UNIVERSITÉ DE GENÈVE

Quantum transport in many-body localized systems

Phys. Rev. B 94, 201112(R) (2016)

Michele Filippone

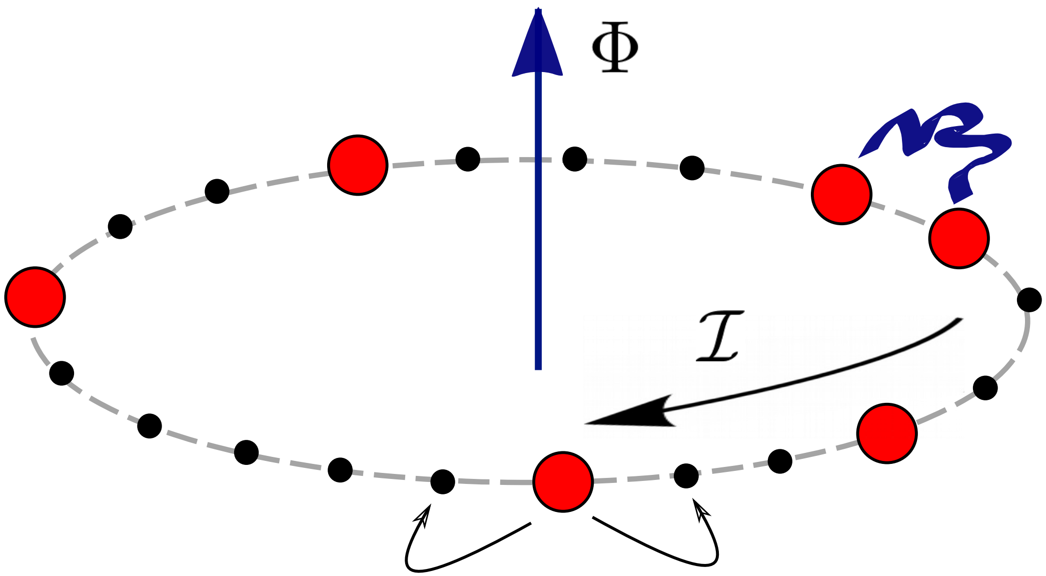
Dahlem Center for Complex Quantum Systems

Freie Universität Berlin

Alexander von Humboldt Stiftung / Foundation

Message

Persistent currents are induced by a magnetic flux Φ in metallic rings ...



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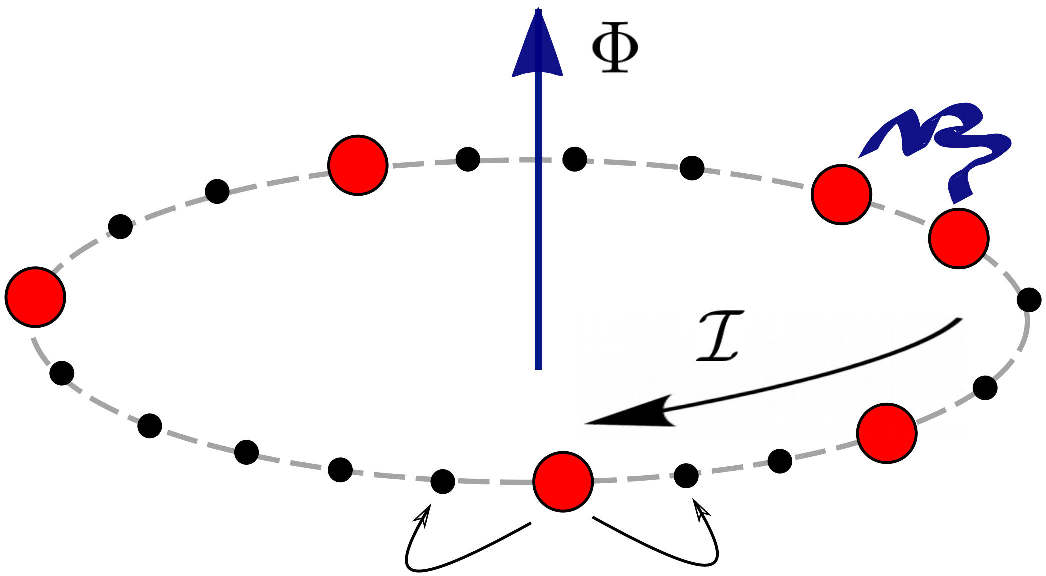

... and they actually are a spectral property

$$\mathcal{I}_n = -\frac{1}{2\pi}\frac{\partial E_n}{\partial \Phi}$$

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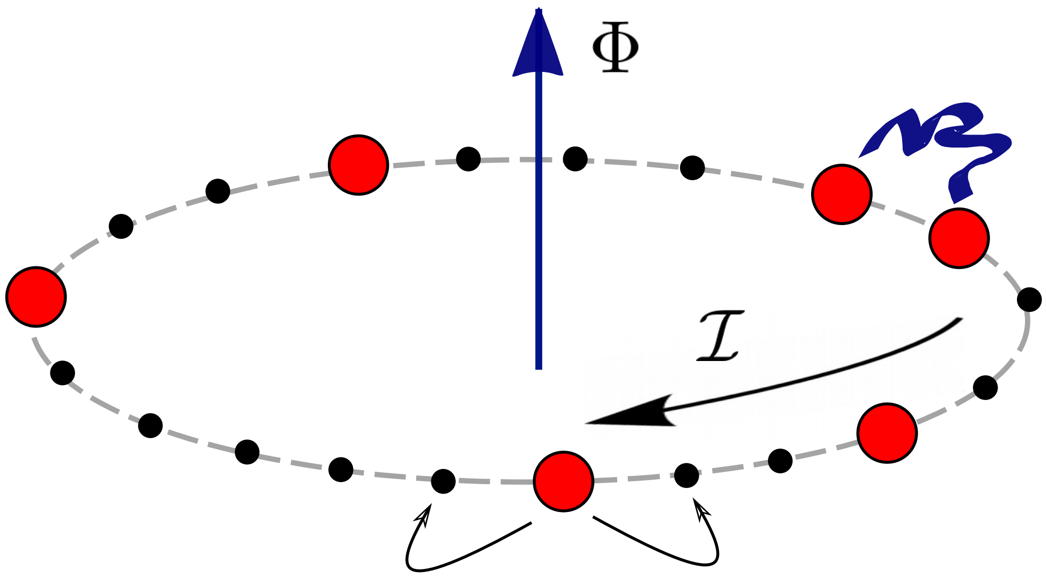

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DRUDE WEIGHT DISTRIBUTION :
a
probe of Many-body localization based on transport properties

$$\mathcal{D}_n = \frac{L}{2}\left.\frac{\partial^2 E_n}{\partial \Phi^2}\right|_{\phi=0}$$

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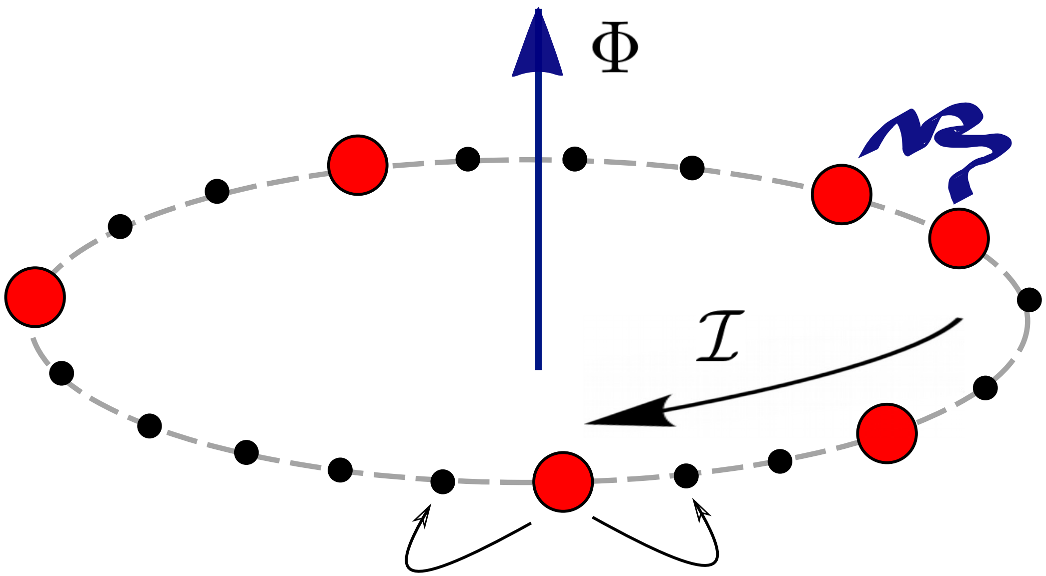

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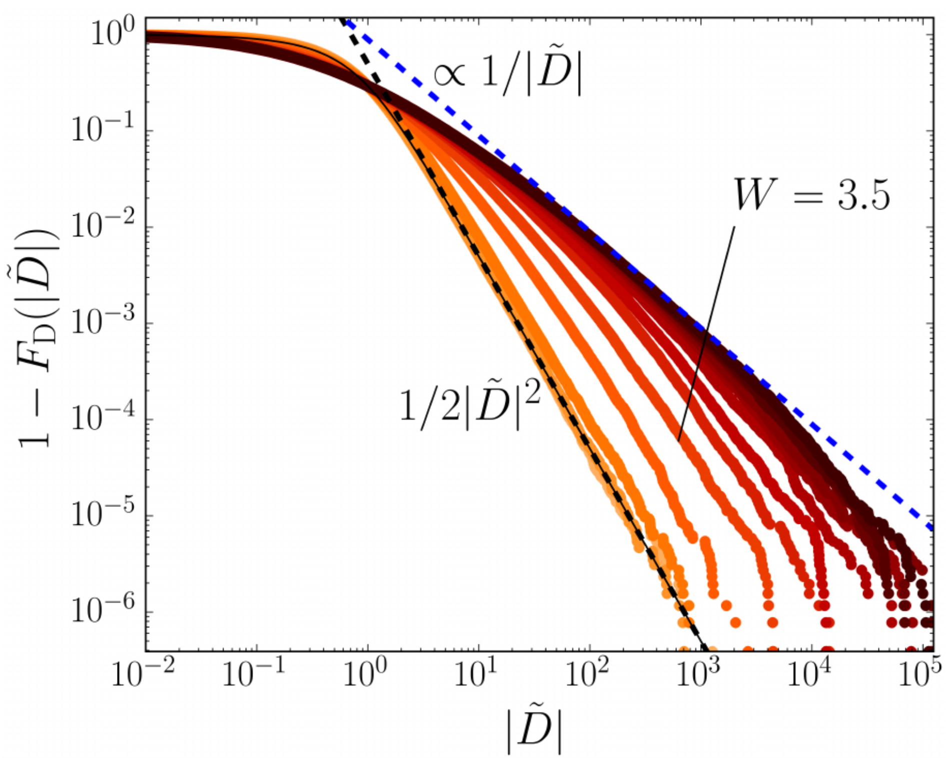

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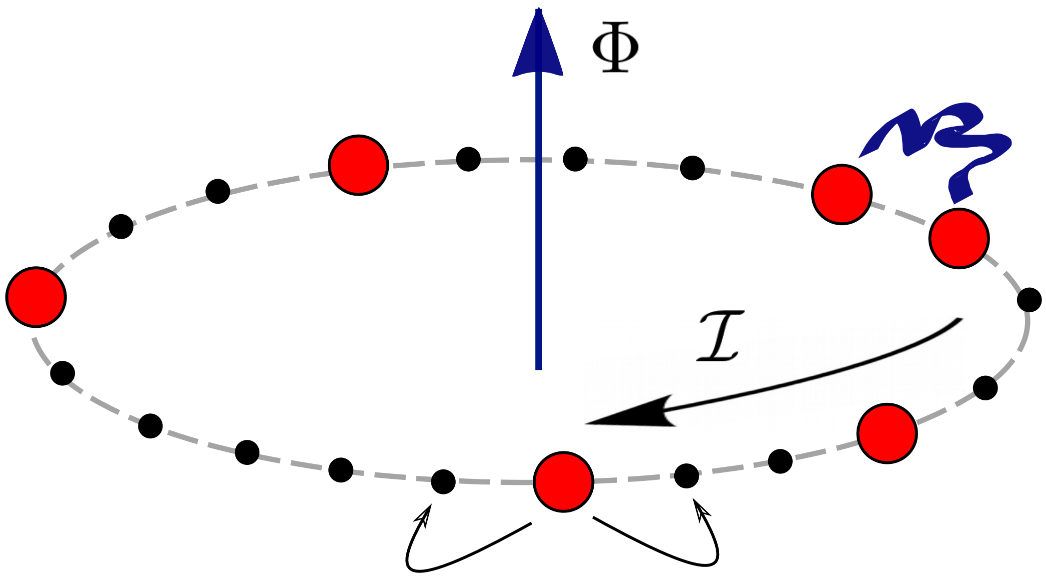

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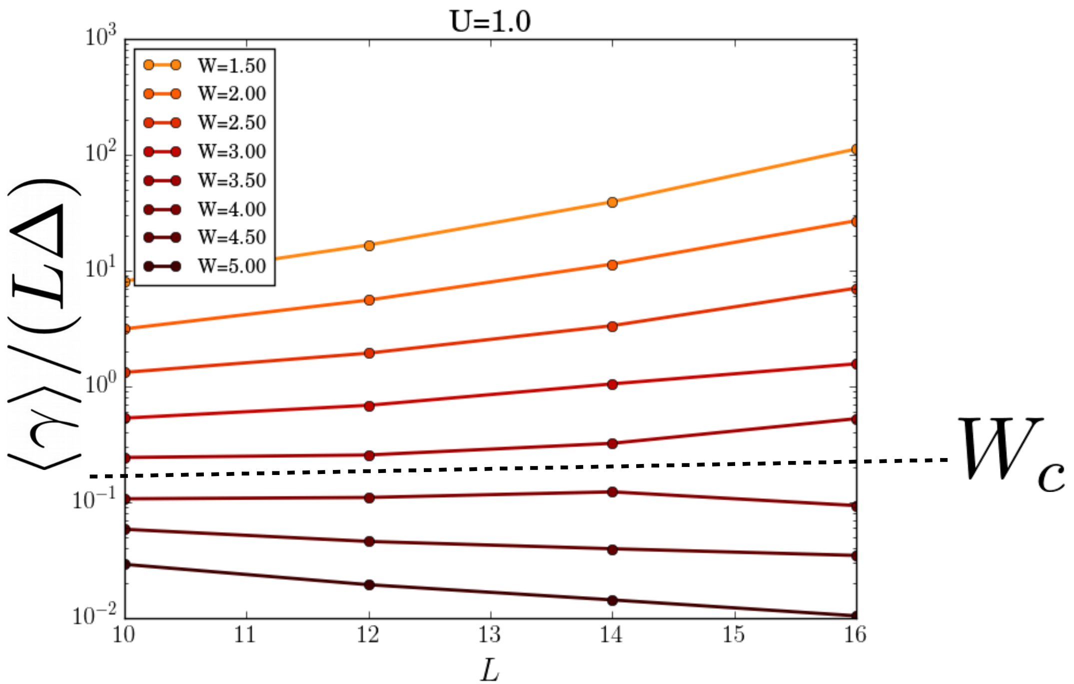


Alexander von Humboldt
Stiftung/Foundation

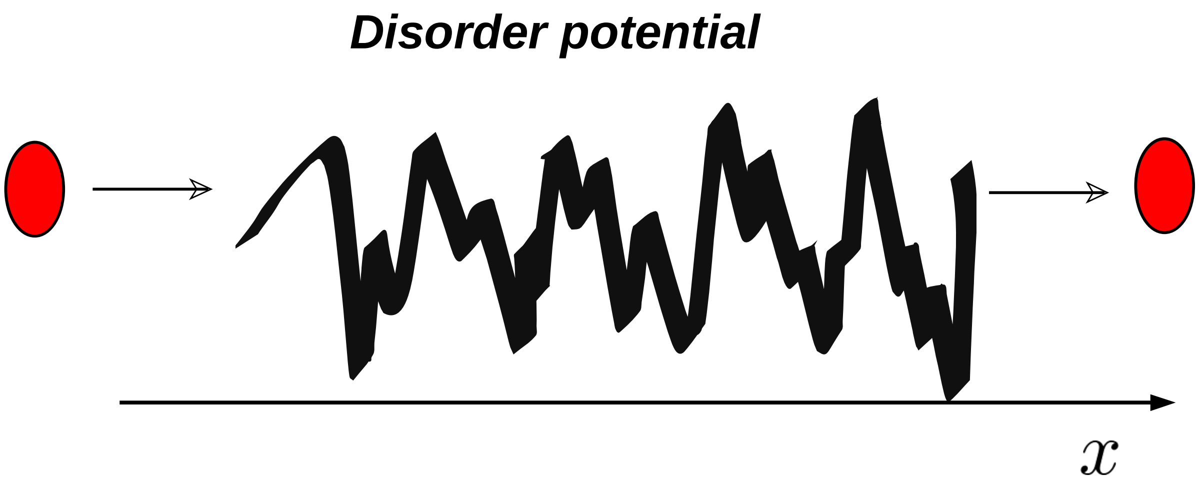
Jens Eisert

Piet Brouwer

Felix von Oppen

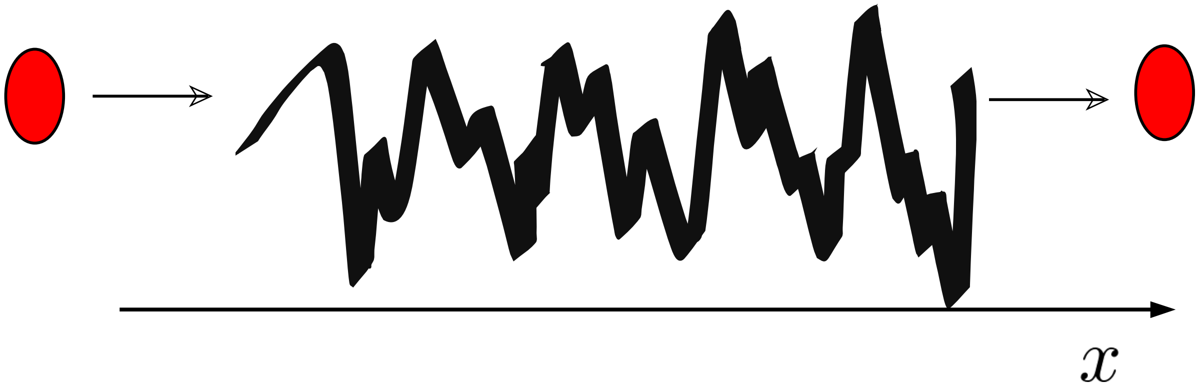
Freie Universität Berlin

Anderson Localization



Anderson Localization

Disorder potential



Exponential suppression of the transmission probability

$$T(\varepsilon) \longrightarrow 0$$

Anderson Localization

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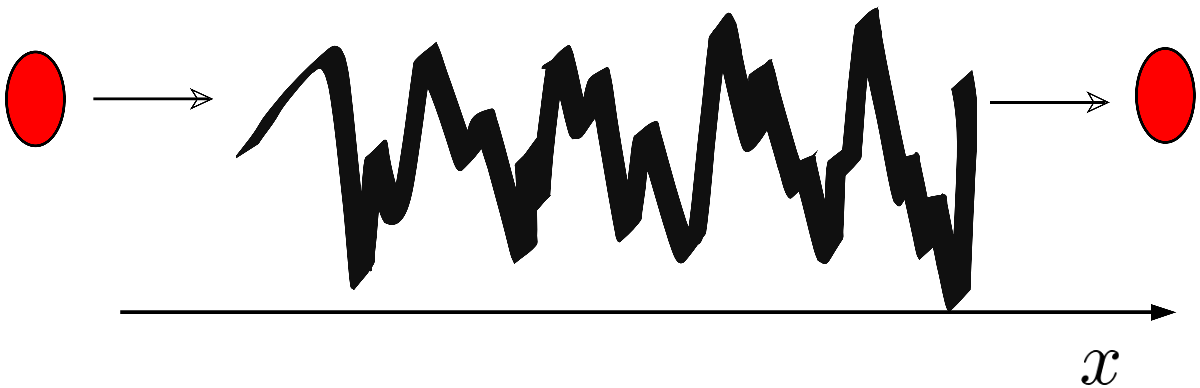

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Landauer Formula :

$$G = \frac{e^2}{h} T$$

Quantum interference leads to particle localization...

