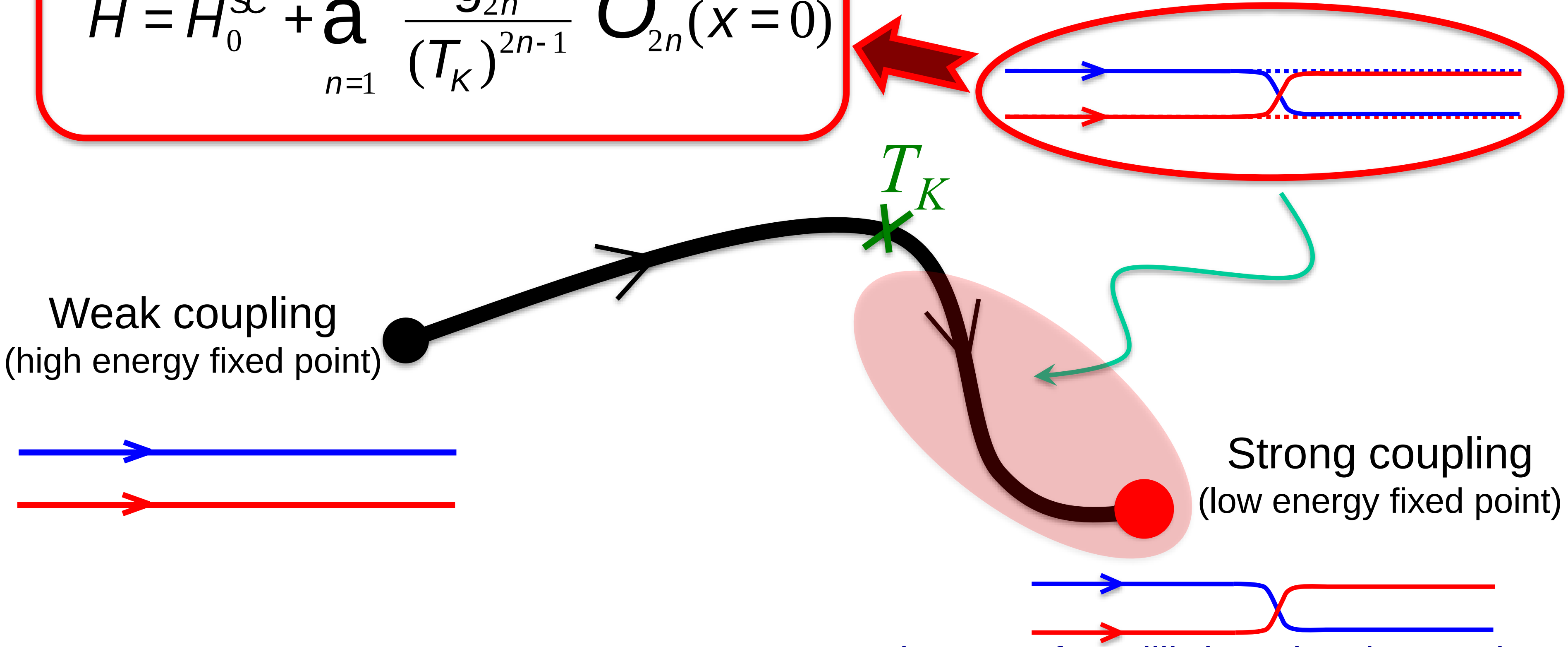
Recover the linear regime for the charge current

Around strong coupling

Integrability ➔ dual hamiltonian

$$H = H_0^{SC} + \sum_{n=1}^{\infty} \frac{g_{2n}}{(T_K)^{2n-1}} \hat{O}_{2n}(x=0)$$

Physics is controlled
by **backscattering** (many body!)



Derive out-of-equilibrium density matrix

➔ Linear regime: $I = \frac{2e^2}{h} V$

Dual theory

$$H = H_0^{SC} + H_B^{SC} \qquad H_B^{SC} = \sum_{n=1}^{\infty} \frac{g_{2n}}{(T_K)^{2n-1}} \hat{O}_{2n}(x=0)$$

- The operators O_{2n} are the (infinitely many) conserved quantities stemming from integrability.
- The couplings g_n are pure numbers, **fixed** by integrability. (Lesage, Saleur 1999)
- Fermi liquid: the least irrelevant operator is $O_2 = T$, an energy momentum tensor.
- Higher order processes have integer dimensions = 4,6,8,…

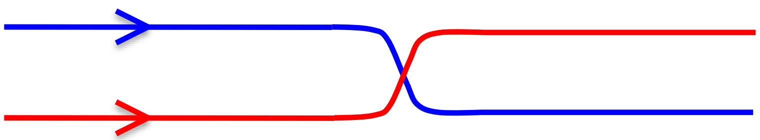
Backscattering transfers integer charges (electrons)

"SUPER FERMI LIQUID"

Keldysh expansion

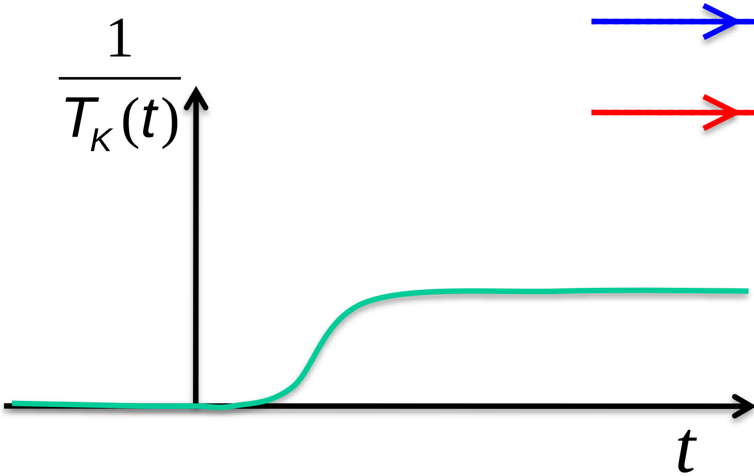
Step 2: Many-body scattering

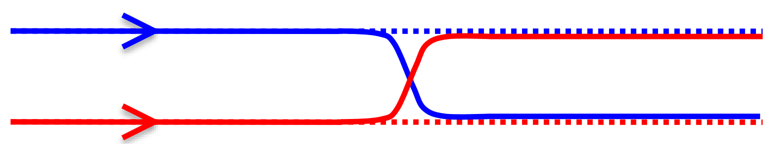
- Start at time $t = -¥$ at the SC fixed point $(T_K = ¥)$



$$r(-¥) = r_{SC} = e^{-H_0^{SC}/k_B T}$$

- Switch on backscattering at time t=0



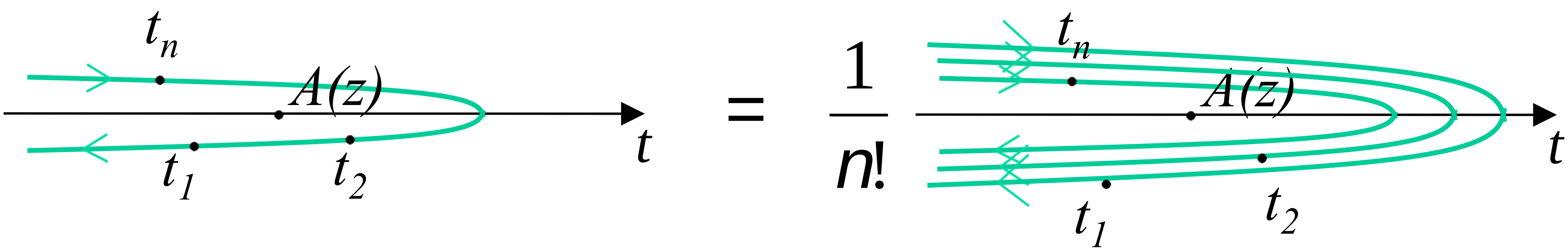


$$r(t) = U(t)\, r_{SC}\, U(t)^{\dagger}$$

$$U(t) = \mathcal{P}_K e^{-i g \grave{o}_{-¥}^{t} dt' H_B^{SC}(t')}$$

Effective operators

In a **super Fermi liquid**, the Keldysh expansion bears a simple form:

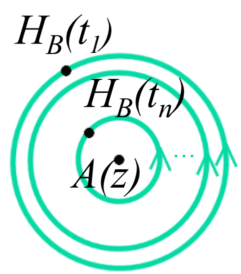


Each (local) operator can be replaced by an *effective* operator:

Complete many-body scattering

$$A^{eff}(z) = \mathcal{U}_{BS}(z) \bullet A(z)$$

$$= e^{-i\oint_z dt\, H_B^{SC}(t)} \bullet A(z) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \oint_z dt_1 \ldots \oint_z dt_n$$