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A. Lagendijk, B. van Tiggelen, and D. S. Wiersma, **Phys. Today** 62, 24 (2009); E. Abrahams, 50 years of Anderson Localization, Vol. 24 (World Scientific, 2010).
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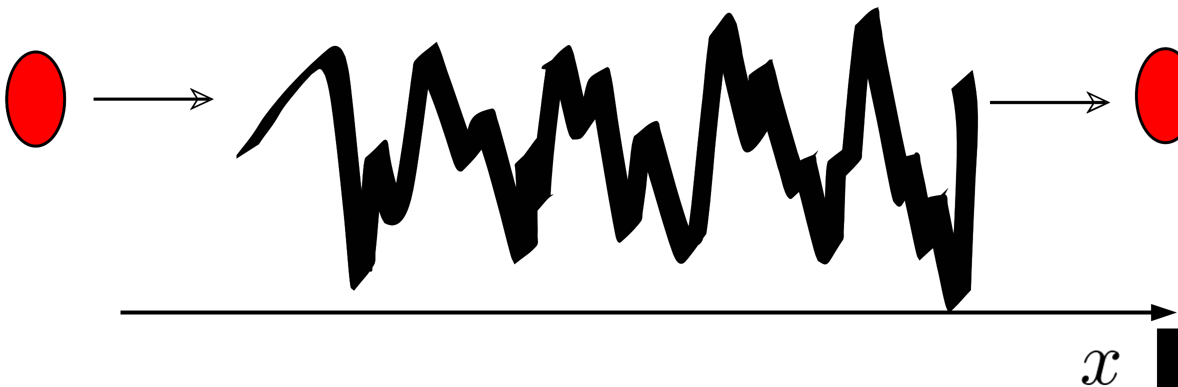

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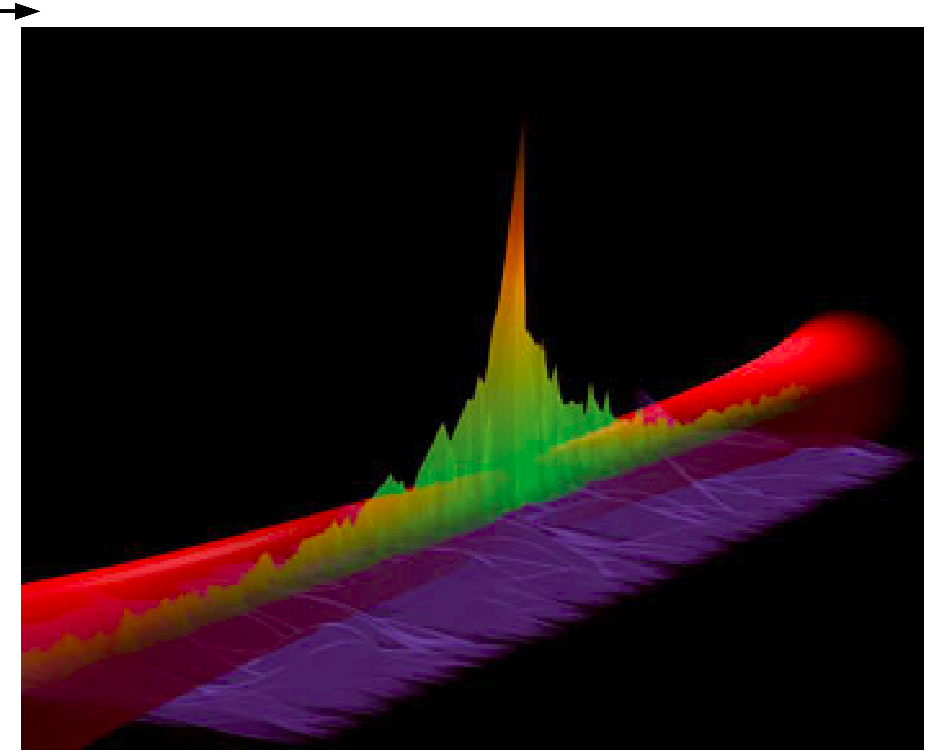
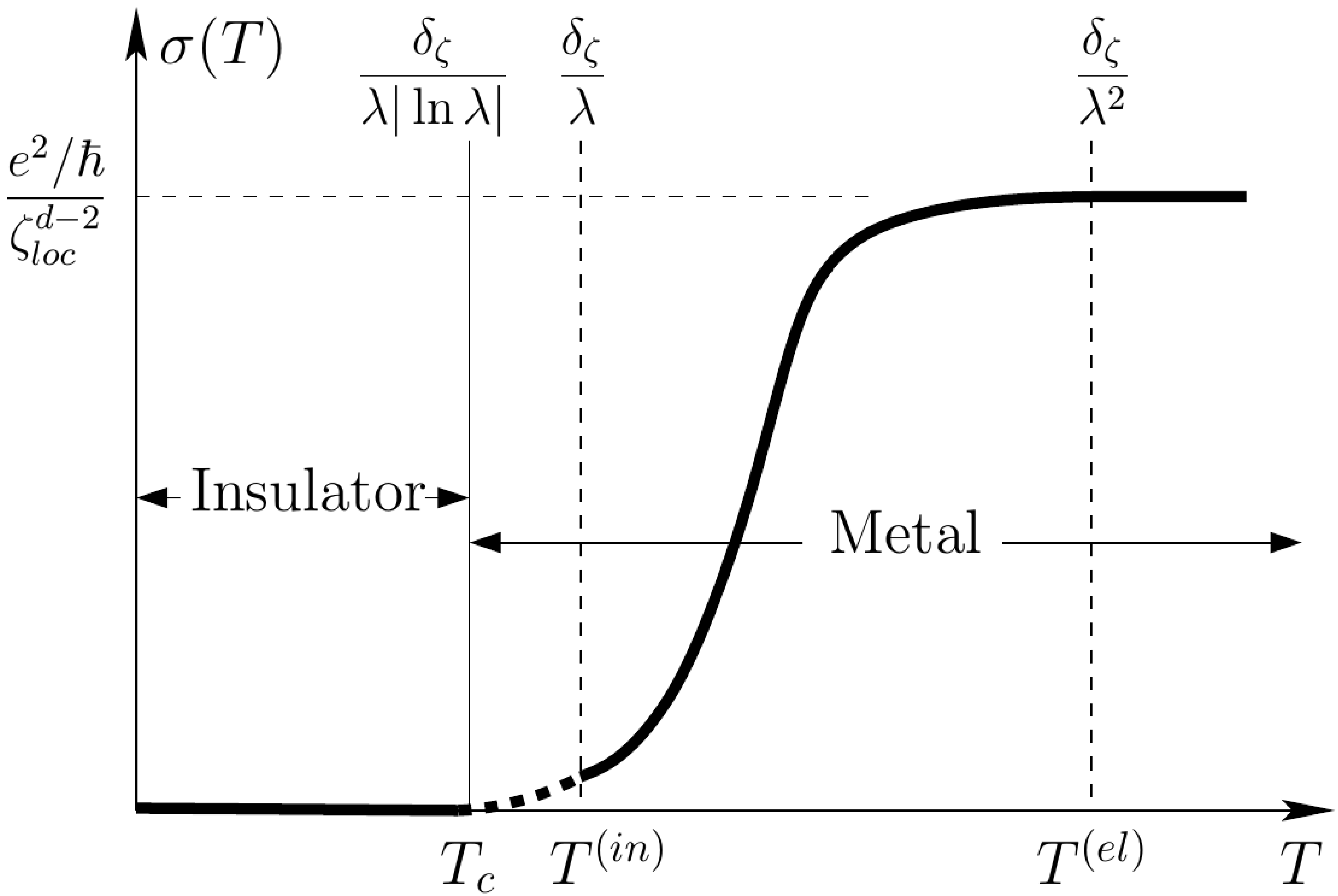


J.Billy *et al., `Direct observation of Anderson localization of matter waves in a controlled disorder',* ***Nature 453,*** *891 (2008)*

Many-Body Localization



D. Basko, I. Aleiner, and B. Altshuler, **Annals of physics 321**, 1126 (2006)
D. Basko, L. Aleiner, and B. Altshuler, **Problems of Condensed Matter Physics , 50** (2006)

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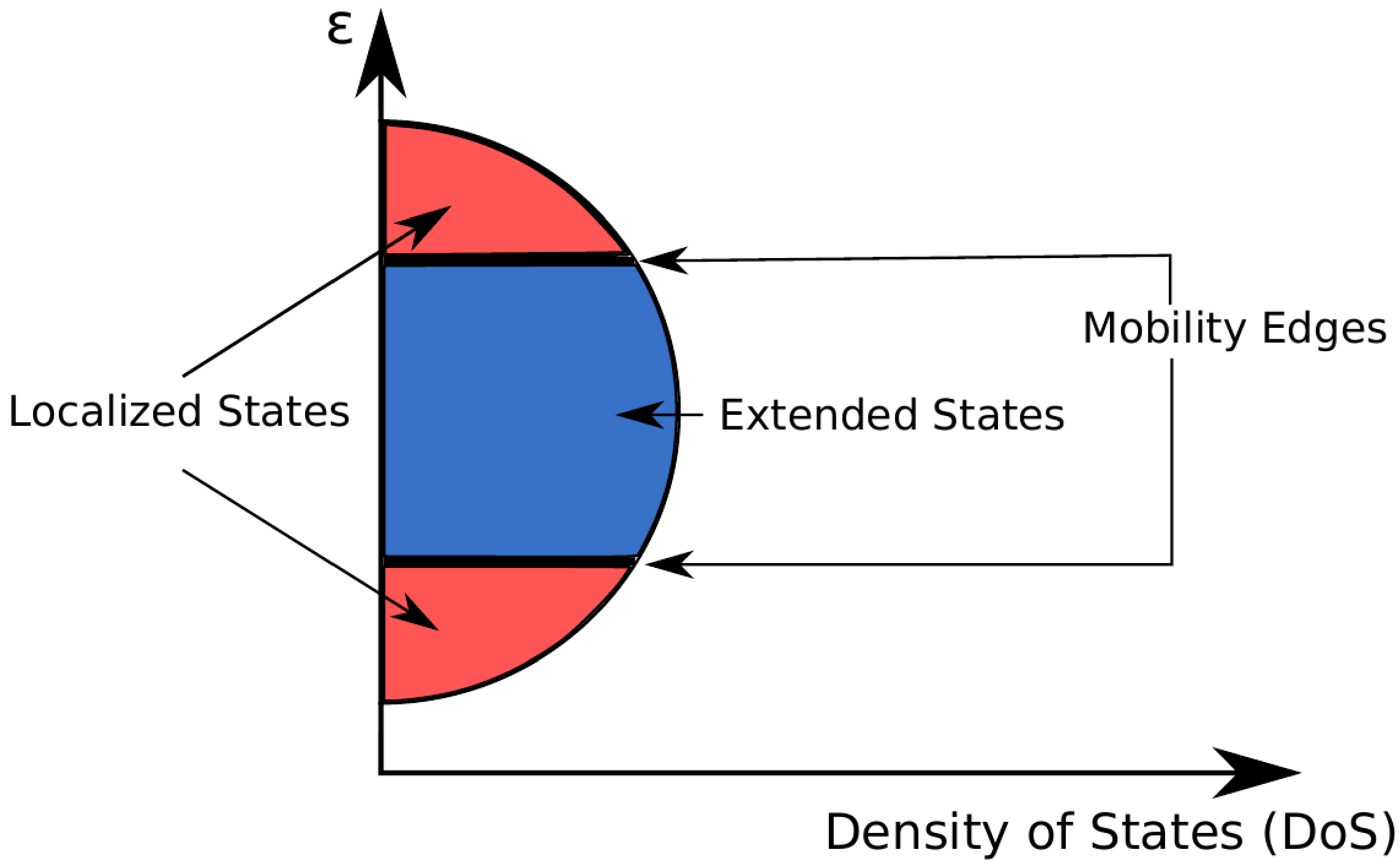
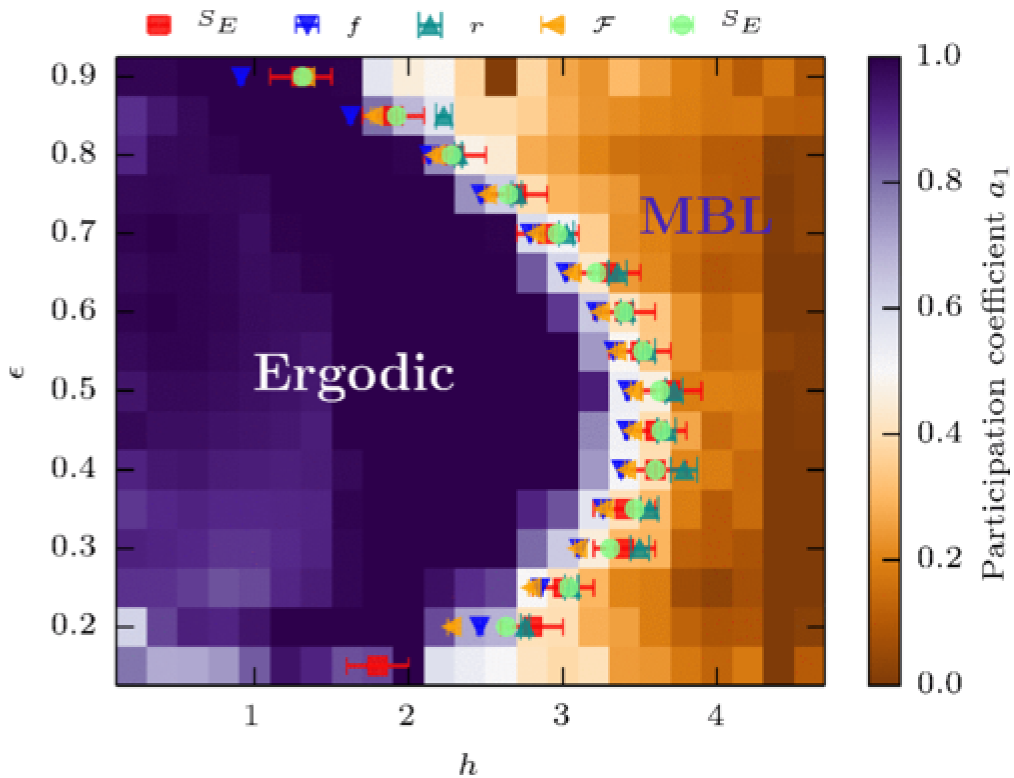


Many-Body Localization

Mobility Edge



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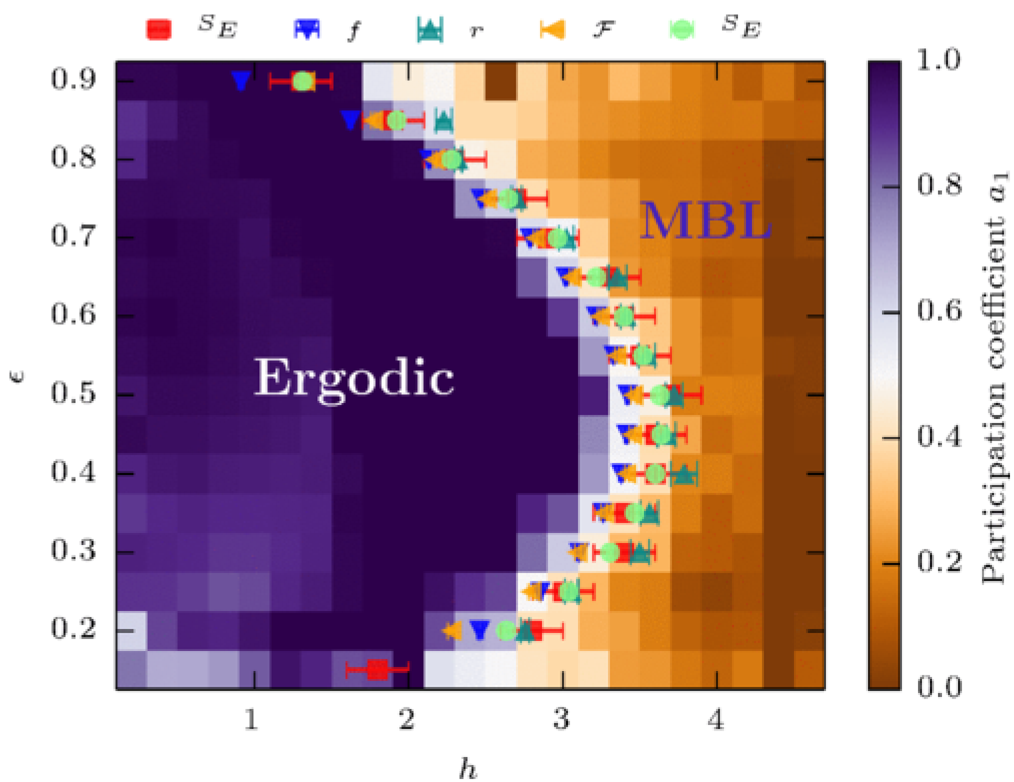

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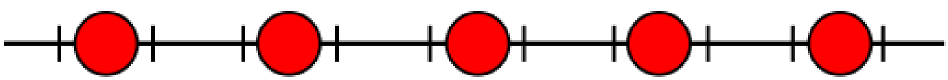
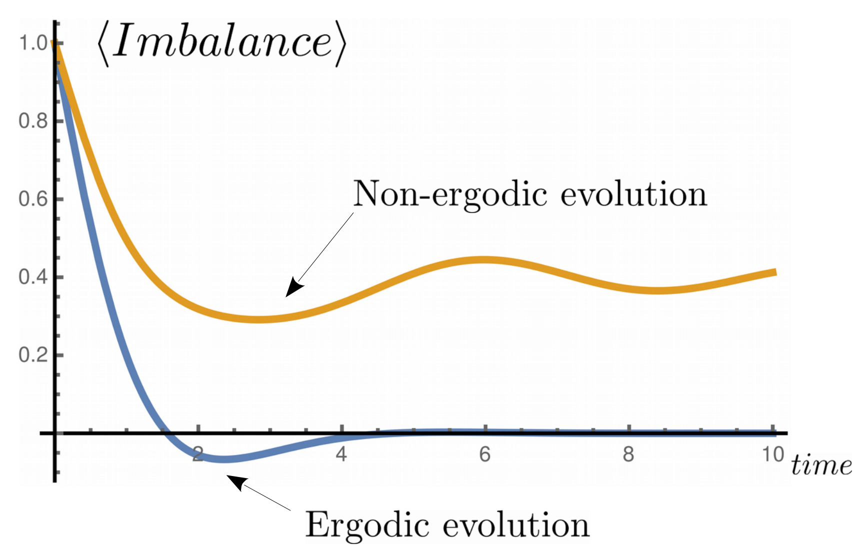
Initial state