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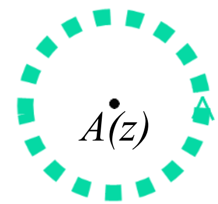
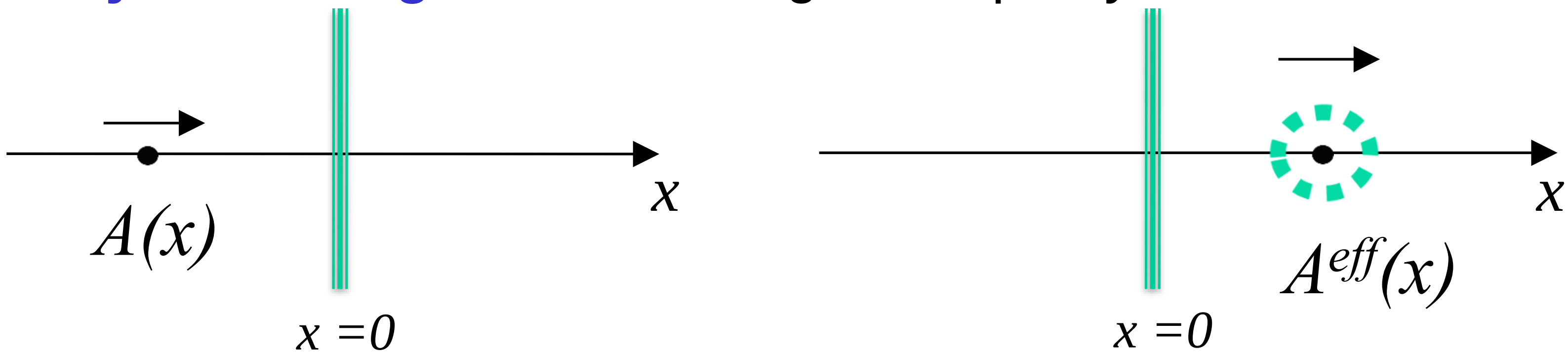
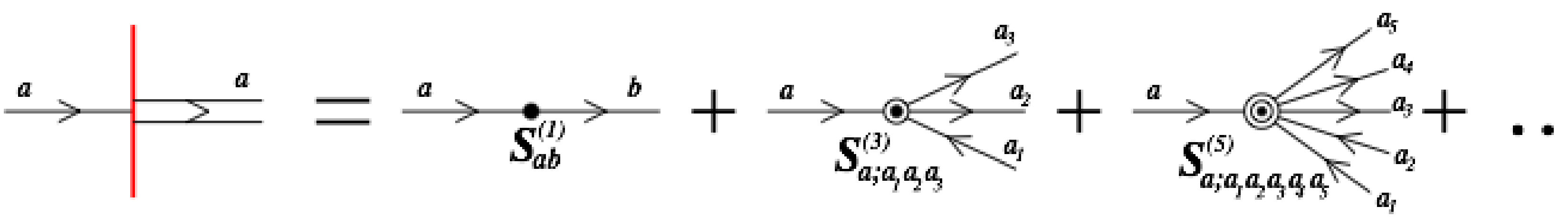


Effective operators

- **ALL** interactions processes are described by a **dressing by scattering** when crossing the impurity:



$$\left\langle A(x,t) \right\rangle_{N.Equ} = \left\langle \mathcal{U}_{N.Equ}(z) \cdot \ \mathcal{U}_{BS}(z) \cdot \ A(z) \right\rangle_0$$



Summary

A simple formula

$$\langle A(t) \rangle_{\mathrm{n.eq.}} = \langle A_{eff} \rangle_0$$

can be expanded in powers of 1/ T_K : non-linear effects to arbitrary order !

Contours + CFT relations :

⇨ Algebraic (computer-friendly) reformulation

\+

PT is *finite*: No UV divergence !

Exact formula ! **Directly gives universal results**

Versatile formulation $V, \mu, \omega, T_i, t, B, \epsilon_d \ldots$

−

Integrable models with integer conserved quantities only

Not always easy to find conserved quantities and couplings

The price to pay…

Electrical current operator : **initial fermions**

$$I_{BS} = 1/2((\psi^\dagger_{1\uparrow}\psi_{1\uparrow}) - (\psi^\dagger_{2\uparrow}\psi_{2\uparrow}) + (\psi^\dagger_{1\downarrow}\psi_{1\downarrow}) - (\psi^\dagger_{2\downarrow}\psi_{2\downarrow}))$$

The price to pay…

Electrical current operator : **Transparent fermions**

Zeroth order

x

I_{BS}

$$
\begin{aligned}
&-\frac{1}{2}\sin(\theta)\cos(\theta)(\cos(\xi)-1)+\frac{1}{2}i\sin(\theta)\sin(\xi)((\Psi_{1\uparrow})^\dagger(\Psi_{2\uparrow}))\\
&+\frac{1}{2}\sin(\theta)\cos(\theta)(\cos(\xi)-1)+\frac{1}{2}i\sin(\theta)\sin(\xi)((\Psi_{1\uparrow})(\Psi_{2\uparrow})^\dagger)\\
&+\frac{1}{2}\left(\sin^2(\theta)\cos(\xi)+\cos^2(\theta)\right)((\Psi_{1\uparrow})^\dagger(\Psi_{1\uparrow}))\\
&+\frac{1}{2}\left(-\sin^2(\theta)\cos(\xi)-\cos^2(\theta)\right)((\Psi_{2\uparrow})^\dagger(\Psi_{2\uparrow}))\\
&+-\frac{1}{2}\sin(\theta)\cos(\theta)(\cos(\xi)-1)-\frac{1}{2}i\sin(\theta)\sin(\xi)((\Psi_{1\downarrow})^\dagger(\Psi_{2\downarrow}))\\
&+\frac{1}{2}\sin(\theta)\cos(\theta)(\cos(\xi)-1)-\frac{1}{2}i\sin(\theta)\sin(\xi)((\Psi_{1\downarrow})(\Psi_{2\downarrow})^\dagger)\\
&+\frac{1}{2}\left(\sin^2(\theta)\cos(\xi)+\cos^2(\theta)\right)((\Psi_{1\downarrow})^\dagger(\Psi_{1\downarrow}))\\
&+\frac{1}{2}\left(-\sin^2(\theta)\cos(\xi)-\cos^2(\theta)\right)((\Psi_{2\downarrow})^\dagger(\Psi_{2\downarrow}))
\end{aligned}
$$

The price to pay…

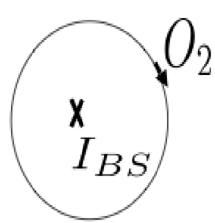
Electrical current operator : **Transparent fermions**

$$-(\sin\theta*(2*\cos[\theta/2]^2*(\cos\xi-i*\sin\xi)+2*\cos[\theta/2]^2*\cos\theta*(\cos\xi-i*\sin\xi)+\sin\theta^2*(\cos\xi-i*\sin\xi)))/16$$
$$+(\sin\theta*(-2*\cos[\theta/2]^2*(\cos\xi+i*\sin\xi)+2*\cos[\theta/2]^2*\cos\theta*(\cos\xi+i*\sin\xi)+\sin\theta^2*(\cos\xi+i*\sin\xi)))/16*(d\psi^\dagger_{1\uparrow}\psi_{2\uparrow})$$
$$+-(\sin\theta*(-2*\sin[\theta/2]^2*(\cos\xi-i*\sin\xi)-2*\cos\theta*\sin[\theta/2]^2*(\cos\xi-i*\sin\xi)+\sin\theta^2*(\cos\xi-i*\sin\xi)))/16$$
$$+(\sin\theta*(2*\sin[\theta/2]^2*(\cos\xi+i*\sin\xi)-2*\cos\theta*\sin[\theta/2]^2*(\cos\xi+i*\sin\xi)+\sin\theta^2*(\cos\xi+i*\sin\xi)))/16*(\psi^\dagger_{1\uparrow}d\psi_{2\uparrow})$$

$$+-((\cos[\theta/2]^2-\cos\theta-\sin[\theta/2]^2)*\sin\theta^2*(\cos\xi-i*\sin\xi))/8+((\cos[\theta/2]^2-\cos\theta$$
$$-\sin[\theta/2]^2)*\sin\theta^2*(\cos\xi+i*\sin\xi))/8*(\psi^\dagger_{1\uparrow}(\psi_{1\uparrow}(\psi^\dagger_{2\uparrow}\psi_{2\uparrow})))$$