Experiments

M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, **Science 349**, 842 (2015)
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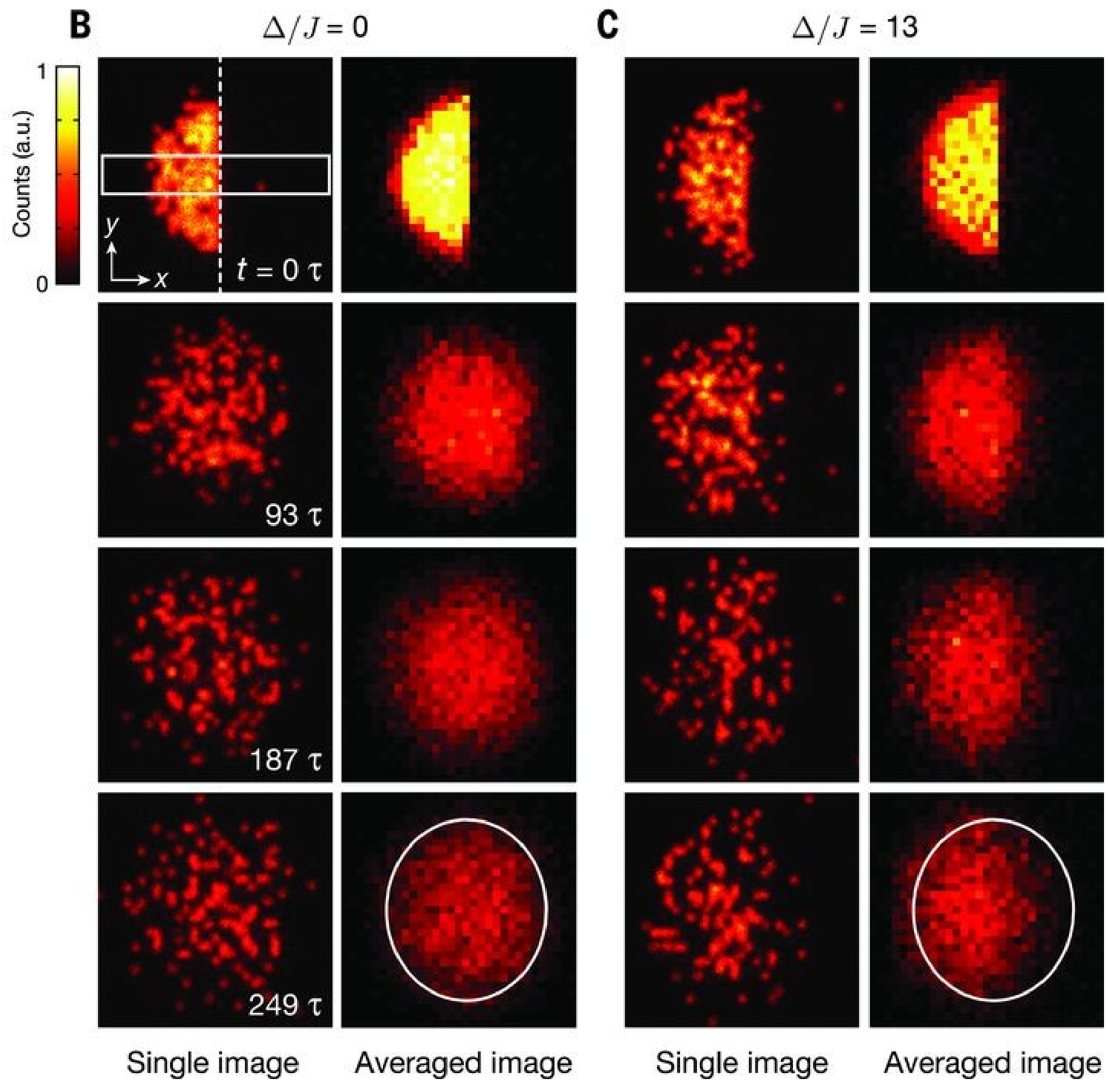

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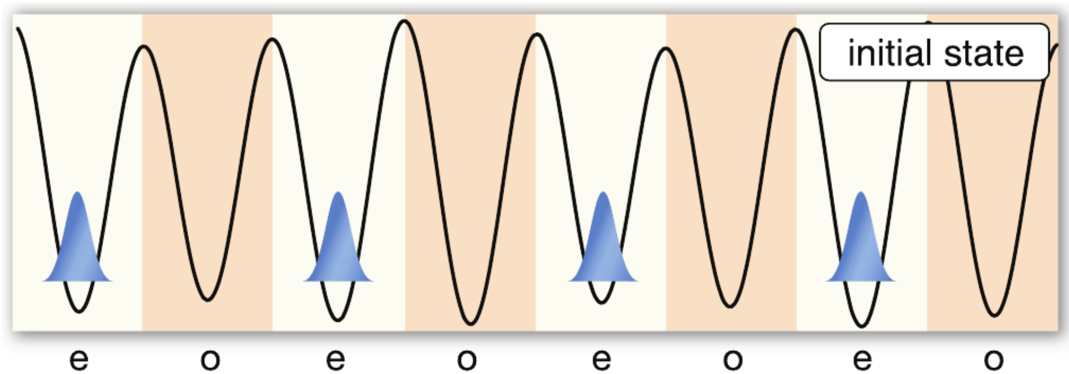
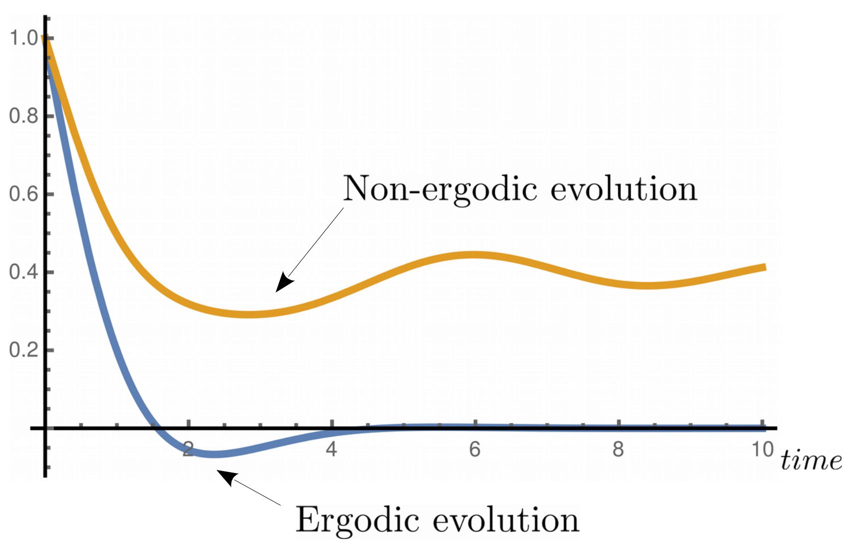


Some generic evolution



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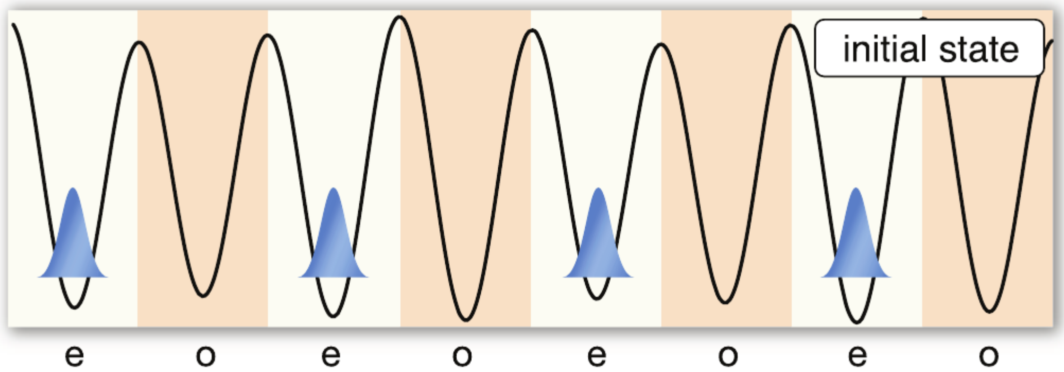

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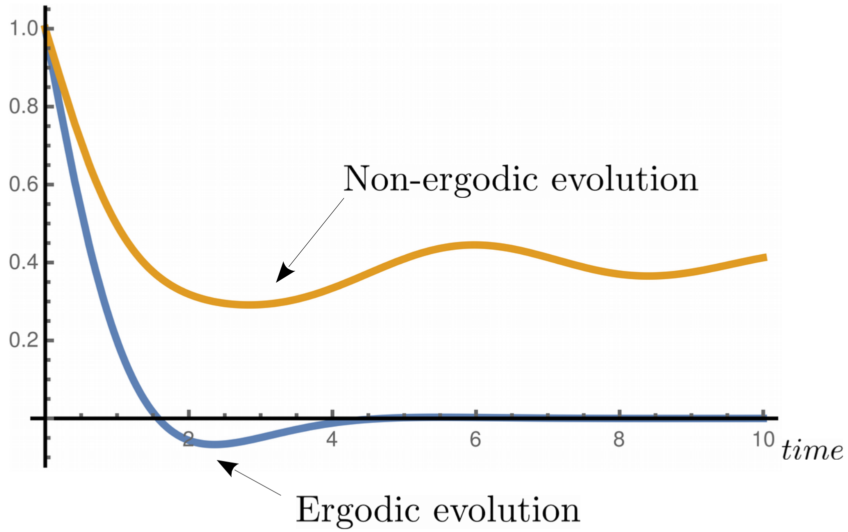

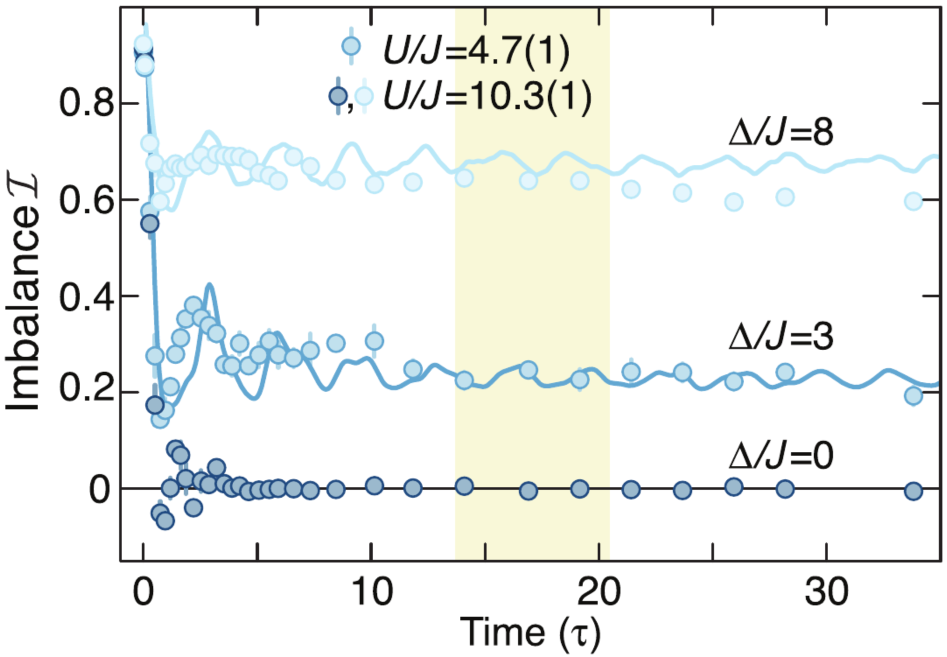

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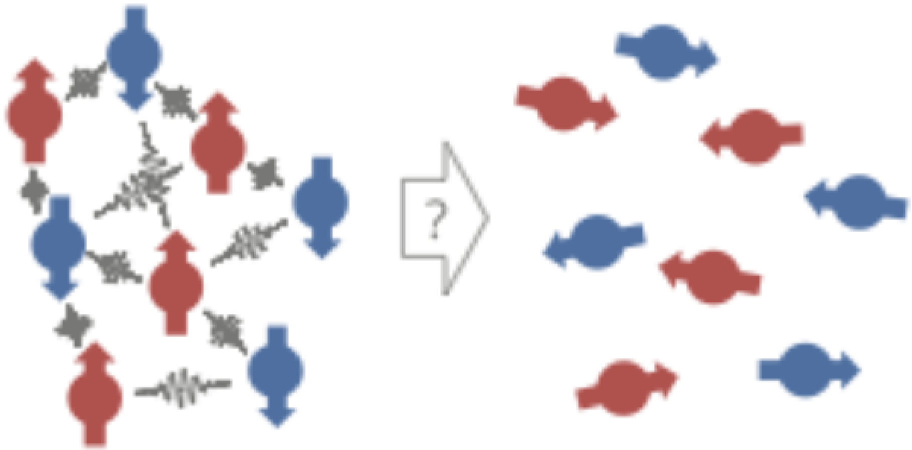
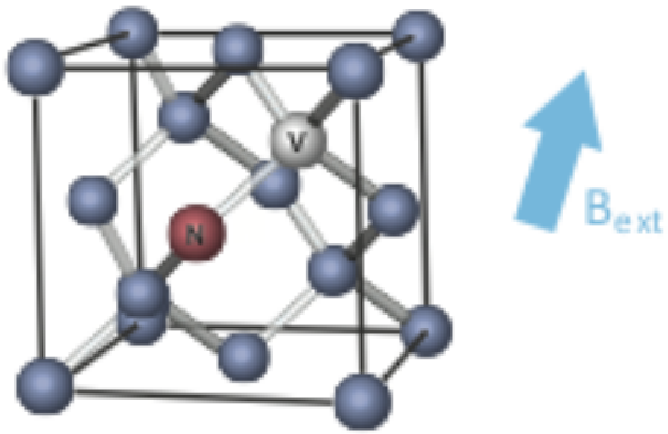
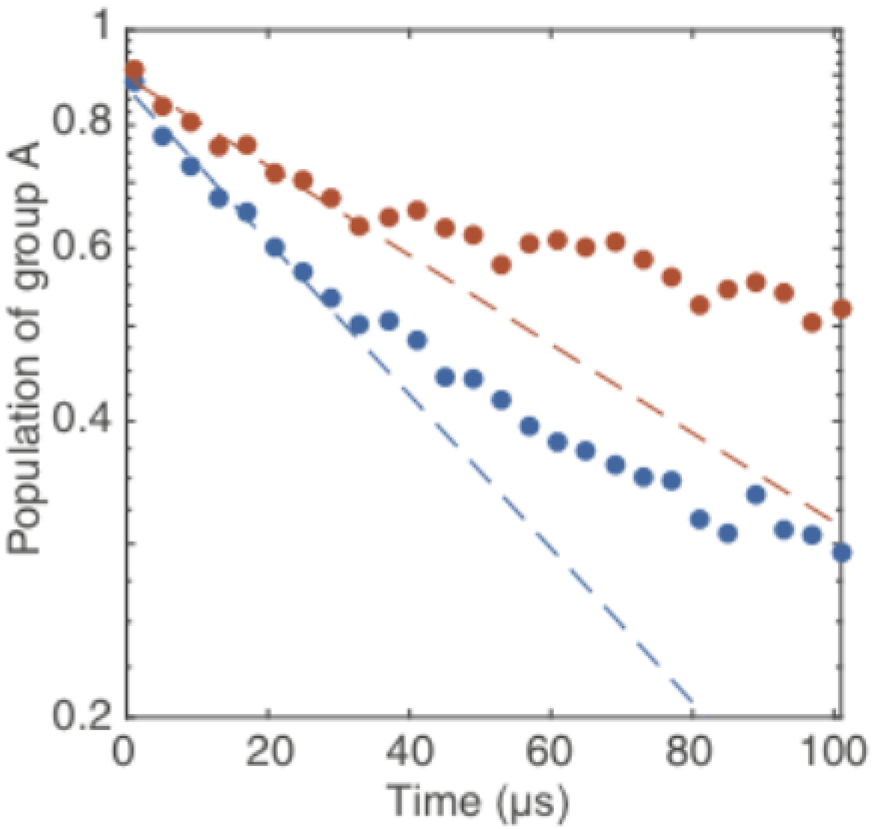
Experiments

Many-Body Localization with Nitrogen Vacancies (NV) in diamonds







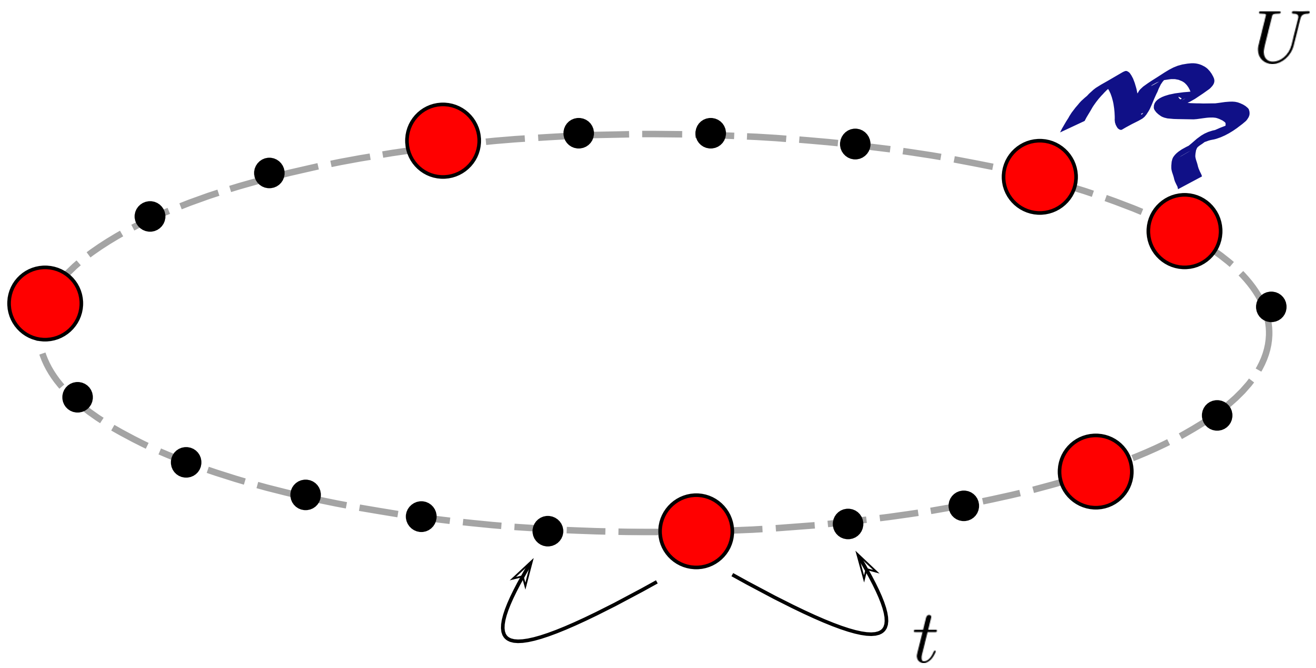
N. Y. Yao, C. R. Laumann, S. Gopalakrishnan, M. Knap, M. Müller, E. A. Demler, and M. D. Lukin, **Phys. Rev. Lett. 113**, 243002 (2014)

Georg Kucsko, Soonwon Choi, Joonhee Choi, Peter C. Maurer, Hitoshi Sumiya, Shinobu Onoda, Junich Isoya, Fedor Jelezko, Eugene Demler, Norman Y. Yao, Mikhail D. Lukin, **arXiv:1609.08216**

What about transport ??

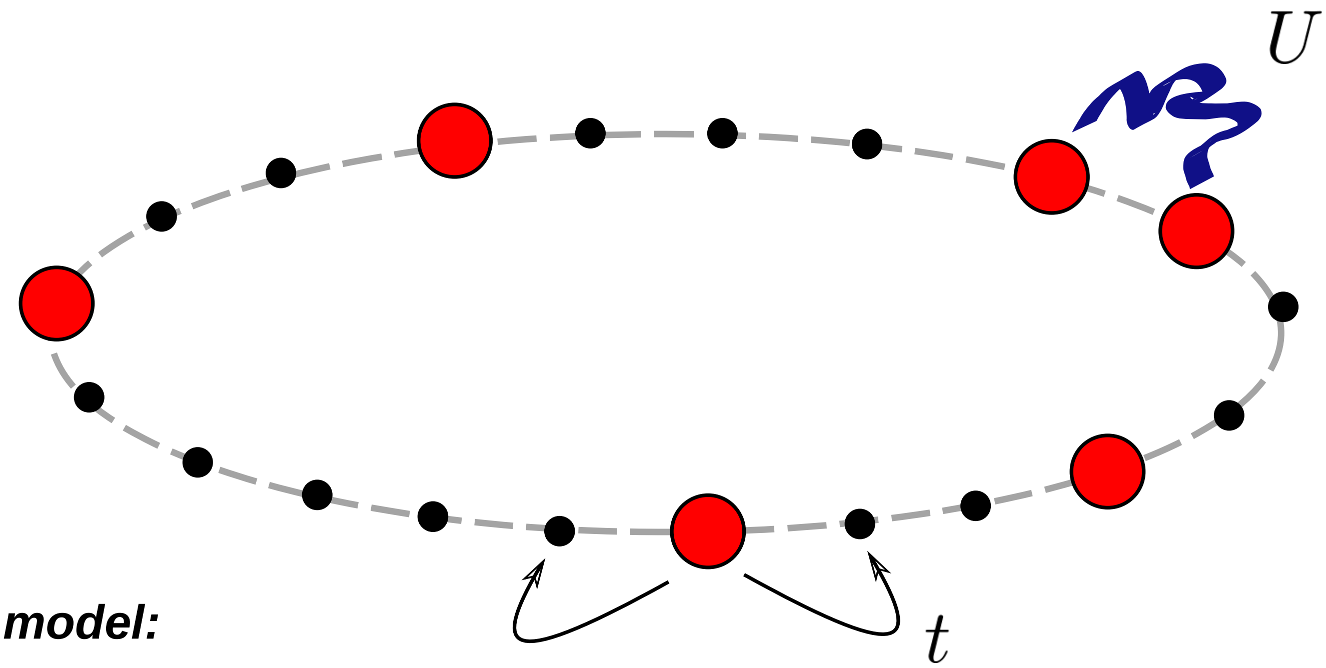
(persistent currents)

Persistent currents



M. Büttiker, Y. Imry and R. Landauer **Phys. Lett. A 96, 365** (1983)

Persistent currents



Hubbard model:

$$\mathcal{H} = -\frac{t}{2}\sum_{j=1}^{L}\left[c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j\right] + U\sum_{j=1}^{L} n_j n_{j+1}$$

Periodic boundary conditions : $c_j = c_{j+L}$

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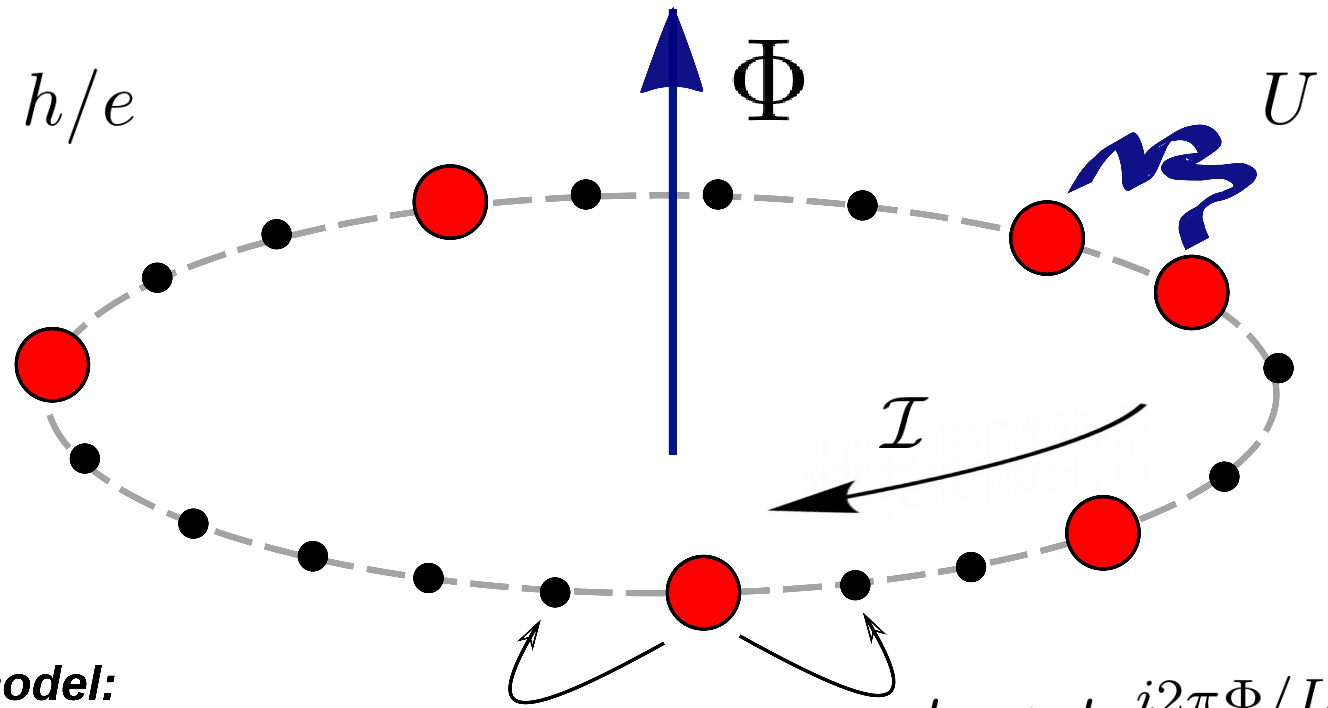
Persistent currents

Flux quantum

$$[\Phi] = \Phi_0 = h/e$$



Hubbard model:

$$t \rightarrow te^{i2\pi\Phi/L}$$

`Peierls phase' factor

$$\mathcal{H} = -\frac{t}{2}\sum_{j=1}^{L}\left[e^{i2\pi\Phi/L}c_j^{\dagger}c_{j+1} + e^{-i2\pi\Phi/L}c_{j+1}^{\dagger}c_j\right] + U\sum_{j=1}^{L}n_j n_{j+1}$$

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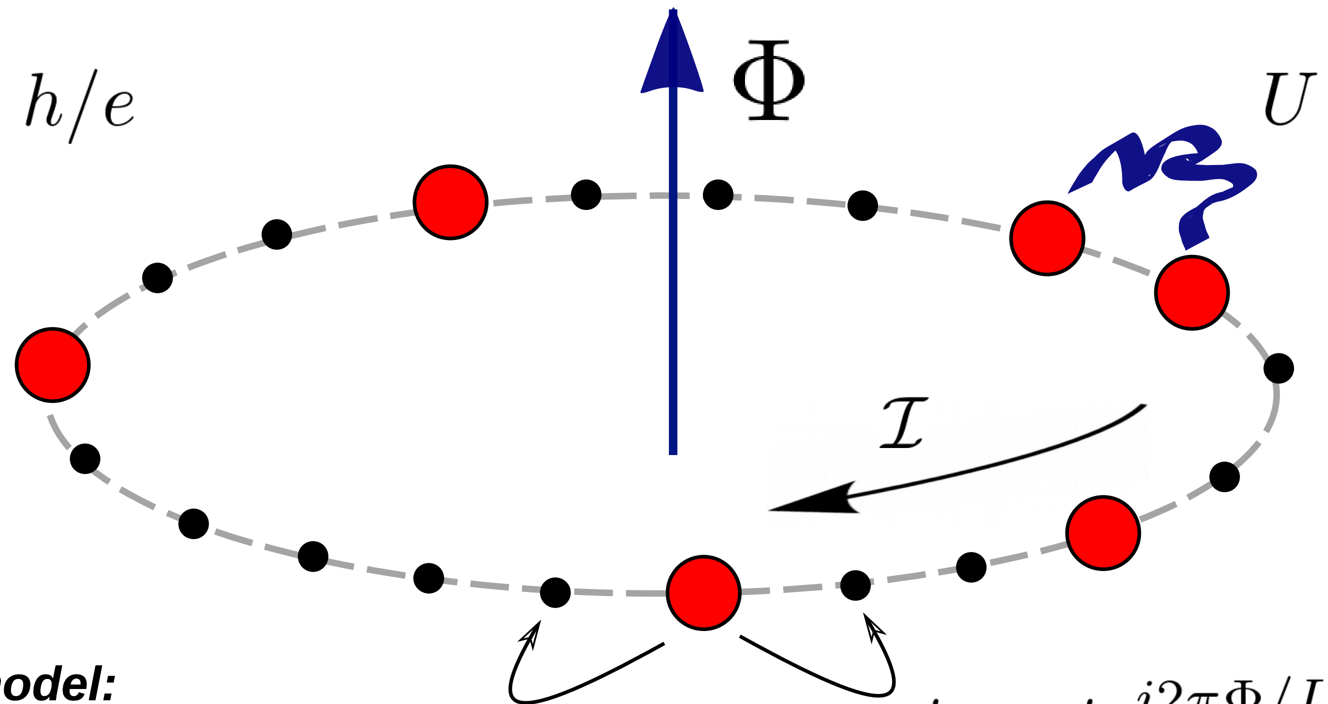

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Current operator

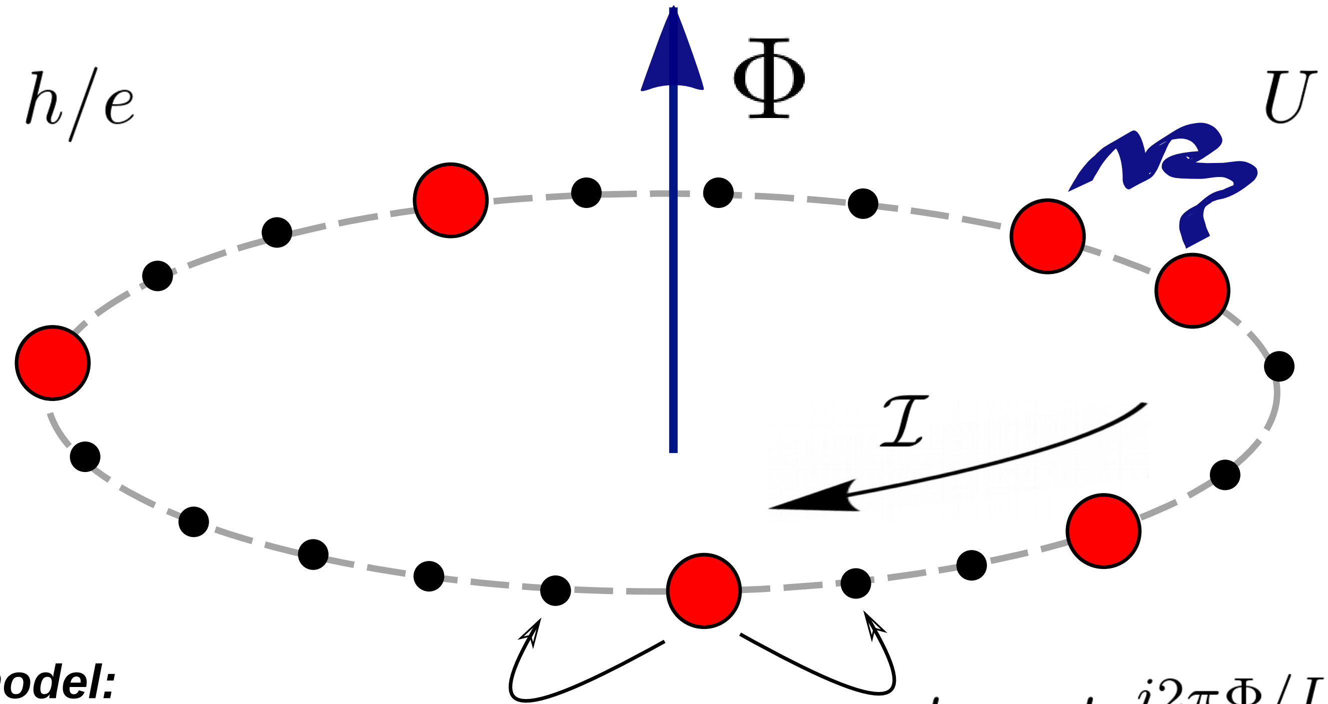
$$\mathcal{I} = \frac{it}{2L}\sum_{j=1}^{L}\left[e^{i2\pi\Phi/L}c_j^\dagger c_{j+1} - e^{-i2\pi\Phi/L}c_{j+1}^\dagger c_j\right]$$

$$\langle\mathcal{I}\rangle = -\frac{1}{2\pi}\frac{\partial\langle H\rangle}{\partial\Phi}$$

Persistent currents

Flux quantum

$$[\Phi] = \Phi_0 = h/e$$



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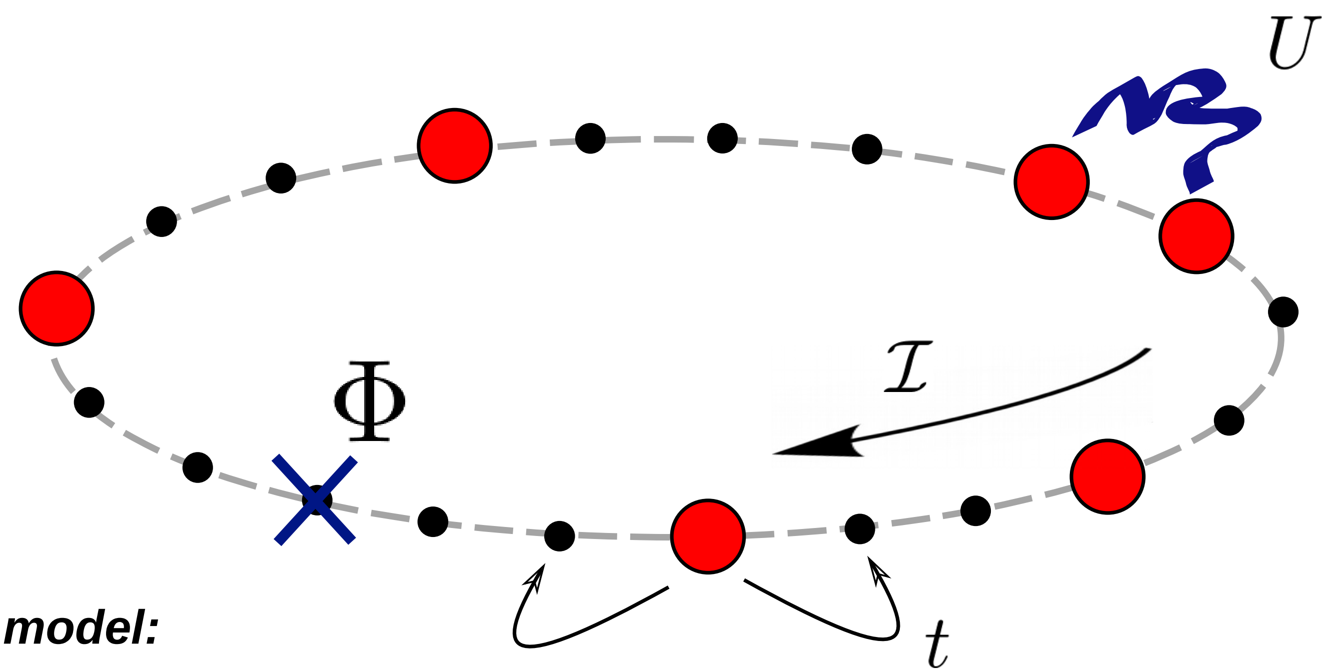
Gauge transformation : $c_j \rightarrow c_j e^{-i2\pi\Phi j/L}$

Persistent currents



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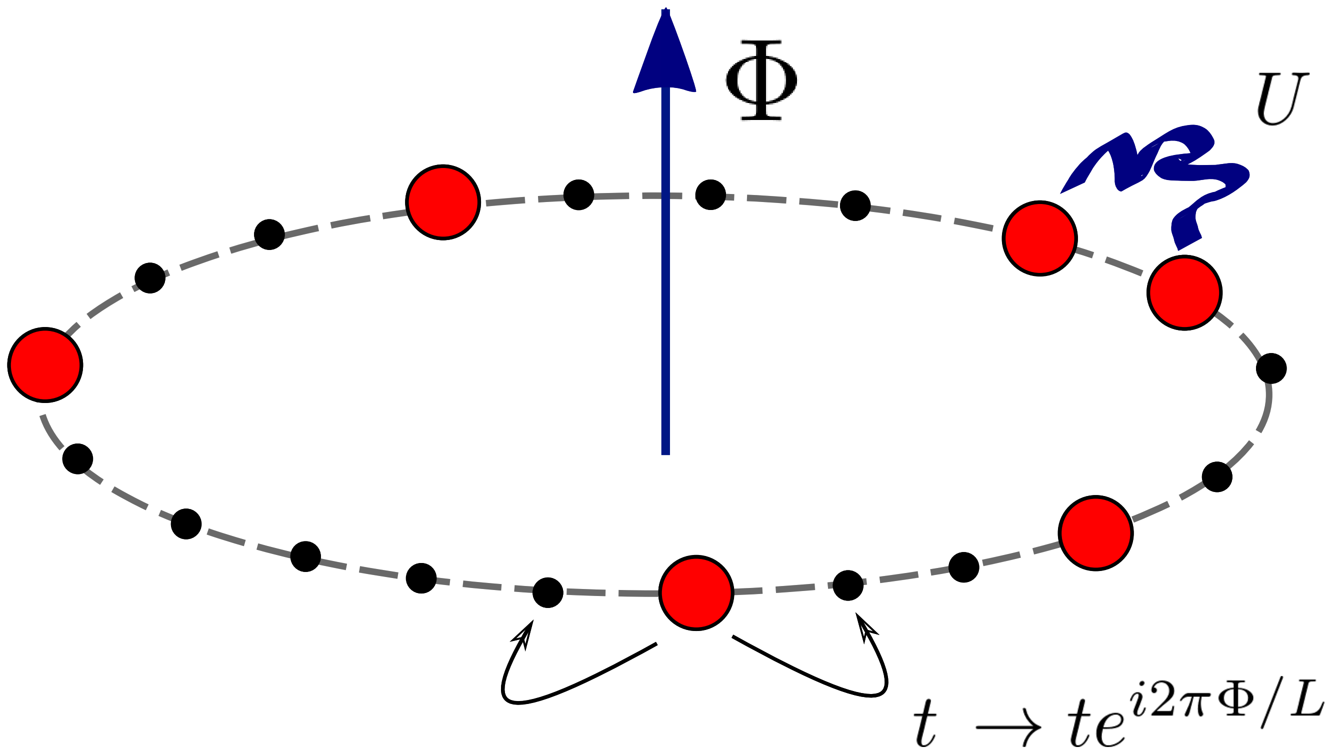
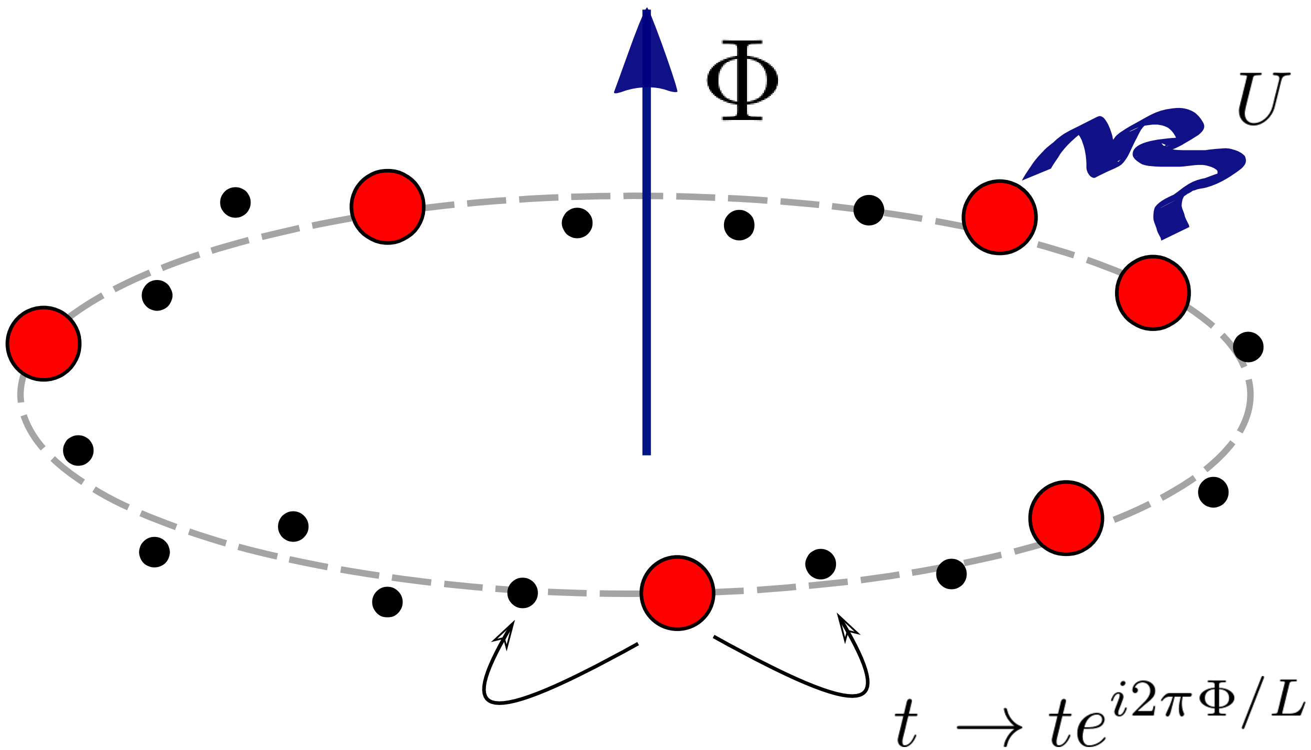
Gauge transformation : $c_j \to c_j e^{-i2\pi\Phi j/L}$