First order: $1/T_K$

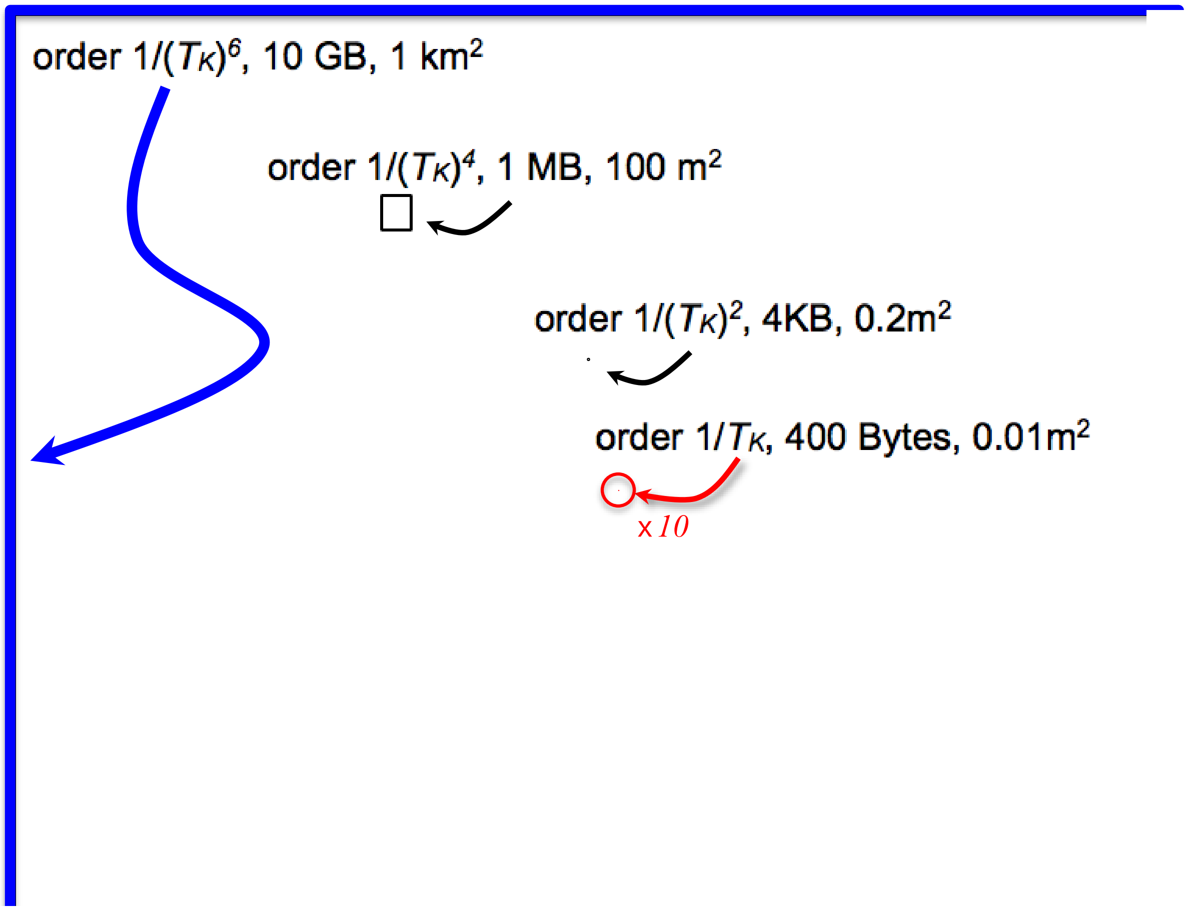
$$+-(\sin\theta^2*(-\cos\xi+2*\cos[\theta/2]^2*(\cos\xi-i*\sin\xi)-\cos\theta*(\cos\xi-i*\sin\xi)+i*\sin\xi))/16+(\sin\theta^2*(\cos\xi$$
$$+2*\cos[\theta/2]^2*(\cos\xi+i*\sin\xi)-\cos\theta*(\cos\xi+i*\sin\xi)+i*\sin\xi))/16*(\psi^\dagger_{1\uparrow}d\psi_{1\uparrow})$$
$$+(i/16)*\sin\theta^2*(i*(\cos\xi-i*\sin\xi)+i*\cos\theta*(\cos\xi-i*\sin\xi)+(2*i)*\sin[\theta/2]^2*(\cos\xi-i*\sin\xi))-(i/16)*\sin\theta^2*((-i)*(\cos\xi+i*\sin\xi)+i*\cos\theta*(\cos\xi+i*\sin\xi)+(2*i)*\sin[\theta/2]^2*(\cos\xi+i*\sin\xi))*(d\psi^\dagger_{2\uparrow}\psi_{2\uparrow})$$
$$+(\sin\theta*(2*\cos[\theta/2]^2*(\cos\xi-i*\sin\xi)+2*\cos[\theta/2]^2*\cos\theta*(\cos\xi-i*\sin\xi)))/16-(\sin\theta*(-2*\cos[\theta/2]^2*(\cos\xi+i*\sin\xi)$$
$$+2*\cos[\theta/2]^2*\cos\theta*(\cos\xi+i*\sin\xi)))/16+(\sin\theta^3*(\cos\xi-i*\sin\xi))/16-(\sin\theta^3*(\cos\xi+i*\sin\xi))/16*(\psi^\dagger_{1\uparrow}(\psi_{2\uparrow}(\psi^\dagger_{1\downarrow}\psi_{1\downarrow})))$$



$$+-(\sin\theta*(-2*\sin[\theta/2]^2*(\cos\xi-i*\sin\xi)-2*\cos\theta*\sin[\theta/2]^2*(\cos\xi-i*\sin\xi)))/16+(\sin\theta*(2*\sin[\theta/2]^2*(\cos\xi$$
$$+i*\sin\xi)-2*\cos\theta*\sin[\theta/2]^2*(\cos\xi+i*\sin\xi)))/16-(\sin\theta^3*(\cos\xi-i*\sin\xi))/16+(\sin\theta^3*(\cos\xi$$
$$+i*\sin\xi))/16*(\psi^\dagger_{1\uparrow}(\psi_{2\uparrow}(\psi^\dagger_{2\downarrow}\psi_{2\downarrow})))$$
$$+(\sin\theta^2*(\cos\xi-\cos\theta*(\cos\xi-i*\sin\xi)-i*\sin\xi))/16+(\sin\theta^2*(\cos\xi+\cos\theta*(\cos\xi-i*\sin\xi)-i*\sin\xi))/16$$
$$-(\sin\theta^2*(-\cos\xi-\cos\theta*(\cos\xi+i*\sin\xi)-i*\sin\xi))/16-(\sin\theta^2*(-\cos\xi+\cos\theta*(\cos\xi+i*\sin\xi)-i*\sin\xi))/16*(\psi^\dagger_{1\uparrow}(\psi_{2\uparrow}(\psi^\dagger_{1\downarrow}\psi_{2\downarrow})))$$
$$+(\sin\theta^2*(\cos\xi+\cos\theta*(\cos\xi-i*\sin\xi)-i*\sin\xi))/16-(\sin\theta^2*(-\cos\xi+\cos\theta*(\cos\xi+i*\sin\xi)-i*\sin\xi))/16$$
$$+(\sin\theta^2*(-\cos\xi-\cos\theta*(\cos\xi-i*\sin\xi)+i*\sin\xi))/16-(\sin\theta^2*(\cos\xi-\cos\theta*(\cos\xi+i*\sin\xi)$$
$$+i*\sin\xi))/16*(\psi^\dagger_{1\uparrow}(\psi_{2\uparrow}(\psi_{1\downarrow}\psi^\dagger_{2\downarrow})))$$
$$+-(\sin\theta*(-2*\cos[\theta/2]^2*(\cos\xi-i*\sin\xi)+2*\cos[\theta/2]^2*\cos\theta*(\cos\xi-i*\sin\xi)+\sin\theta^2*(\cos\xi-i*\sin\xi)))/16+(\sin\theta*(2*\cos[\theta/2]^2*(\cos\xi+i*\sin\xi)+2*\cos[\theta/2]^2*\cos\theta*(\cos\xi+i*\sin\xi)+\sin\theta^2*(\cos\xi+i*\sin\xi)))/16*(d\psi_{1\uparrow}\psi^\dagger_{2\uparrow})$$
$$+-(\sin\theta*(2*\sin[\theta/2]^2*(\cos\xi-i*\sin\xi)-2*\cos\theta*\sin[\theta/2]^2*(\cos\xi-i*\sin\xi)+\sin\theta^2*(\cos\xi-i*\sin\xi)))/16+(\sin\theta*(-2*\sin[\theta/2]^2*(\cos\xi+i*\sin\xi)-2*\cos\theta*\sin[\theta/2]^2*(\cos\xi+i*\sin\xi)+\sin\theta^2*(\cos\xi+i*\sin\xi)))/16*(\psi_{1\uparrow}d\psi^\dagger_{2\uparrow})$$
$$+(\sin\theta^2*(\cos\xi+2*\cos[\theta/2]^2*(\cos\xi-i*\sin\xi)-\cos\theta*(\cos\xi-i*\sin\xi)-i*\sin\xi))/16-(\sin\theta^2*(-\cos\xi+2*\cos[\theta/2]^2*(\cos\xi+i*\sin\xi)-\cos\theta*(\cos\xi+i*\sin\xi)-i*\sin\xi))/16*(d\psi^\dagger_{1\uparrow}\psi_{1\uparrow})$$
$$+(\sin\theta^2*(-\cos\xi+\cos\theta*(\cos\xi-i*\sin\xi)+2*\sin[\theta/2]^2*(\cos\xi-i*\sin\xi)+i*\sin\xi))/16-(\sin\theta^2*(\cos\xi+\cos\theta*(\cos\xi+i*\sin\xi)+2*\sin[\theta/2]^2*(\cos\xi+i*\sin\xi)+i*\sin\xi))/16*(\psi^\dagger_{2\uparrow}d\psi_{2\uparrow})$$
$$+-(\sin\theta*(-2*\cos[\theta/2]^2*(\cos\xi-i*\sin\xi)+2*\cos[\theta/2]^2*\cos\theta*(\cos\xi-i*\sin\xi)))/16+(\sin\theta*(2*\cos[\theta/2]^2*(\cos\xi+i*\sin\xi)+2*\cos[\theta/2]^2*\cos\theta*(\cos\xi+i*\sin\xi)))/16-(\sin\theta^3*(\cos\xi-i*\sin\xi))/16+(\sin\theta^3*(\cos\xi+i*\sin\xi))/16*(\psi_{1\uparrow}(\psi^\dagger_{2\uparrow}(\psi^\dagger_{1\downarrow}\psi_{1\downarrow})))$$
$$+(\sin\theta*(2*\sin[\theta/2]^2*(\cos\xi-i*\sin\xi)-2*\cos\theta*\sin[\theta/2]^2*(\cos\xi-i*\sin\xi)))/16-(\sin\theta*(-2*\sin[\theta/2]^2*(\cos\xi+i*\sin\xi)-2*\cos\theta*\sin[\theta/2]^2*(\cos\xi+i*\sin\xi)))/16+(\sin\theta^3*(\cos\xi-i*\sin\xi))/16-(\sin\theta^3*(\cos\xi+i*\sin\xi))/16*(\psi_{1\uparrow}(\psi^\dagger_{2\uparrow}(\psi^\dagger_{2\downarrow}\psi_{2\downarrow})))$$