Twisted boundary condition : $c_j = c_{j+L} e^{-i2\pi\Phi}$

Disorder



Disorder



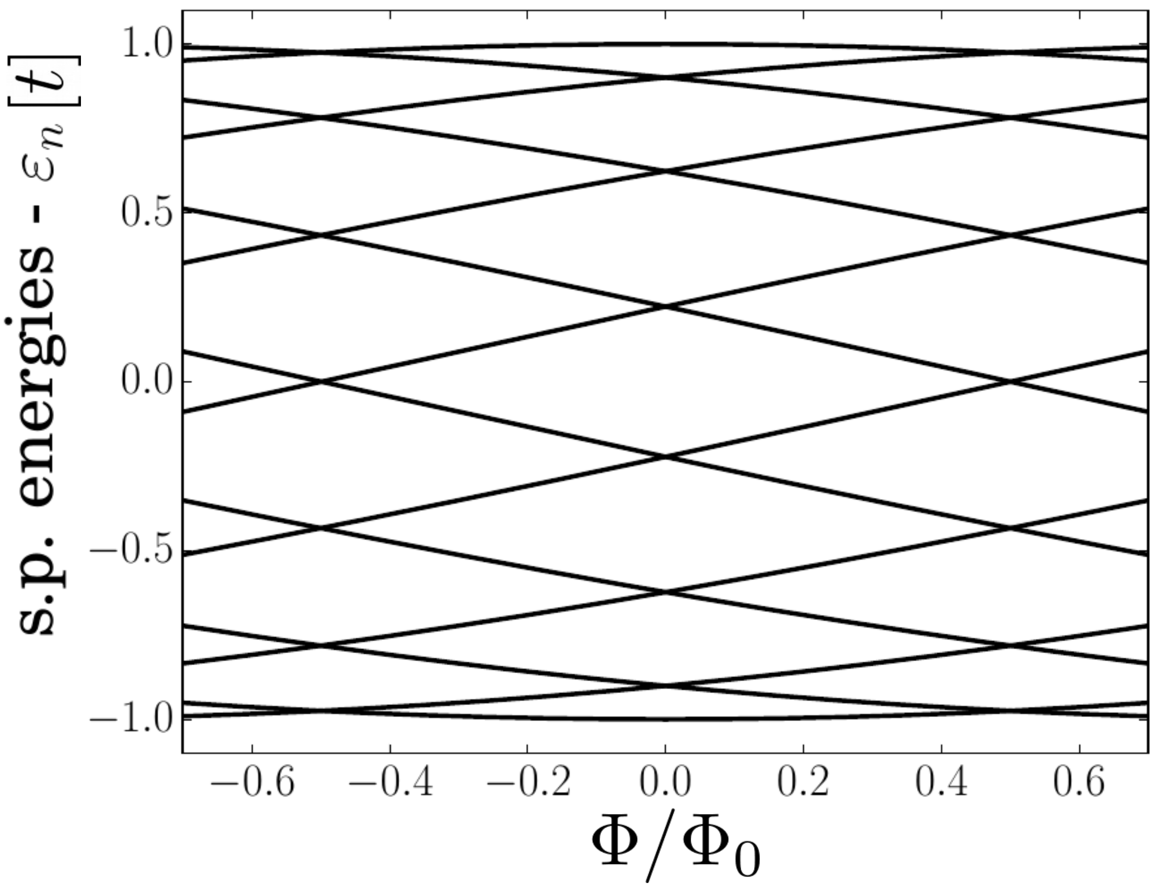
On site disorder

$$\mathcal{H} \to \mathcal{H} + \mathcal{H}_D$$

$$\mathcal{H}_D = \sum_{j=1}^{L} \varepsilon_j n_j \qquad \varepsilon_j \in [-W, W]$$

Persistent currents

Single particle spectrum

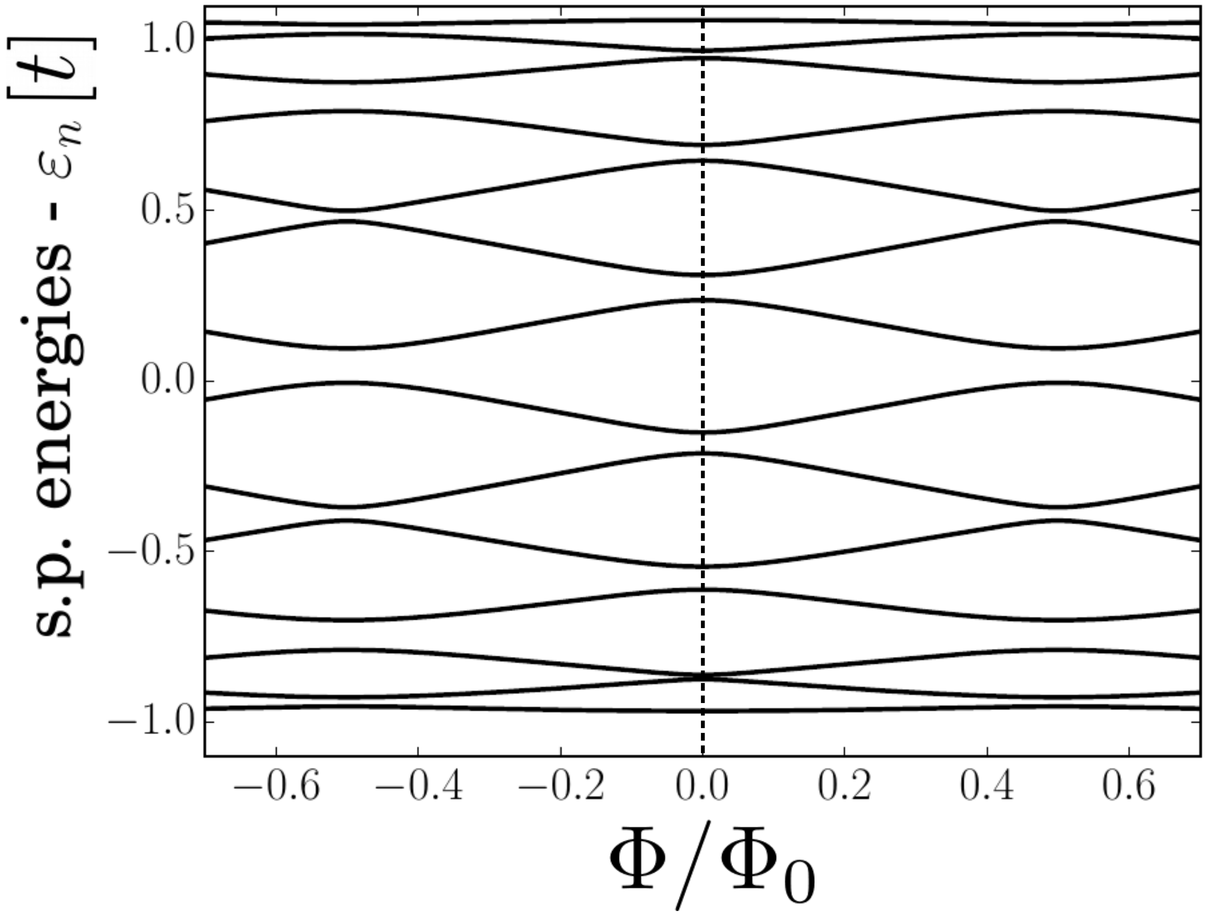


$$\varepsilon_n = -t\cos\left(\frac{2\pi n}{L} + \frac{2\pi\Phi}{L}\right)$$

$$i_n = \frac{e}{h}\frac{t}{L}\sin\left(\frac{2\pi n}{L} + \frac{2\pi\Phi}{L}\right)$$

Disorder

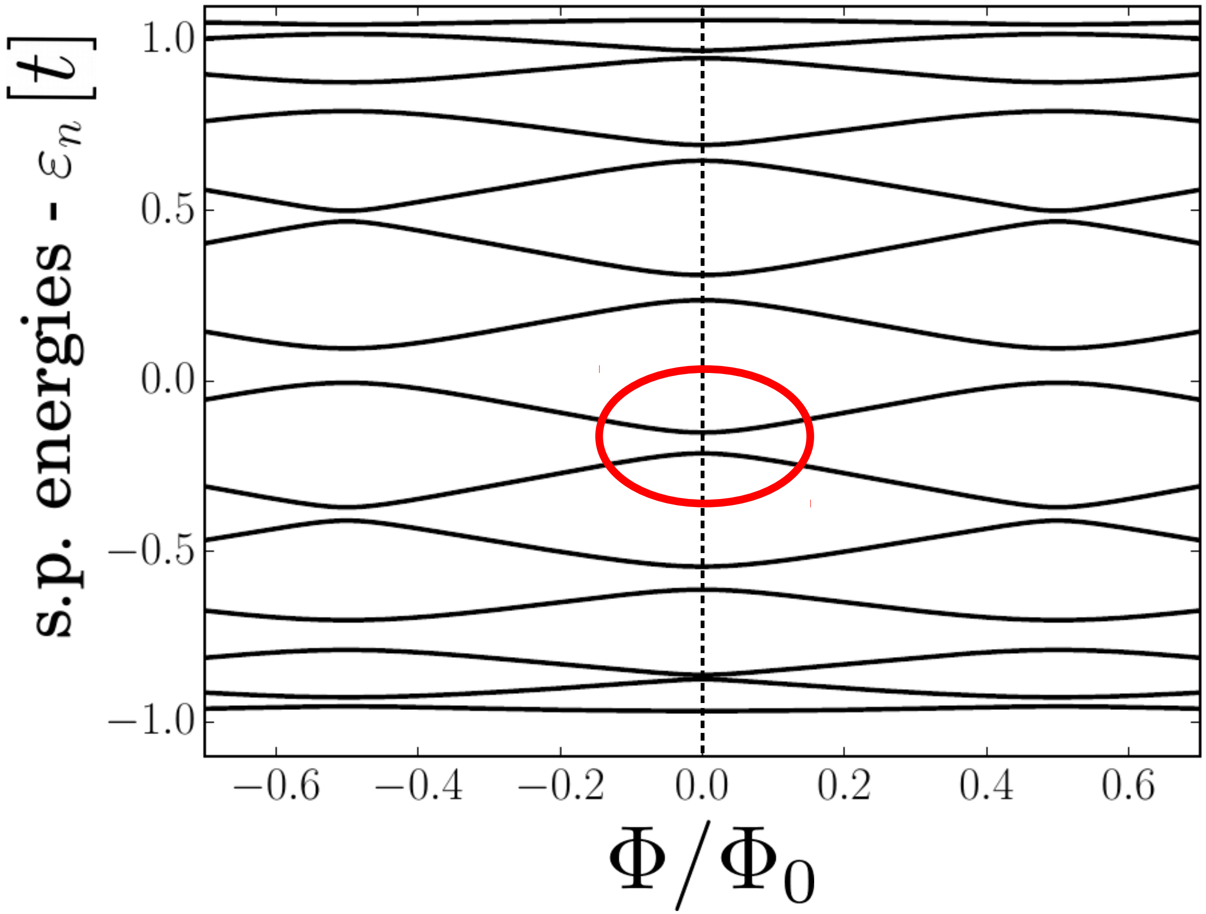
Single particle spectrum



$$W = 0.25t$$

Disorder

Single particle spectrum



$W = 0.25t$

$\mathcal{I} \to 0$ ***for*** $\Phi \to 0$

$$\mathcal{I} \sim \Phi \left. \frac{\partial \mathcal{I}}{\partial \Phi} \right|_{\Phi=0}$$

$$\left. \frac{\partial \mathcal{I}}{\partial \Phi} \right|_{\Phi=0} \neq 0$$

Drude weights

$$\mathcal{D}_n = \frac{L}{2}\,\frac{\partial^2 E_n}{\partial \Phi^2}\bigg|_{\phi=0}$$

Drude weight and zero Φ limit

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localization

$D \to 0$

W. Kohn, *Theory of the Insulating State*, **Physical Review 133**, A171 (1964)
B.S. Shastry and B. Sutherland, **Phys. Rev. Lett. 65,** 243 (1990)
D. J. Scalapino *et al.*, **Phys. Rev. Lett. 68,** 2830 (1992)
T. Giamarchi and B.S. Shastry , **Phys. Rev. B 51,** 10915 (1995)

Drude weight and zero Φ limit

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localization

$$D \to 0$$

delocalization

$$D \to \frac{\rho}{m^*}$$

ρ *: density*

m^* *: effective mass*

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Small flux expansion :

$$\mathcal{H}(\Phi) = \mathcal{H}(0) - \frac{2\pi\Phi}{L}\, j$$

Drude weight and zero Φ limit

Hubbard Model :

$$\mathcal{H}(\Phi) = -\frac{t}{2}\sum_{j=1}^{L}\left[e^{i2\pi\Phi/L}c_j^{\dagger}c_{j+1} + e^{-i2\pi\Phi/L}c_{j+1}^{\dagger}c_j\right] + U\sum_{j=1}^{L}n_j n_{j+1}$$

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$$j = \frac{it}{2}\sum_{j=1}^{L}\left[c_j^{\dagger}c_{j+1} - c_{j+1}^{\dagger}c_j\right]$$

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Small flux expansion :

$$\mathcal{H}(\Phi) = \mathcal{H}(0) - \frac{2\pi\Phi}{L}\,j - \frac{1}{2}\left(\frac{2\pi\Phi}{L}\right)^2 \mathcal{T} + O(\Phi^3)$$

Current density:

$$j = \frac{it}{2}\sum_{j=1}^{L}\left[c_j^\dagger c_{j+1} - c_{j+1}^\dagger c_j\right]$$

Kinetic energy :

$$\mathcal{T} = -\frac{t}{2}\sum_{j=1}^{L}\left[c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j\right]$$

Drude weight and zero Φ limit

Energy correction to second order :

$$E_n(\Phi) - E_n(0) = \frac{\Phi^2}{e^2 L} D_n$$

W. Kohn, *Theory of the Insulating State*, **Physical Review 133**, A171 (1964)
B.S. Shastry and B. Sutherland, **Phys. Rev. Lett. 65,** 243 (1990)
D. J. Scalapino *et al.*, **Phys. Rev. Lett. 68,** 2830 (1992)
T. Giamarchi and B.S. Shastry , **Phys. Rev. B 51,** 10915 (1995)

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$$\mathcal{D}_n = \frac{L}{2} \left. \frac{\partial^2 E_n}{\partial \Phi^2} \right|_{\phi=0}$$

$$D_n = e^2 \frac{4\pi^2}{L} \left[-\frac{1}{2} \langle \mathcal{T} \rangle + L^2 \sum_{m \neq n} \frac{|\langle \psi_n | \mathcal{I} | \psi_m \rangle|^2}{E_n - E_m} \right]$$

W. Kohn, *Theory of the Insulating State*, **Physical Review 133**, A171 (1964)
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Kubo Formula :

$$\mathcal{I}_n(\omega) = \sigma_n(\omega) E(\omega)$$

$$\mathrm{Re}\big[\sigma_n(\omega)\big] = D_n \delta(\omega) + \sigma_{n,\mathrm{reg}}$$

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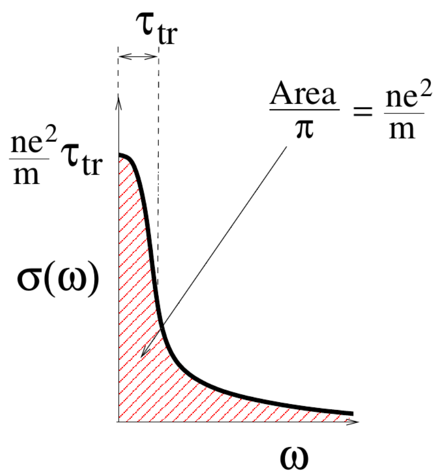

Drude weights and Thouless Conductance

J. Edwards and D. J. Thouless, **J. Phys. C: Sol. State Phys. 5**, 807 (1972)
D. J. Thouless, **Phys. Rep. 13,** 93 (1974)

Thouless' conjecture

$$D_n = e^2 \frac{4\pi^2}{L} \left[-\frac{1}{2} \langle \mathcal{T} \rangle + L^2 \sum_{m \neq n} \frac{|\langle \psi_n | \mathcal{I} | \psi_m \rangle|^2}{E_n - E_m} \right]$$

Thouless conductance (single particle !!!) :

$$\langle \sigma_{Thouless} \rangle = \frac{1}{\Delta} \left\langle \left| \frac{\partial^2 \varepsilon_n}{\partial \phi^2} \right| \right\rangle$$

Δ : *Average level spacing*

ε_n : *single particle spectrum*

J. Edwards and D. J. Thouless, **J. Phys. C: Sol. State Phys. 5**, 807 (1972)s
D. J. Thouless, **Phys. Rep. 13,** 93 (1974)
Y. V. Fyodorov and H.-J. Sommers, **Phys. Rev. E 51**, R2719 (1995).