The price to pay…

The price to pay…



DC current

DC bias applied to the two baths

Net analytical result : current

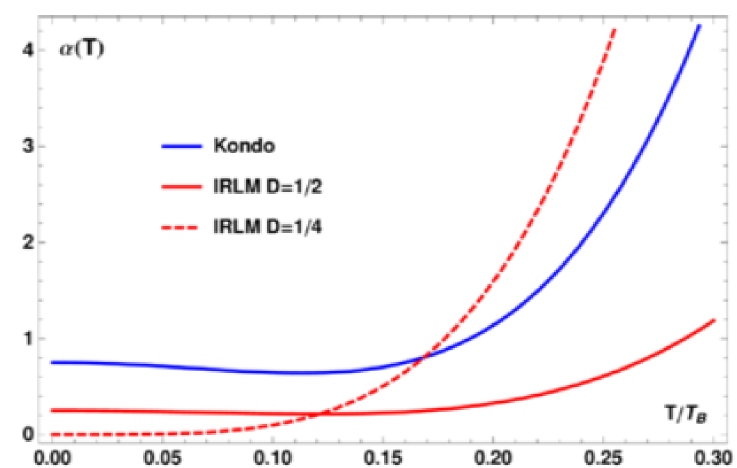
$$I(V) = 2\sin^2\theta V - \frac{\sin^2\theta V^3}{4T_B^2} + \frac{\left(252\sqrt{3}+65\pi+5\pi\cos 2\theta\right)\sin[\theta]^2 V^5}{1920\pi T_B^4} + O[V^7/T_B^7]$$

linear response

Fermi Liquid result

Temperature dependence of non linearity captured !

$$I(V)/I_0 V = (1 + \alpha(T) V^2/T_B^2 + \ldots)$$



DC conductance

DC bias: Universal differential conductance G=dI/dV

Rescaled quantities

$$\overline{X} = X / T_K$$

Fixed point

Fermi liquid corrections

$$\frac{G(\overline{V},\overline{T},\overline{h})}{G_0} = 1 - c_T\overline{T}^2 - \alpha_V c_T \overline{V}^2 - \alpha_h c_T \overline{h}^2$$

$$+\overline{V}^2 c_T^2(\gamma_V\overline{V}^2 + \gamma_T\overline{T}^2 + \gamma_h\overline{h}^2) + \rho_{Th}c_T^2\overline{T}^2\overline{h}^2 + \rho_h c_T^2\overline{h}^4 + \rho_T c_T^2\overline{T}^4$$

$$-\overline{V}^4 c_T^3(\kappa_V\overline{V}^2 + \kappa_T\overline{T}^2 + \kappa_h\overline{h}^2) - \overline{T}^4 c_T^3(\beta_V\overline{V}^2 + \beta_T\overline{T}^2 + \beta_h\overline{h}^2)$$

$$-\overline{h}^4 c_T^3(\chi_V\overline{V}^2 + \chi_T\overline{T}^2 + \chi_h\overline{h}^2) - \gamma_{Th}c_T^3\overline{V}^2\overline{T}^2\overline{h}^2 + \mathcal{O}(T_K^{-7})$$

NEW !

$\alpha_V = 3/2\pi^2 \approx 0.15$ $\quad c_T = \pi^2/4 \approx 2.46$ $\quad \alpha_h = 1/\pi^2 \approx 0.10$

$\gamma_V = \frac{252\sqrt{3}+65\pi+5\pi\cos 2\theta}{48\pi^5} \approx 0.04$ $\quad \gamma_T = \frac{72\sqrt{3}+17\pi+\pi\cos 2\theta}{4\pi^3} \approx 1.44^{\pm 0.03}$ $\quad \gamma_h = 3(5\sqrt{3}+\pi)/\pi^5 \approx 0.12$

$\rho_{Th} = 2(5\sqrt{3}+\pi)/\pi^3 \approx 1.52$ $\quad \rho_T = \frac{5}{3} + \frac{36\sqrt{3}}{5\pi} \approx 5.64$ $\quad \rho_h = (6\sqrt{3}+\pi)/3\pi^5 \approx 0.02$

DC conductance

DC bias: Universal differential conductance G=dI/dV

Fixed point — Fermi liquid corrections

$$\frac{G(\bar{V},\bar{T},\bar{h})}{G_0} = 1 - c_T\bar{T}^2 - \alpha_V c_T\bar{V}^2 - \alpha_h c_T\bar{h}^2$$

NEW !

$$+\bar{V}^2 c_T^2(\gamma_V\bar{V}^2 + \gamma_T\bar{T}^2 + \gamma_h\bar{h}^2) + \rho_{Th}c_T^2\bar{T}^2\bar{h}^2 + \rho_h c_T^2\bar{h}^4 + \rho_T c_T^2\bar{T}^4$$

$$-\bar{V}^4 c_T^3(\kappa_V\bar{V}^2 + \kappa_T\bar{T}^2 + \kappa_h\bar{h}^2) - \bar{T}^4 c_T^3(\beta_V\bar{V}^2 + \beta_T\bar{T}^2 + \beta_h\bar{h}^2)$$

$$-\bar{h}^4 c_T^3(\chi_V\bar{V}^2 + \chi_T\bar{T}^2 + \chi_h\bar{h}^2) - \gamma_{Th}c_T^3\bar{V}^2\bar{T}^2\bar{h}^2 + \mathcal{O}(T_K^{-7})$$

$\kappa_V = \frac{12960+19025\sqrt{5}+7308\sqrt{3}\pi+420\pi^2}{720\pi^8} \approx 0.01$

$\kappa_T = \frac{5265+7375\sqrt{5}+2826\sqrt{3}\pi+150\pi^2}{36\pi^6} \approx 1.11$

$\kappa_h = \frac{1215+1850\sqrt{5}+756\sqrt{3}\pi+35\pi^2}{120\pi^8} \approx 0.08$

$\beta_V = \frac{4455+6300\sqrt{5}+2376\sqrt{3}\pi+121\pi^2}{15\pi^4} \approx 22.36$

$\beta_T = \frac{2(14580+21250\sqrt{5}+8316\sqrt{3}\pi+427\pi^2)}{315\pi^2} \approx 71.77$

$\beta_h = \frac{2(3240+4600\sqrt{5}+1674\sqrt{3}\pi+75\pi^2)}{45\pi^4} \approx 10.66$