Curvature distributions

$$D_n = e^2 \frac{4\pi^2}{L} \left[-\frac{1}{2}\langle \mathcal{T} \rangle + L^2 \sum_{m \neq n} \frac{|\langle \psi_n | \mathcal{I} | \psi_m \rangle|^2}{E_n - E_m} \right]$$

For large value of D :

$$D \propto \frac{1}{s}$$

s : *level spacing*

$$P(D \to \infty)\ dD = P(s \to 0)\ ds$$

$$\left| \frac{ds}{dD} \right| = \frac{1}{D^2}$$

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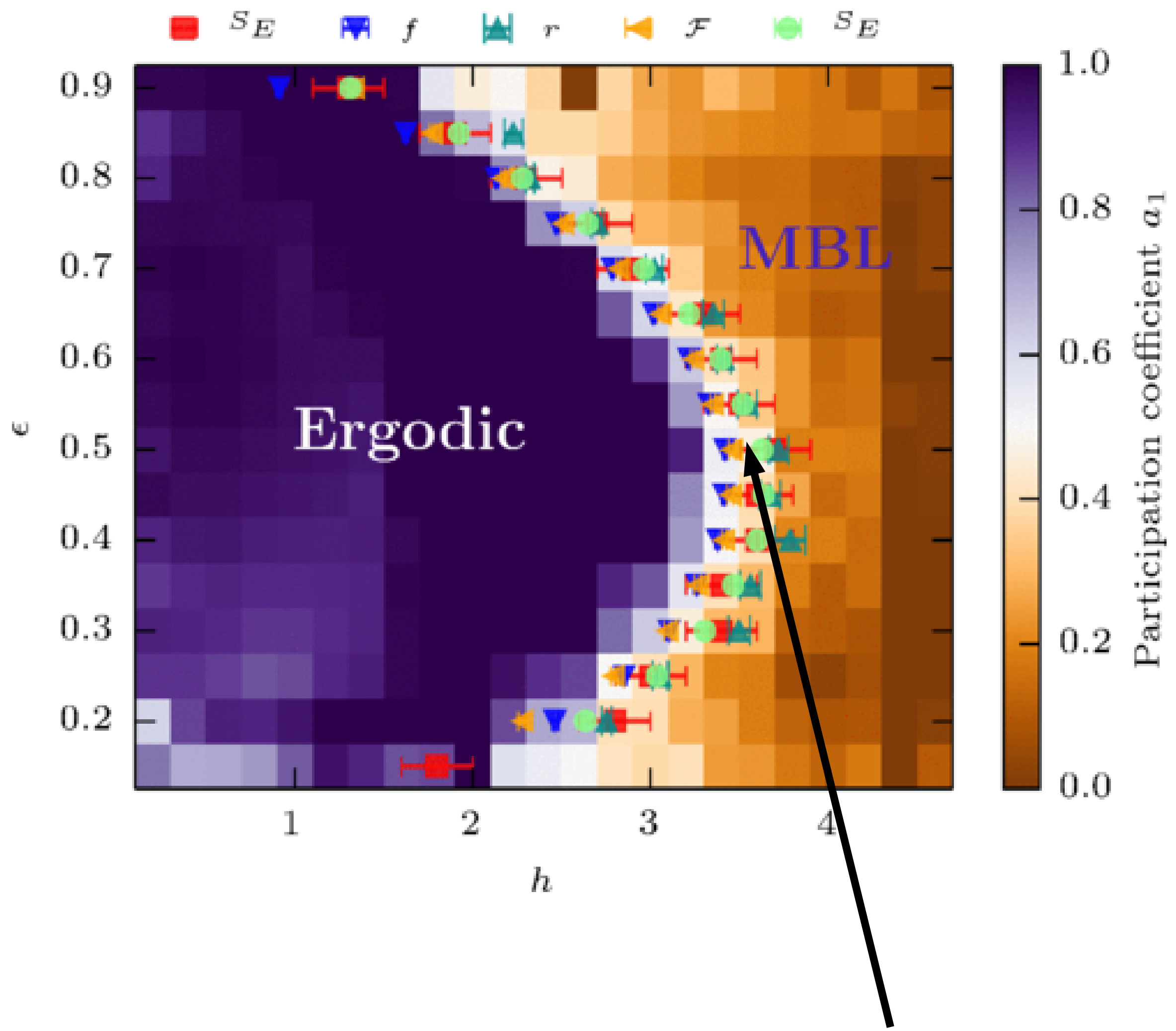
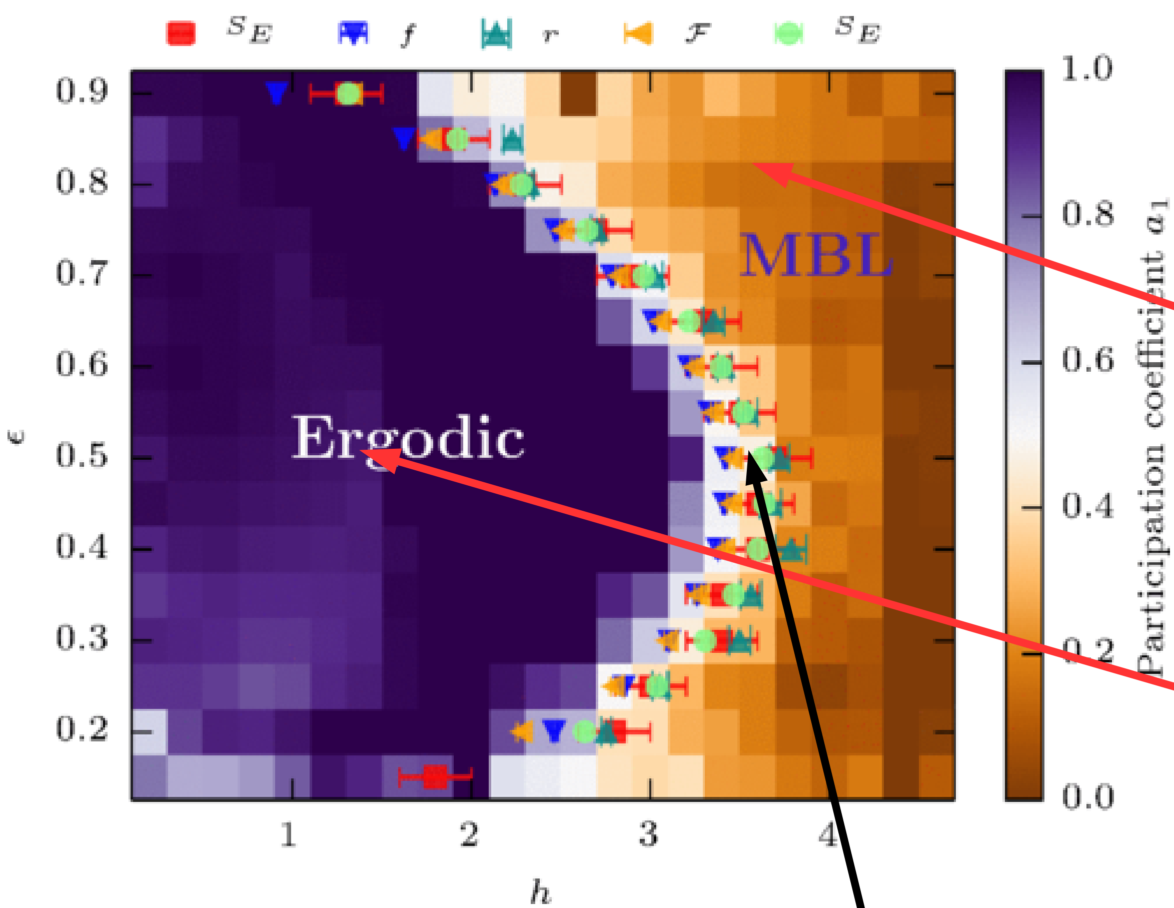


$$W_c = 3.6t$$

D. Luitz, N. Laflorencie and F. Alet,
Phys. Rev. B 91, 081103(R) (2015)

M. Serbyn, Z. Papic and D. Abanin,
Phys. Rev. X 5, 041047 (2015)

Curvature distributions



Uncorrelated spectrum
–
Poissonian statistics

$$P(s) = \frac{1}{\Delta} e^{-s/\Delta}$$

Correlated spectrum
–
Wigner-Dyson statistics

$$P(s) = \frac{\pi}{2}\frac{s}{\Delta} e^{-\frac{\pi s^2}{4\Delta^2}}$$

$$W_c = 3.6t$$

D. Luitz, N. Laflorencie and F. Alet,
Phys. Rev. B 91, 081103(R) (2015)

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Phys. Rev. X 5, 041047 (2015)

What we expect …

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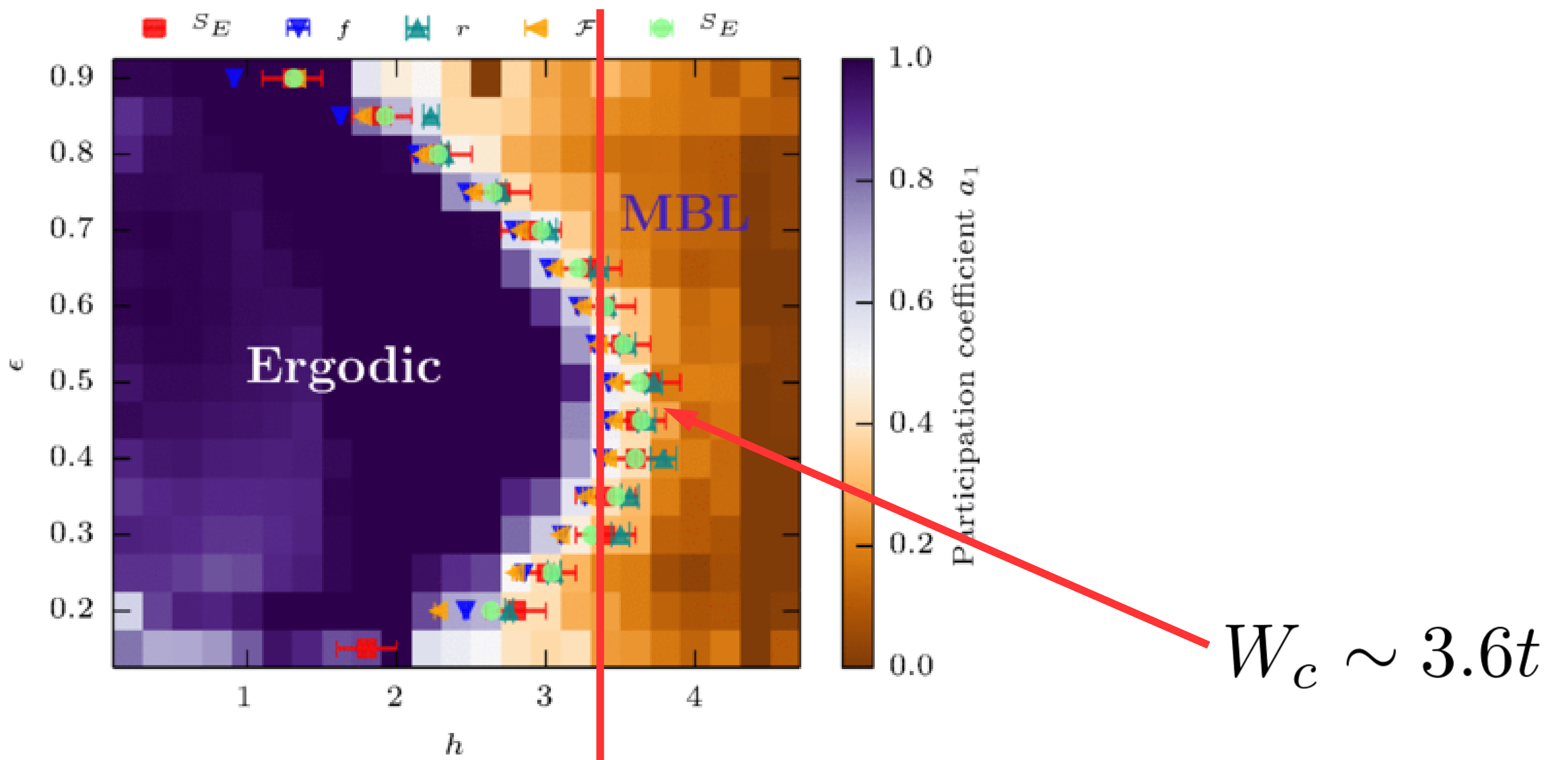


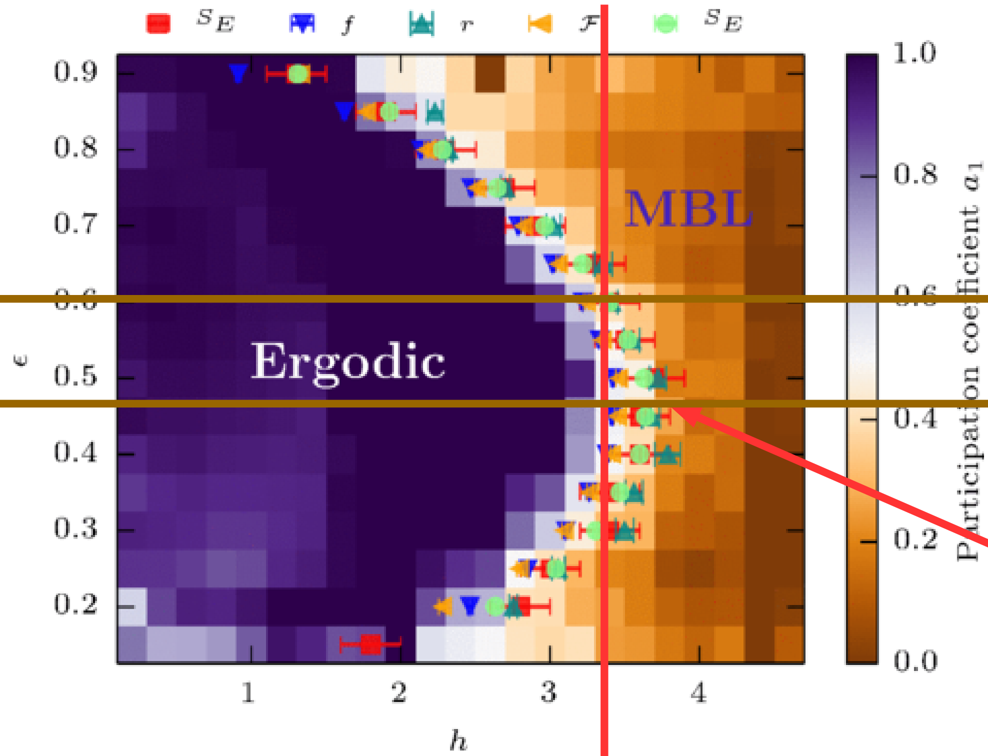
Levy distribution

$$P(D) = \frac{\gamma^2}{(\gamma^2 + D^2)^{3/2}} \xrightarrow{D\to\infty} \frac{1}{D^3}$$

Cauchy distribution

$$P(D) = \frac{\gamma/\pi}{(\gamma^2 + D^2)} \xrightarrow{D\to\infty} \frac{1}{D^2}$$

What we expect …



$L = 16$

$N = 8$

I consider

2554 *states*

in the middle of the many-body spectrum

$W_c \sim 3.6t$

Levy distribution

$$P(D) = \frac{\gamma^2}{(\gamma^2 + D^2)^{3/2}} \xrightarrow{D \to \infty} \frac{1}{D^3}$$

Cauchy distribution

$$P(D) = \frac{\gamma/\pi}{(\gamma^2 + D^2)} \xrightarrow{D \to \infty} \frac{1}{D^2}$$

Technical note

Cumulative distribution functions

$$F_D(D) = \int_{-|D|}^{|D|} dx P_D(x)$$

Levy distribution

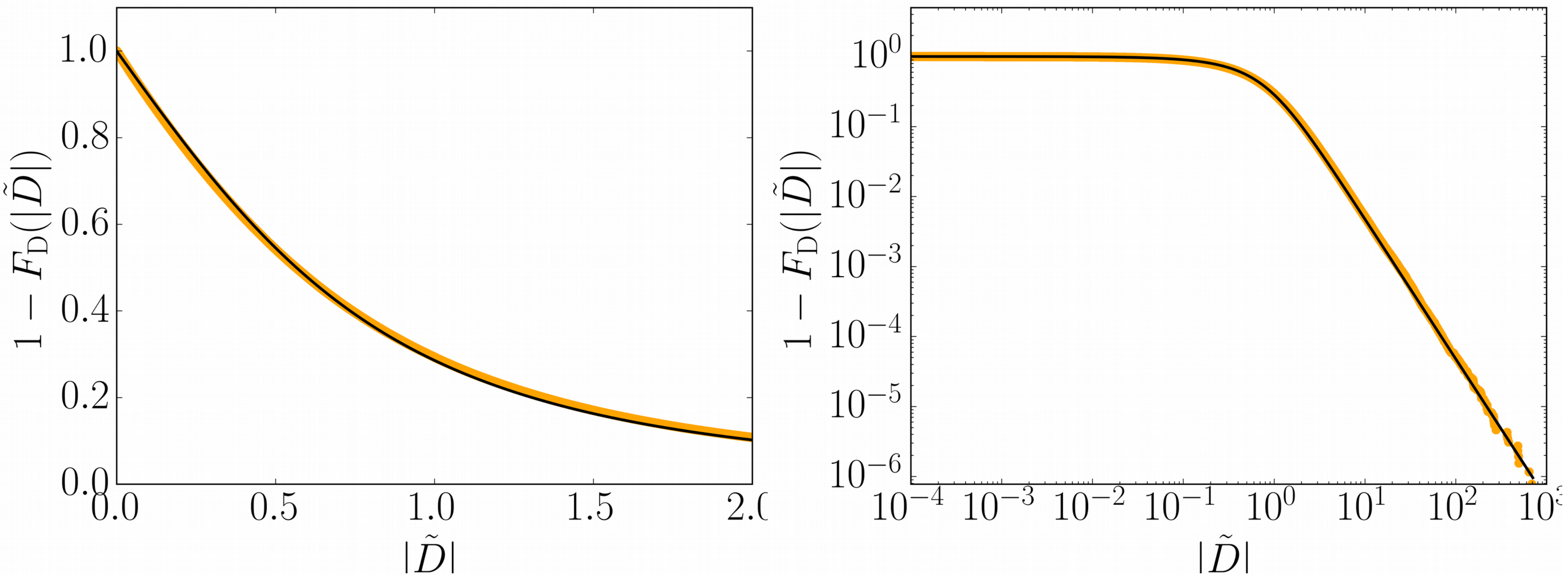
$$F_{\mathrm{D,RMT}}(D) = \frac{|D|}{\sqrt{\gamma^2 + D^2}} \xrightarrow{D \to \infty} 1 - \frac{\gamma^2}{2D^2}$$

Cauchy distribution

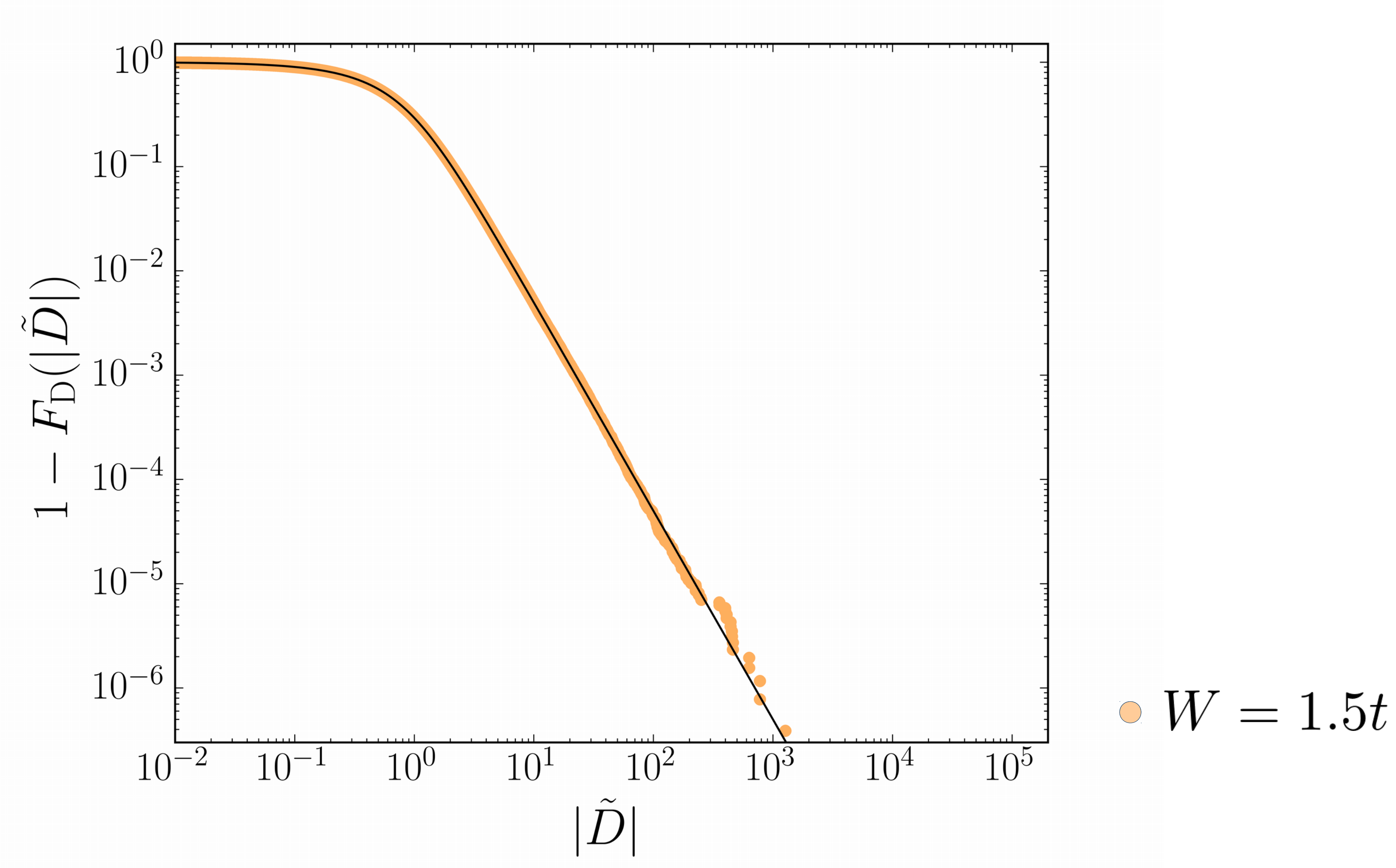
$$F_{\mathrm{D,Cauchy}}(D) = \frac{2}{\pi} \arctan\left(\frac{D}{\gamma}\right) \xrightarrow{D \to \infty} 1 - \frac{2\gamma}{\pi D}$$

Rescaling

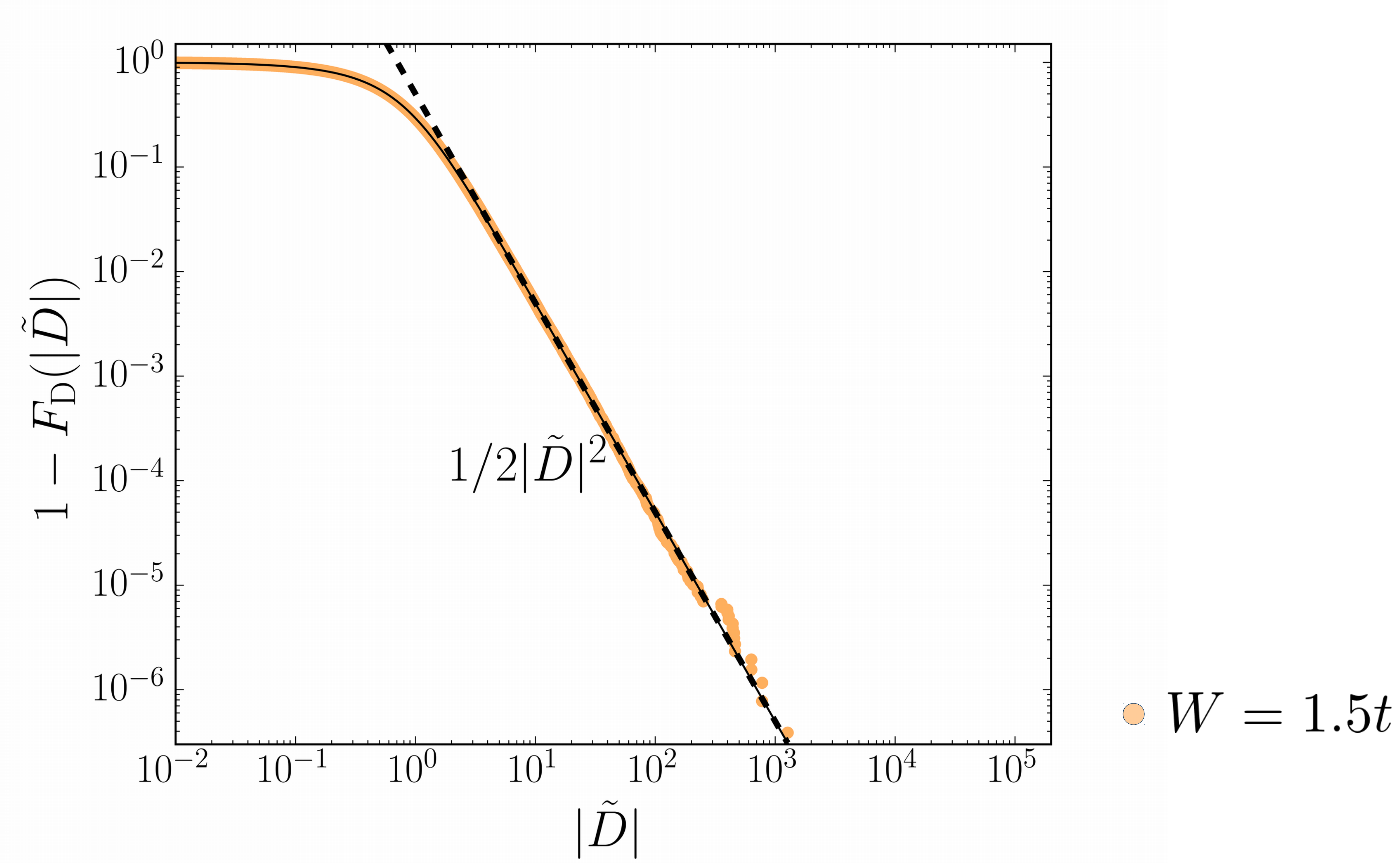
Delocalized phase $W = 2.0t$



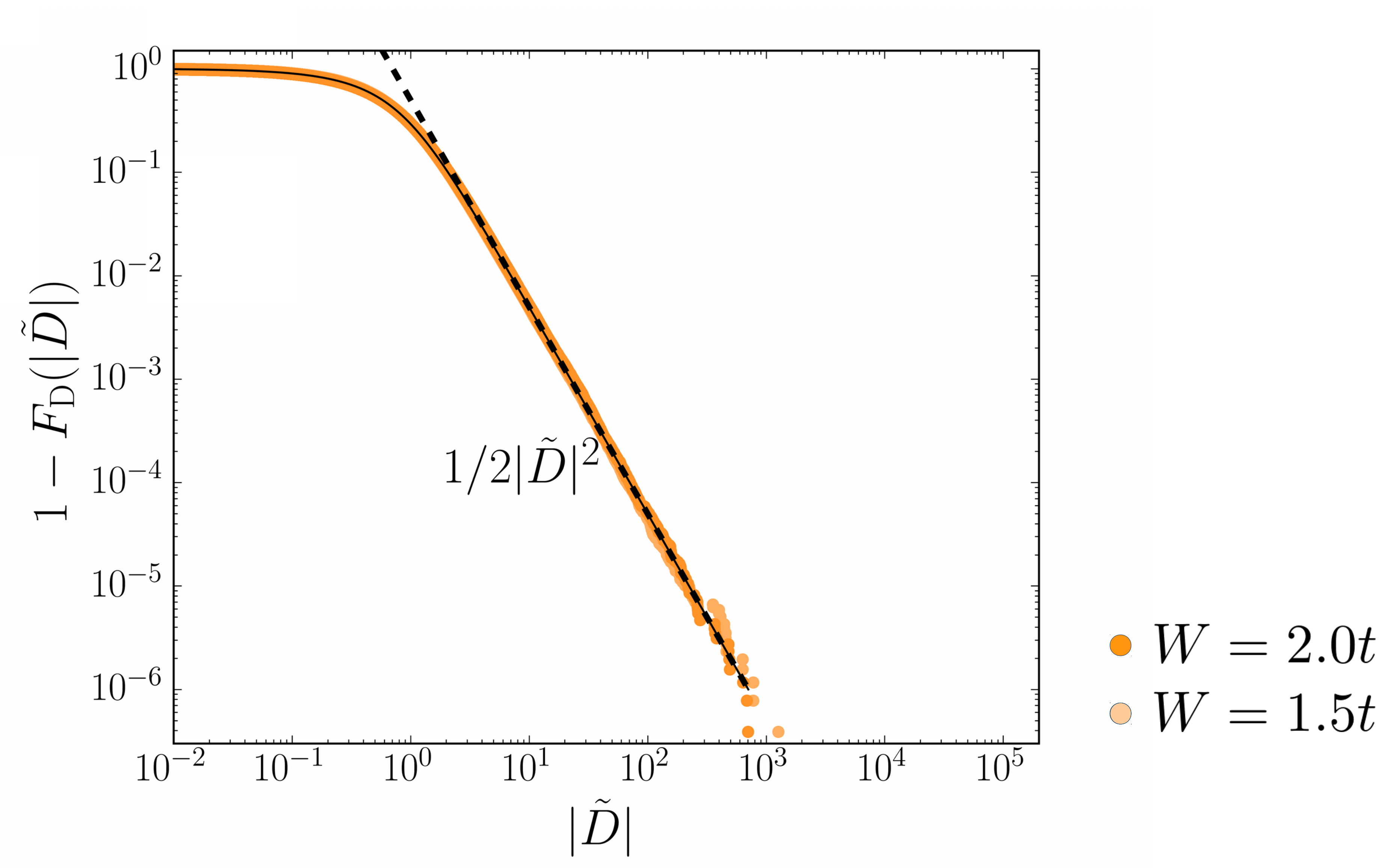
Distribution across the MBL transition



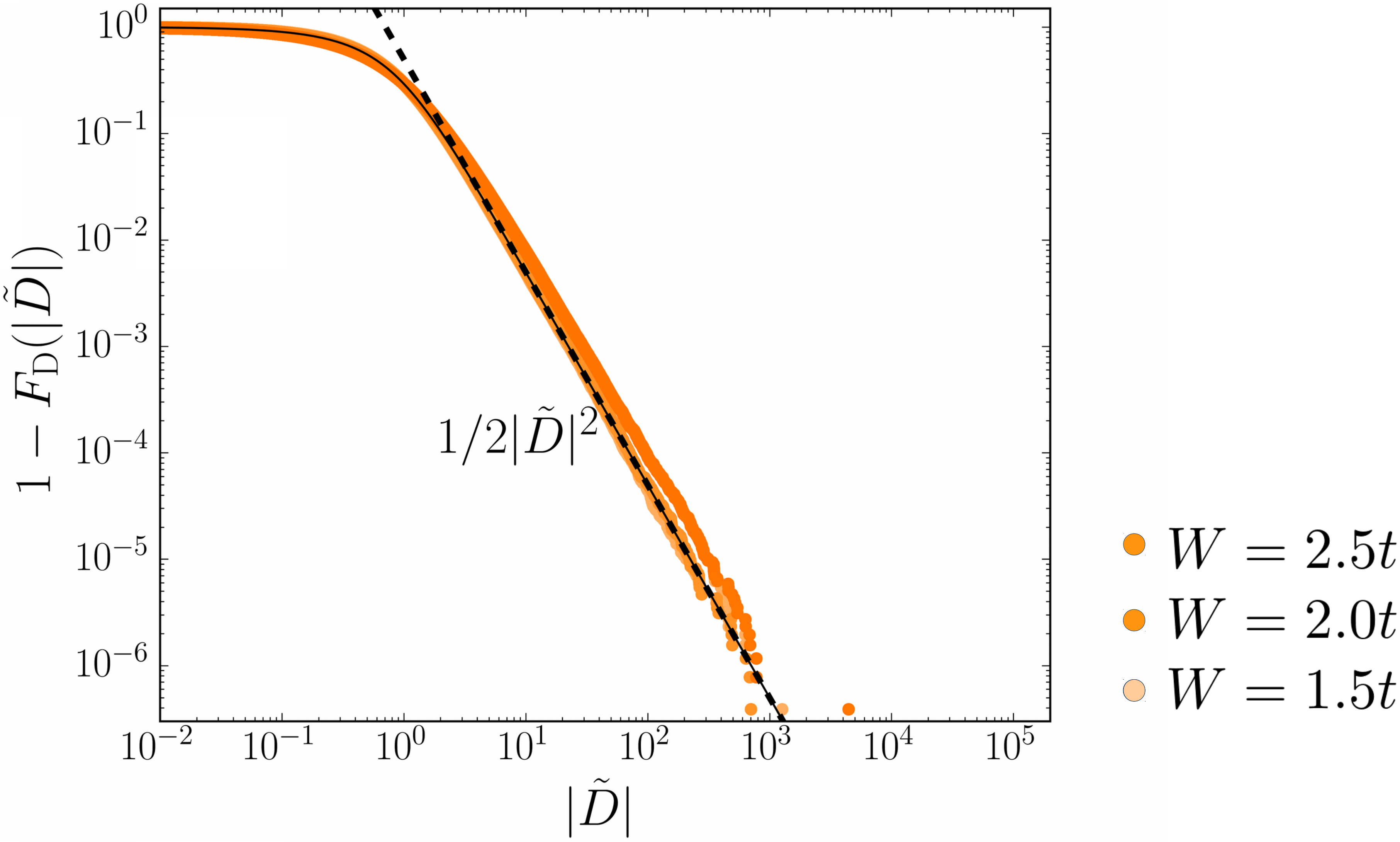
Distribution across the MBL transition



Distribution across the MBL transition



Distribution across the MBL transition



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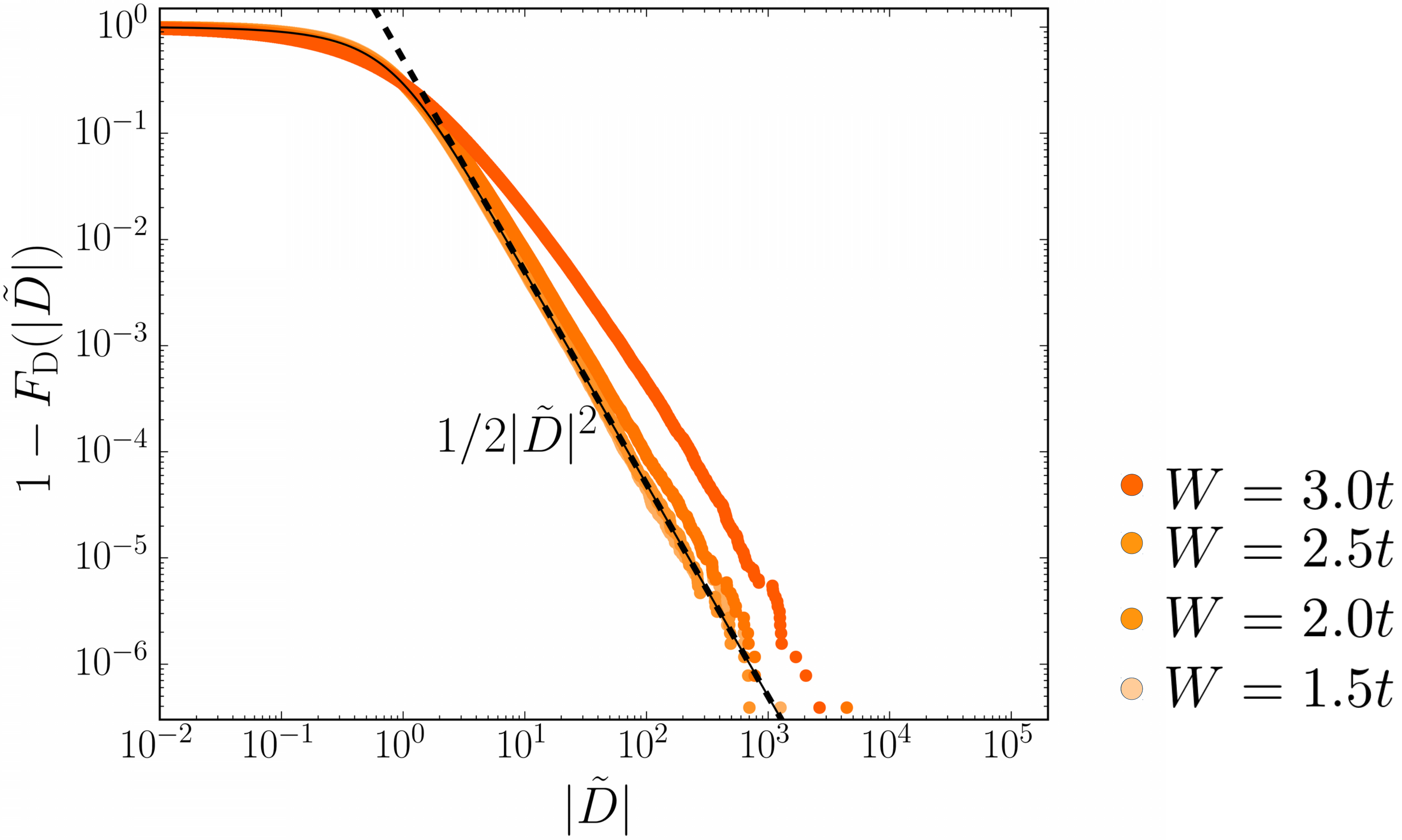

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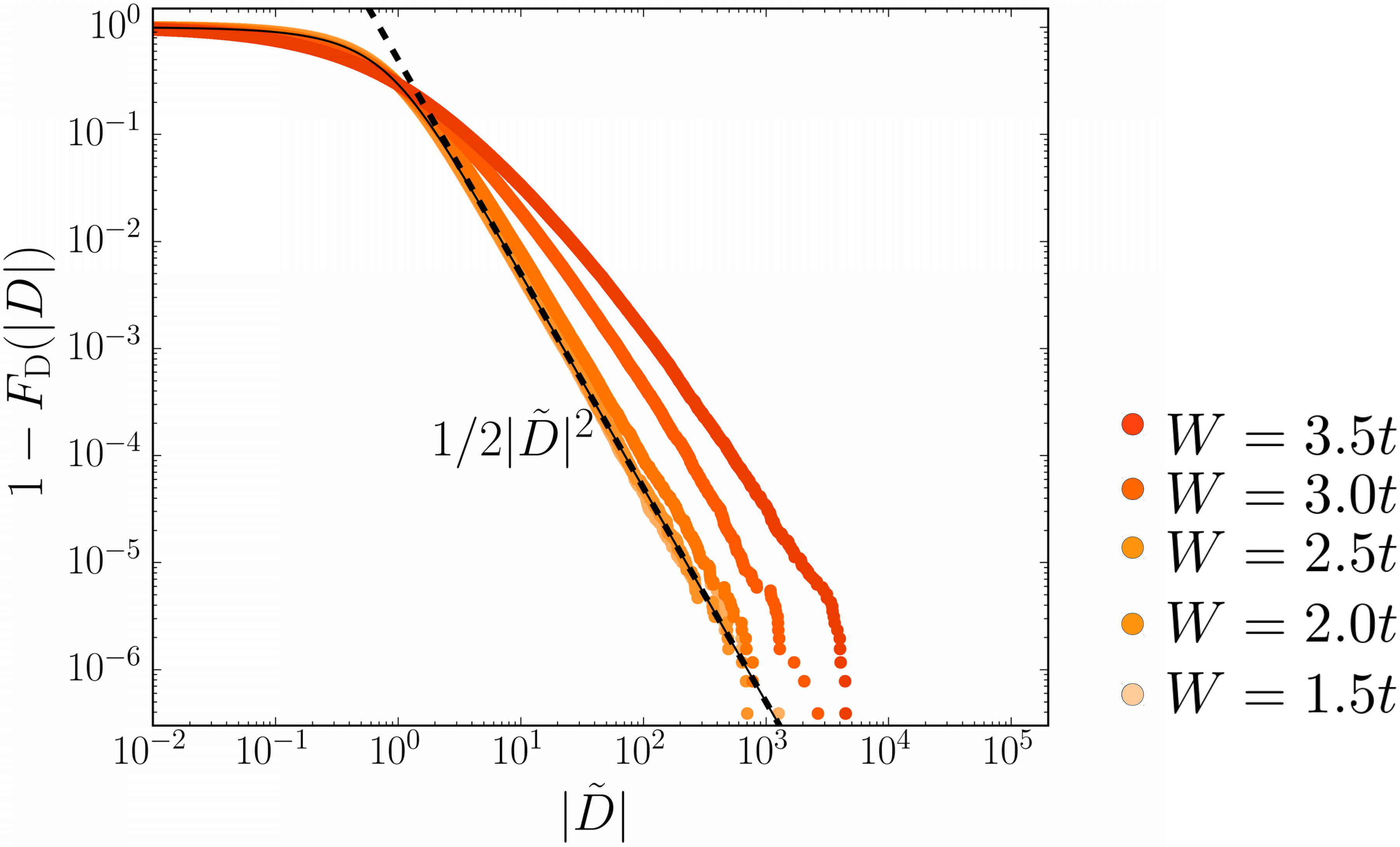