$\chi_V = \frac{8(117+175\sqrt{5}+56\sqrt{3}\pi+2\pi^2)}{\pi^8} \approx 0.70$

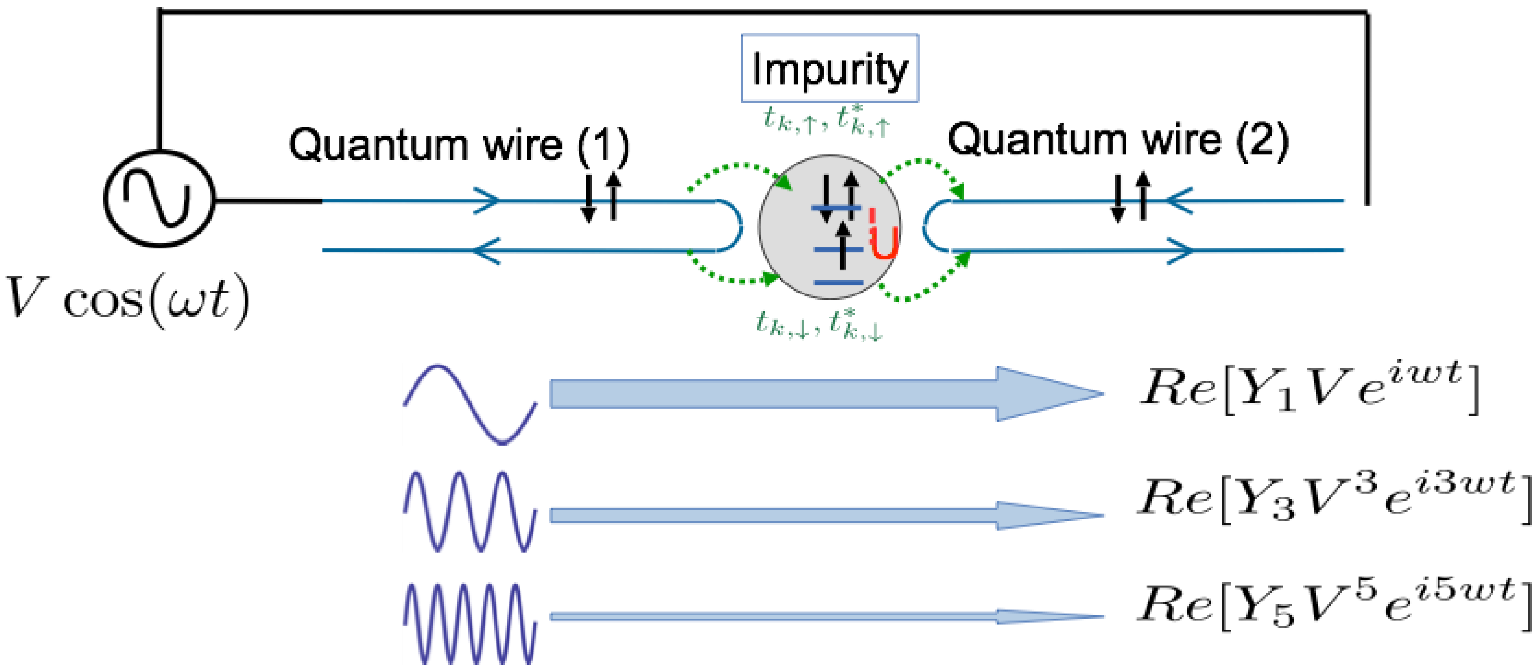
$\chi_T = \frac{16(117+175\sqrt{5}+56\sqrt{3}\pi+2\pi^2)}{3\pi^6} \approx 4.62$

$\chi_h = \frac{(45(3+5\sqrt{5})+60\sqrt{3}\pi+2\pi^2)}{45\pi^8} \approx 0.002$

$\gamma_{Th} = \frac{4428+6050\sqrt{5}+2160\sqrt{3}\pi+96\pi^2}{12\pi^6} \approx 2.66$

AC forcing

AC bias applied to the two baths

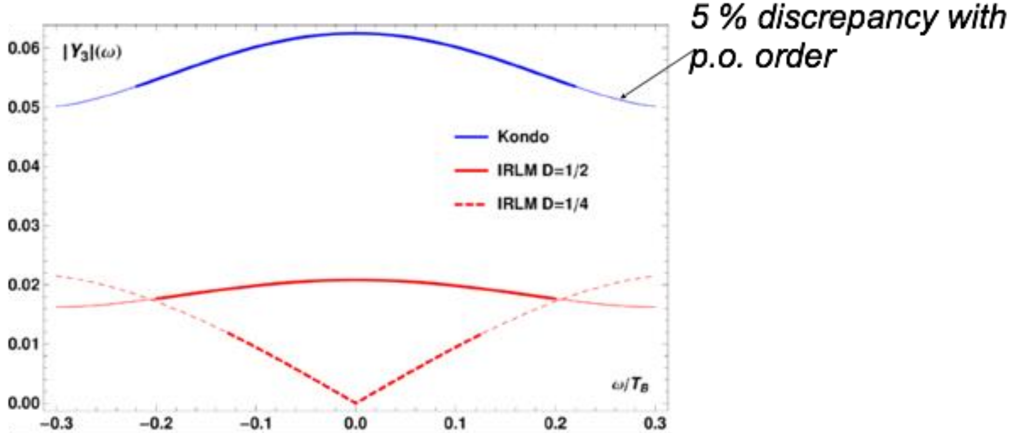


Higher harmonics can be captured ! $Y_a(V, \omega, T)$

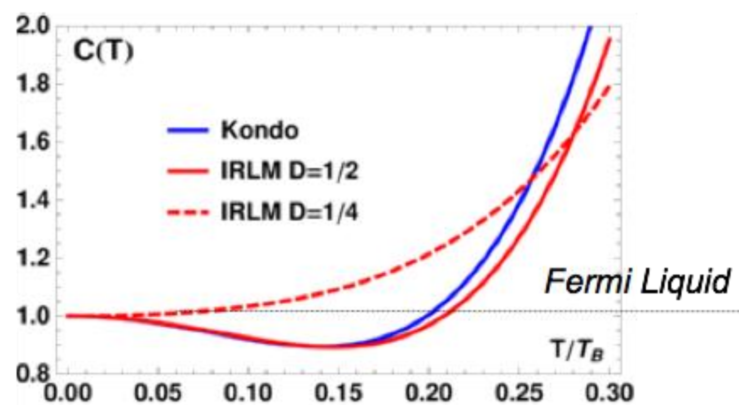
AC forcing

AC bias applied to the two baths

3rd harmonic :



Capacitance C(T) , imaginary part of $Y_1(T)$



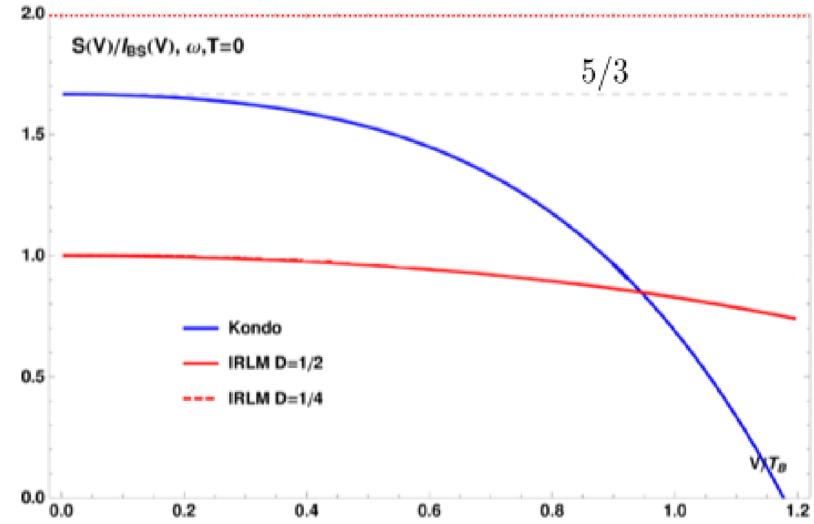
Noise

Noise power spectrum:

$$S(\omega) = \int dt\, e^{i\omega t}\left(\langle I(t)I(0)\rangle - \langle I\rangle^2\right)$$

Fano factor (« effective charge »)

$$F = S(0)/eI_{BS}$$



- At $V = 0$: $F = \dfrac{5}{3}$ *Sela et al. + Gogolin et al. (PRL 2006)*

- Finite voltage corrections $(C = \cos\theta)$:

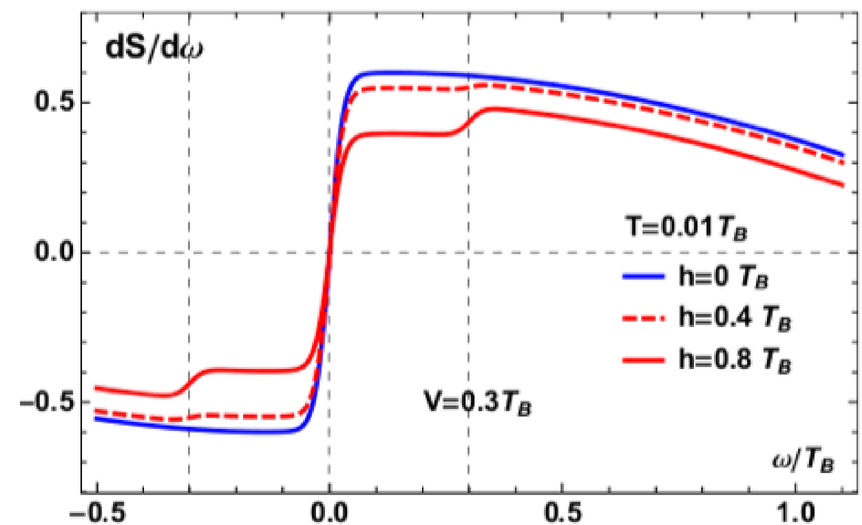
$$F = \frac{1}{3}\left(5 - 8C^2\right) + \frac{\left(C^2-1\right)\left(\pi\left(25C^2+153\right) - 18\sqrt{3}\right)V^2}{360\pi T_B^2}$$
$$+ \frac{\left(\left(513297 + 782875\sqrt{5} + 376866\sqrt{3}\pi + 37275\pi^2\right)C^2 - 42630\pi^2 - 313866\sqrt{3}\pi - 475625\sqrt{5} - 276372\right)V^4}{604800\pi^2 T_B^4}$$
$$+ \frac{\left(875\pi^2C^6 - 70\left(972 + 500\sqrt{5} - 882\sqrt{3}\pi - 163\pi^2\right)C^4\right)V^4}{604800\pi^2 T_B^4}$$

Noise

At T=0 : non-analycities

$$S^{(0)}(\omega, V, 0) = \frac{4\sin^2(\theta)}{\pi\left(W^2+4\right)^2}\left(\left(|V-\omega|+|V+\omega|\right)\left(4\cos^2(\theta)+W^2\right)+8|\omega|\sin^2(\theta)\right)$$
$$-\frac{W\sin^2(\theta)\cos(\theta)}{\pi\left(W^2+4\right)^2 T_B}\left(|V-\omega|\left(8V\cos(2\theta)+W^2(V-\omega)-4(V+\omega)\right)\right.$$
$$\left.+|V+\omega|\left(8V\cos(2\theta)+W^2(V+\omega)-4(V-\omega)\right)+32V|\omega|\sin^2(\theta)\right)$$

Derivative of the noise at low temperature :
Non analycities at T=0 are rounded by temperature



Conclusions

✓ Kondo physics is rich and … not exhausted
(both for experiments and theory)

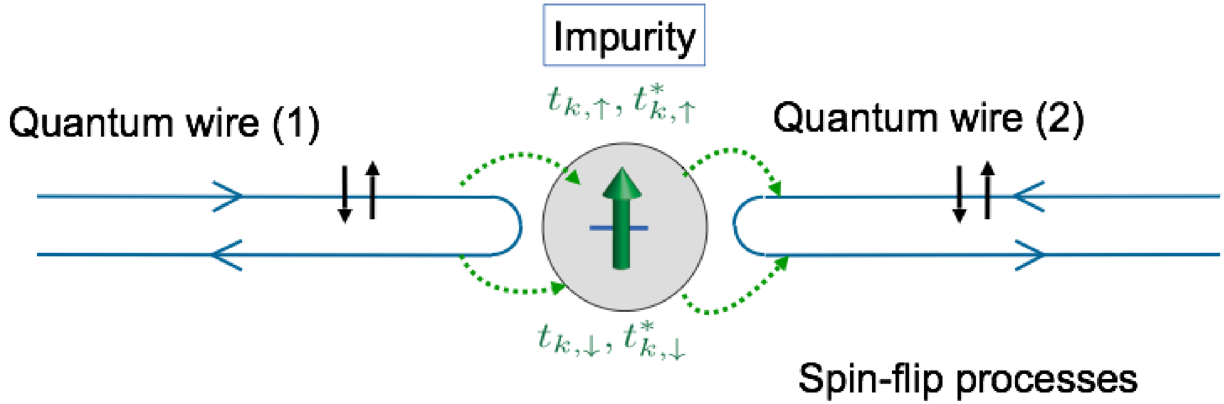
✓ Kondo effect revealed by high-frequency noise?

✓ The super Fermi liquid approach yields exact results for generic perturbations: T, frequency, magnetic field, particle-hole asymmetry

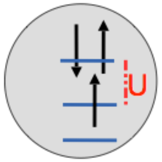
✓ Yet charge fluctuations on the dot are NOT described !

Thank you!

The s=½ Kondo model



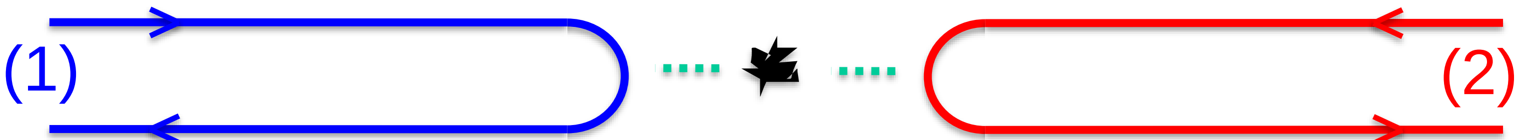
Kondo stems from Anderson model



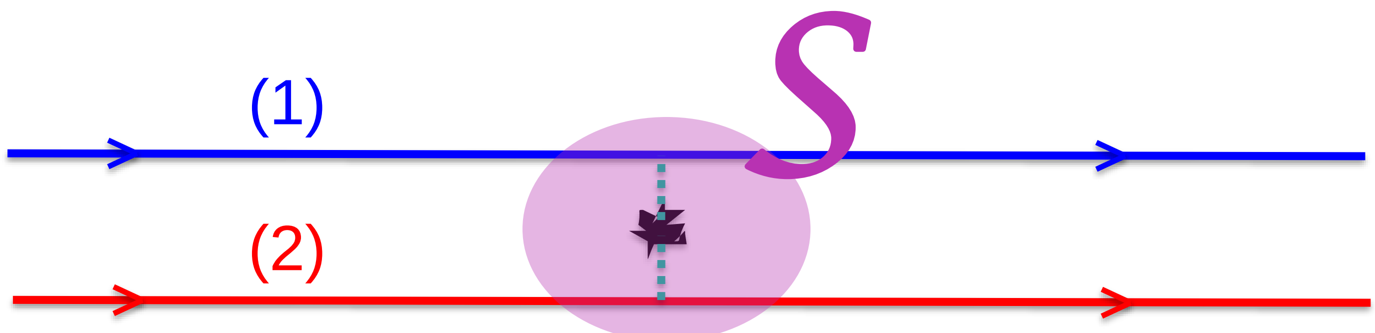
Parameters: - bare exchange coupling J
- anisotropy of couplings to the wires θ

Modeling the baths

- Modes that couple to the impurity are 1D (conduction channel)
- Linearize the spectrum



$$H_0[\Psi] = \sum_{a=1,2} -iv_F \int_{-\infty}^{0} dx \left[\Psi^\dagger_{aR}(x)\partial_x\Psi_{aR}(x) - \Psi^\dagger_{aL}(x)\partial_x\Psi_{aL}(x)\right]$$

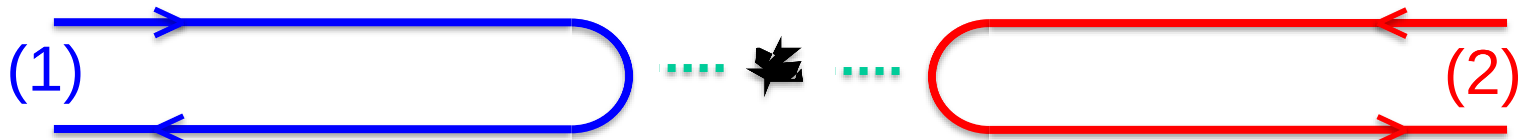


$$H_0[\Psi] = \sum_{a=1,2} -iv_F \int_{-\infty}^{\infty} dx \, \Psi^\dagger_{a}(x)\partial_x\Psi_{a}(x)$$

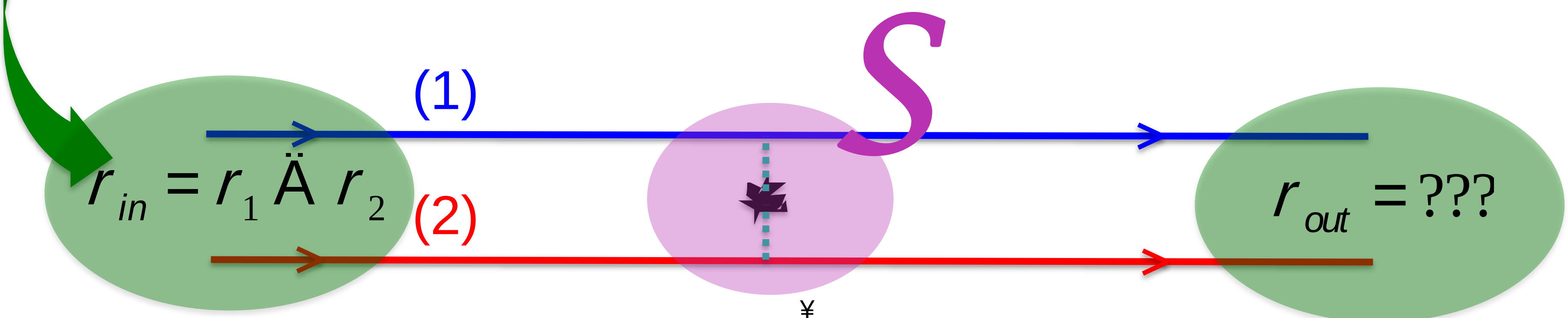
Chiral theory involving only right-moving fields: **scattering problem**

Modeling the baths

- Modes that couple to the impurity are 1D (conduction channel)
- Linearize the spectrum



$$H_0[\Psi] = \sum_{a=1,2} -iv_F \int_{-\infty}^{0} dx \left[\Psi^\dagger_{aR}(x)\partial_x\Psi_{aR}(x) - \Psi^\dagger_{aL}(x)\partial_x\Psi_{aL}(x)\right]$$

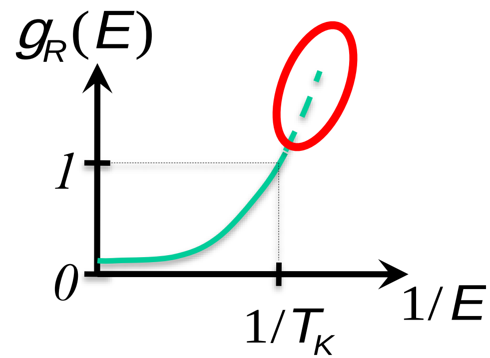


$$H_0[\Psi] = \sum_{a=1,2} -iv_F \int_{-\infty}^{\infty} dx \; \Psi^\dagger_{a}(x)\partial_x\Psi_{a}(x)$$

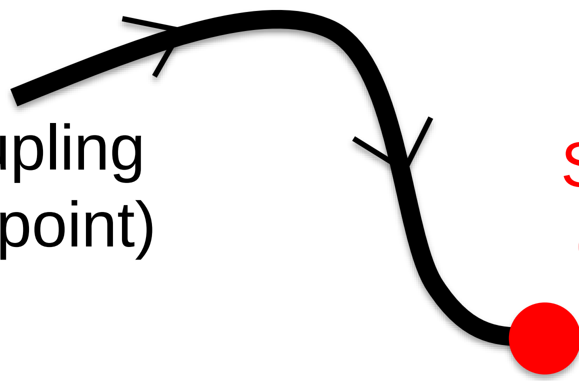
Chiral theory involving only right-moving fields: **scattering problem**

Strong coupling fixed point

- Perturbation is **relevant**
- Strong coupling fixed point described by BCFT



Weak coupling
(UV fixed point)



Strong coupling
(IR fixed point)

- Step 1: Out-of-equilibrium SC fixed point $(T_K = ¥)$

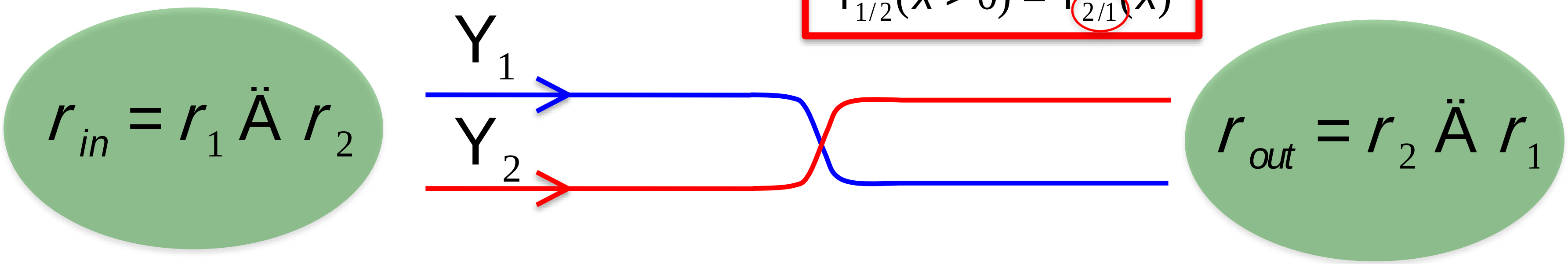
Strong coupling fixed point

- Boundary conditions: $F(x=0^-) = B \times F(x=0^+)$
- **"Transparent fields"**: $\tilde{F}(x<0) = F(x)$; $\tilde{F}(x>0) = B \times F(x)$

  They don't see the impurity!

BC for fermions: $\pi/2$ phase shift

$$\tilde{Y}_{1/2}(x<0) = Y_{1/2}(x)$$
$$\tilde{Y}_{1/2}(x>0) = Y_{2/1}(x)$$



- Forcing out-of-equilibrium easily represented!
- Amounts to a gauge transformation $\mathcal{U}_{N.Equ}(z)$ for the transparent fields

$$r_{in}\ µ\ e^{-\frac{H_0[Y_1] - m_1 Q_1}{T_1}}\ Ä\ e^{-\frac{H_0[Y_2] - m_2 Q_2}{T_2}}$$

$$\langle I \rangle = \left(2e^2/h\right)\left(m_1(t) - m_2(t)\right)$$

Recover the linear regime for the charge current

"Doping" a CFT

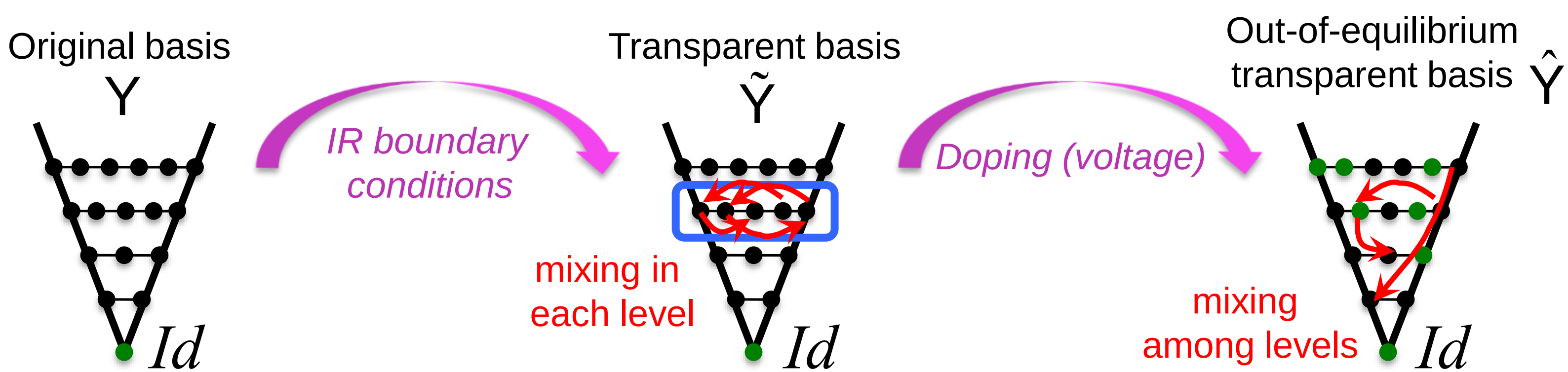
- The strong coupling fixed point has conformal symmetry ; transparent fields $\tilde{F}$ are holomorphic (functions of $z = i(t-x)$)

- The forcing out of equilibrium can be absorbed by a gauge transformation (« doping ») $\hat{Y}(z) = \mathcal{U}_{N.Equ}(z) \times \tilde{Y}(z)$

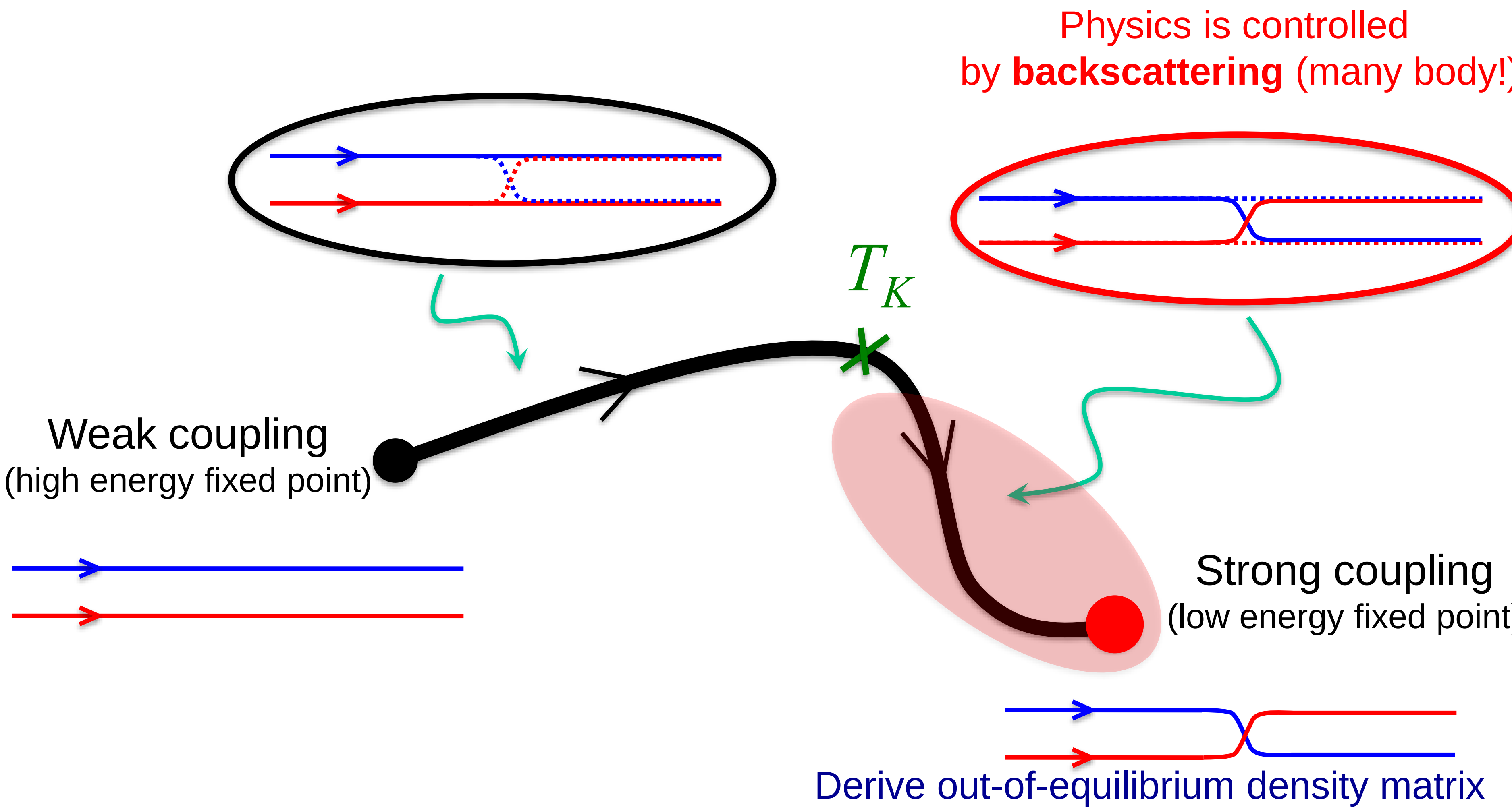
$$\mathcal{U}_{N.Equ}(z) = e^{i\grave{o}_z dw\, X_a(w)\, \tilde{Q}_a(w)} \quad ; \quad X_a(z = i(t - x)) = \grave{O}_0^{t-x} dt'\, m_a(t')$$

$$\left\langle A_1(x_1,t_1)\, A_2(x_2,t_2)\ldots \right\rangle_{N.Equ} = \left\langle \hat{A}_1(x_1,t_1)\, \hat{A}_2(x_2,t_2)\ldots \right\rangle_{Equ.} \quad ; \quad \hat{A} = \mathcal{U}_{N.Equ} \times A$$

- It's a deformation of the CFT (no geometrical interpretation unlike finite temperature CFT)



From weak to strong coupling



Dual theory

$$H = H_0^{SC} + H_B^{SC} \qquad H_B^{SC} = \sum_{n=1}^{\infty} \frac{g_{2n}}{(T_K)^{2n-1}} \hat{O}_{2n}(x=0)$$

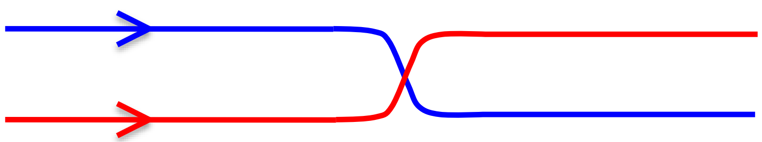
- The operators O_{2n} are the (infinitely many) conserved quantities stemming from integrability.
- The couplings g_n are pure numbers, **fixed** by integrability. (Lesage, Saleur 1999)
- Fermi liquid: the least irrelevant operator is $O_2 = T$, an energy momentum tensor.
- Higher order processes have integer dimensions = 4,6,8,…

Backscattering transfers integer charges (electrons)

"SUPER FERMI LIQUID"

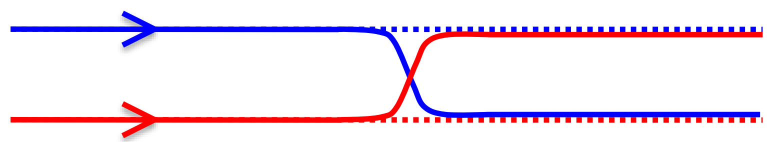
Keldysh expansion

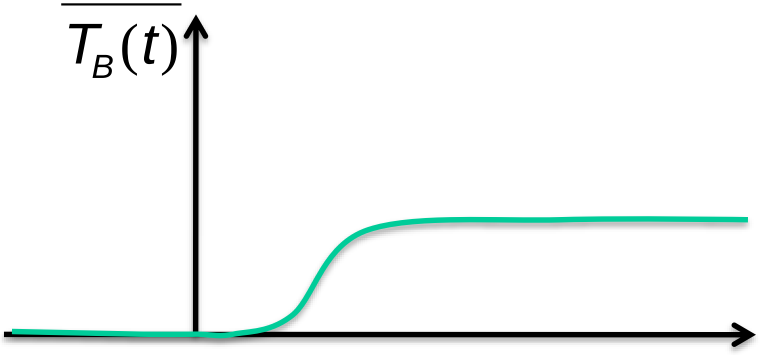
- Start at time $t = -\infty$ at the SC fixed point $(T_K = \infty)$



$$\rho(-\infty) = \rho_{SC} = e^{-H_0^{SC}/k_B T}$$

- Switch on backscattering at time t=0



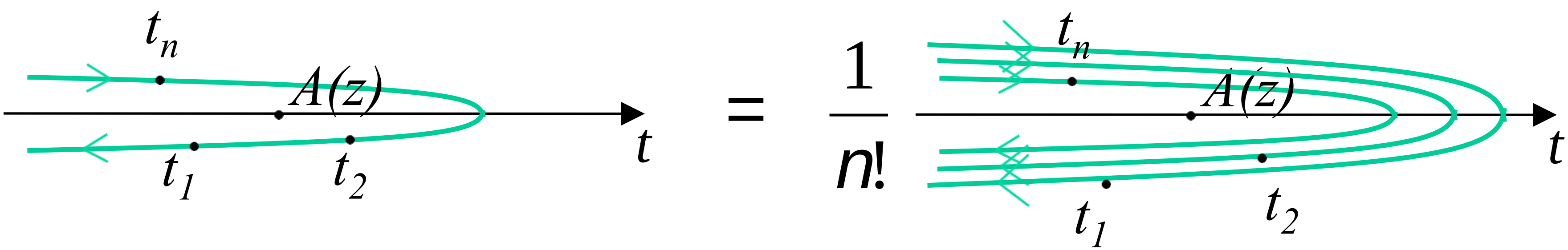


$$\rho(t) = U(t)\rho_{SC}U(t)^{\dagger}$$

$$U(t) = \mathcal{P}_K e^{-i g \int_{-\infty}^{t} dt' H_B^{SC}(t')}$$

Effective operators

In a **super Fermi liquid**, the Keldysh expansion bears a simple form:



Each (local) operator can be replaced by an *effective* operator:

Complete many-body scattering

$$A^{eff}(z) = \mathcal{U}_{BS}(z) \cdot A(z)$$

$$= e^{-i\int_z dt\, H_B(t)} \cdot A(z) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_z dt_1 \ldots \int_z dt_n$$