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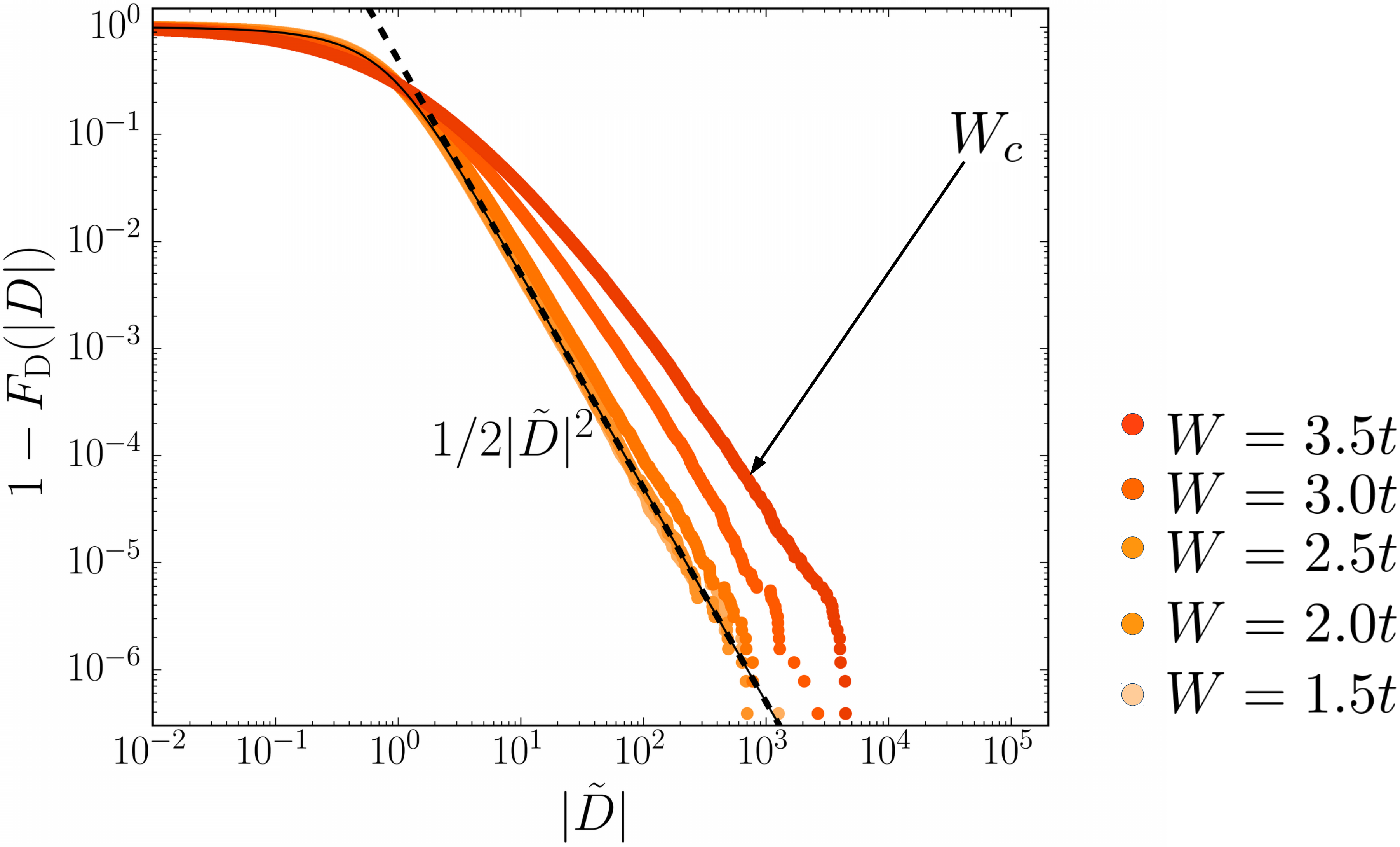

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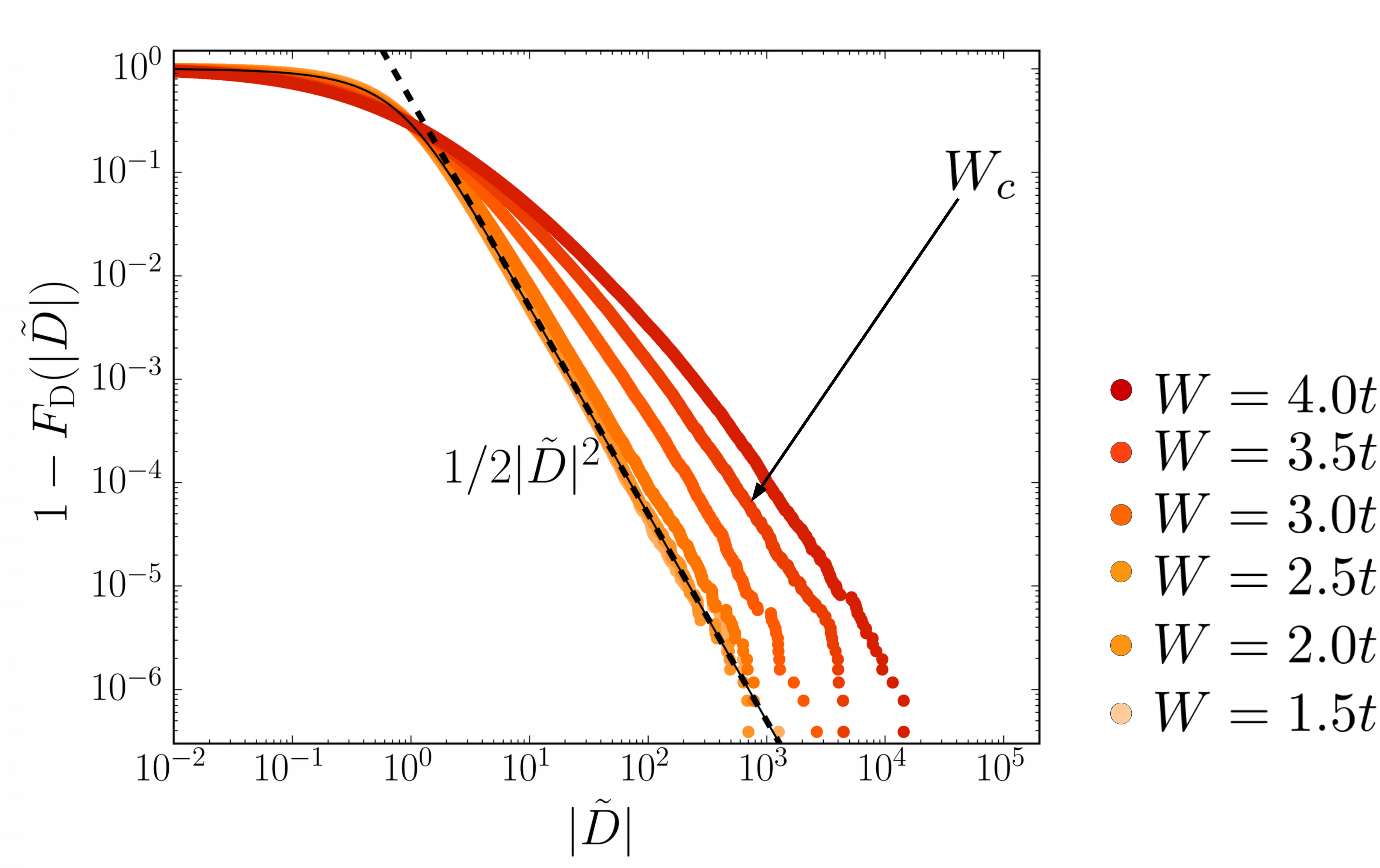

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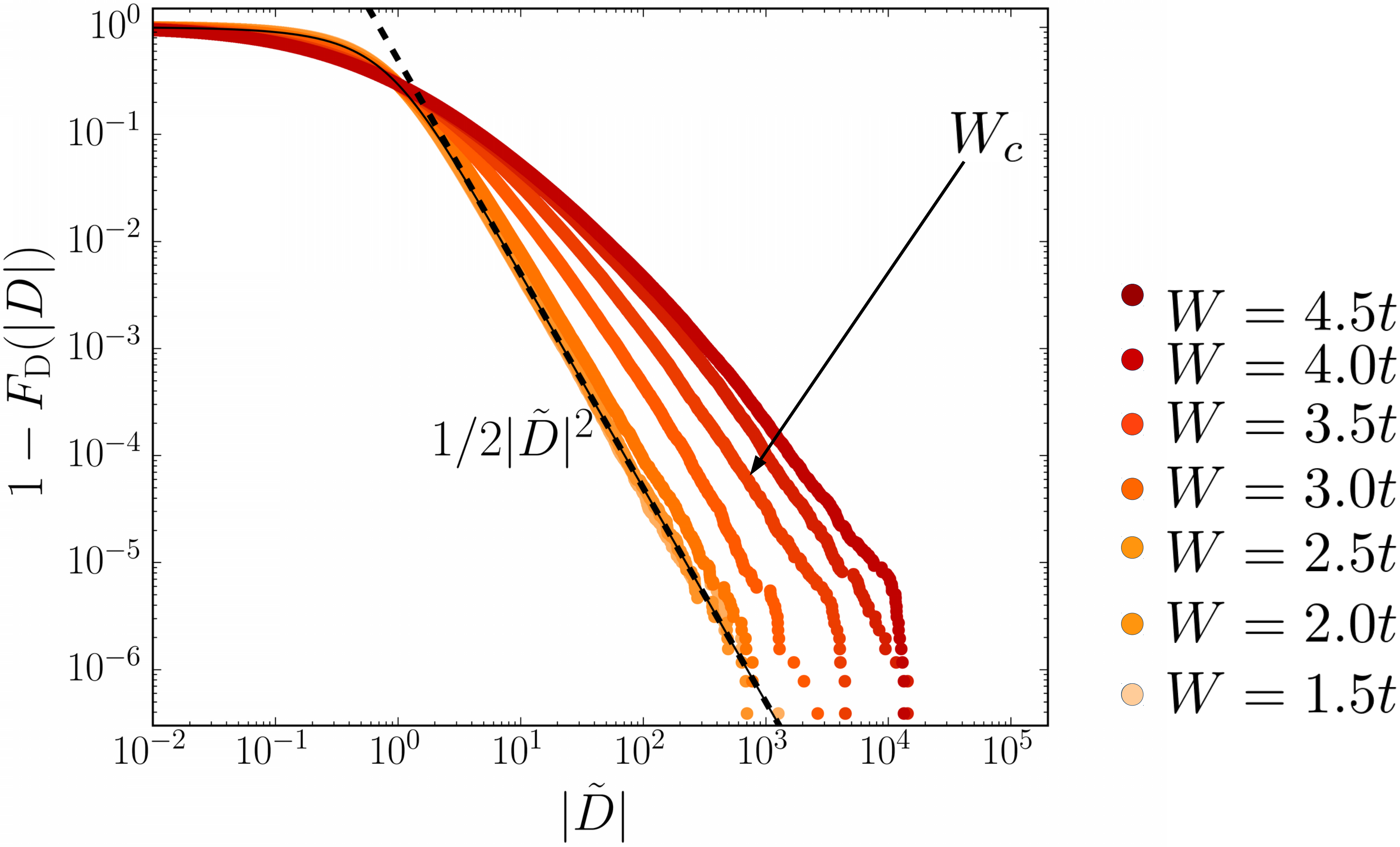

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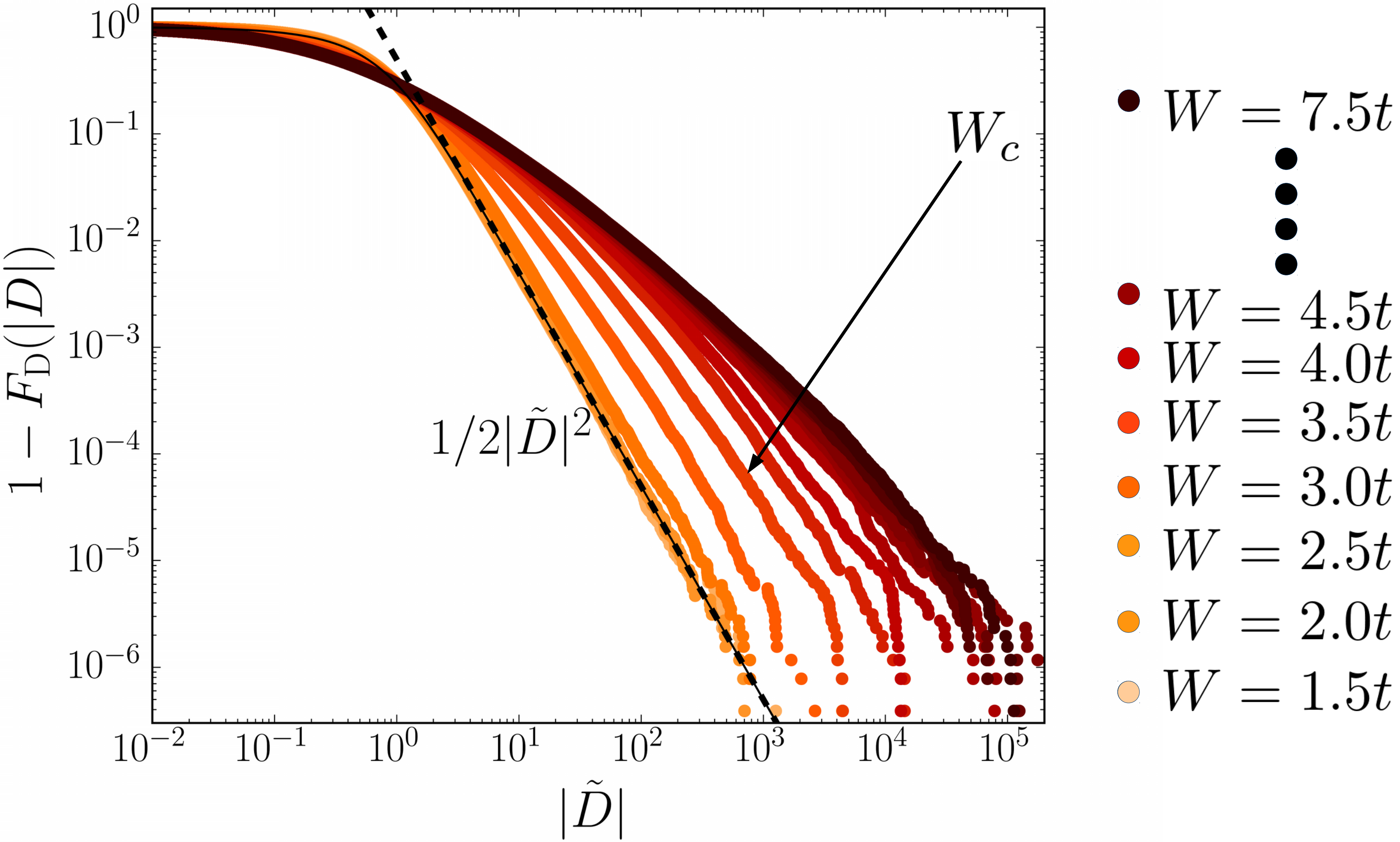
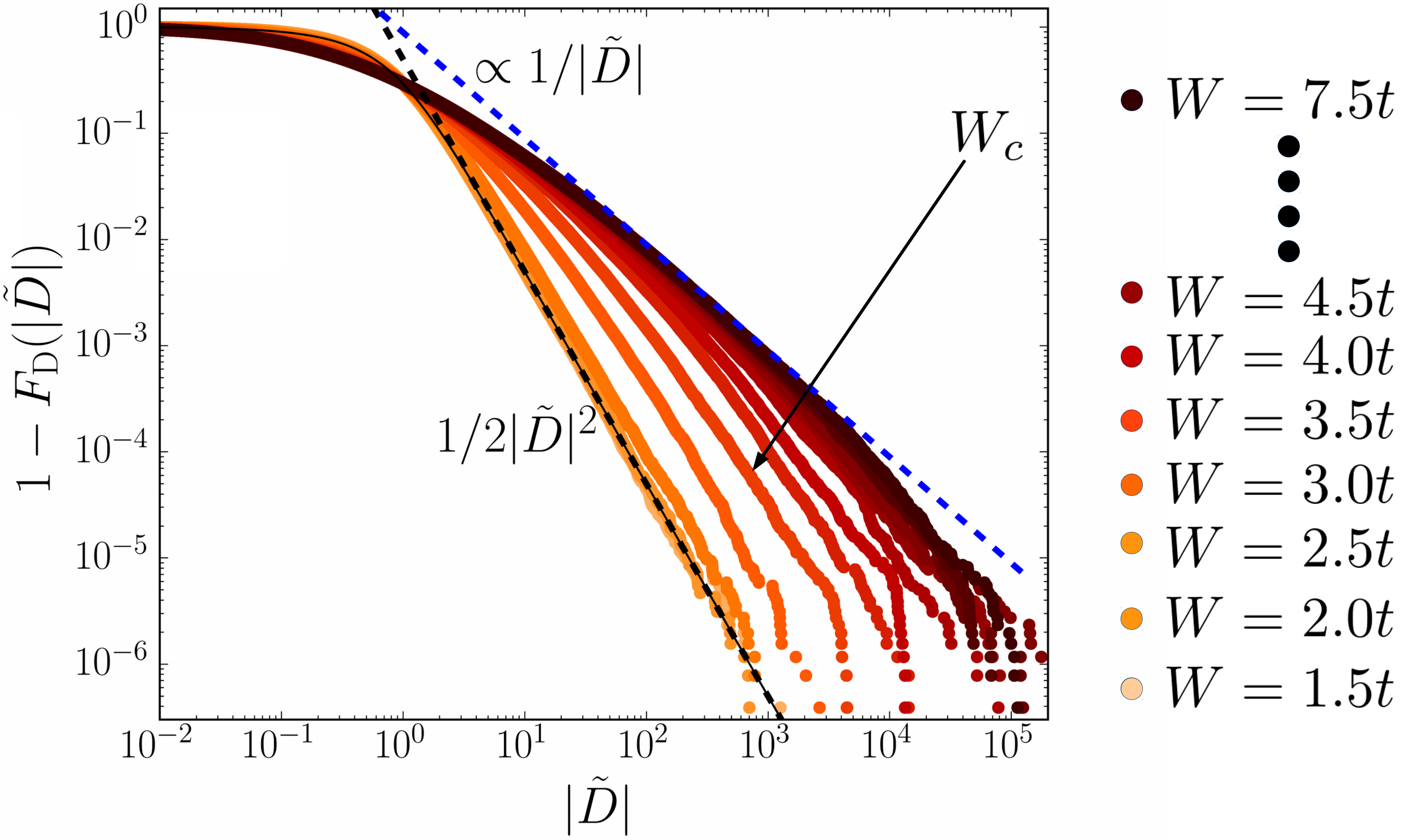
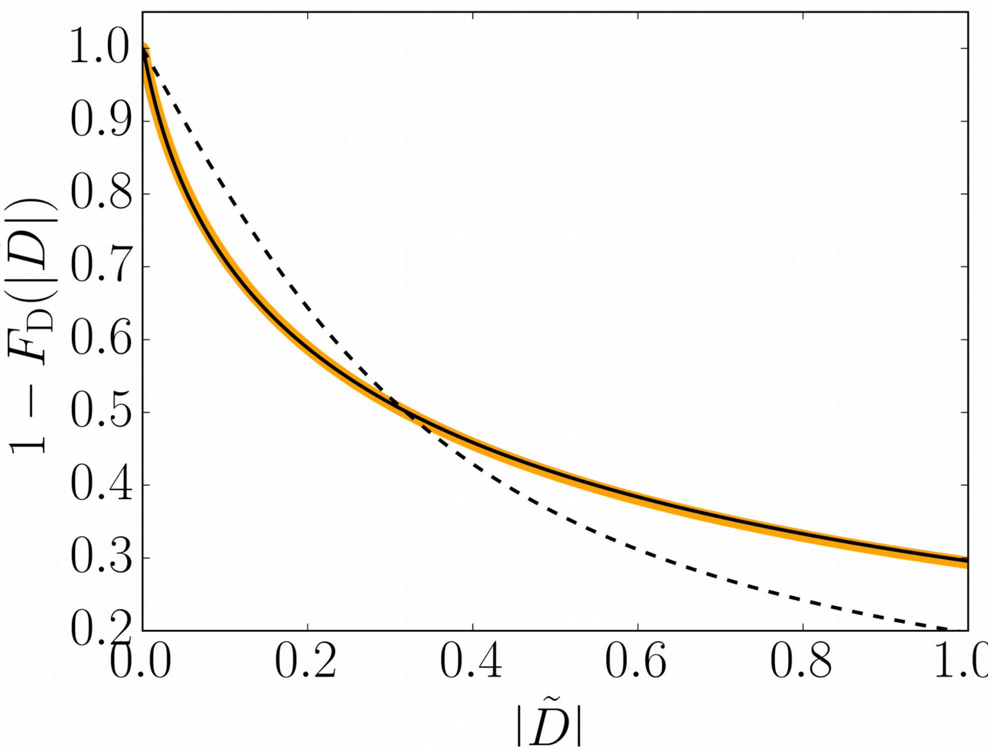


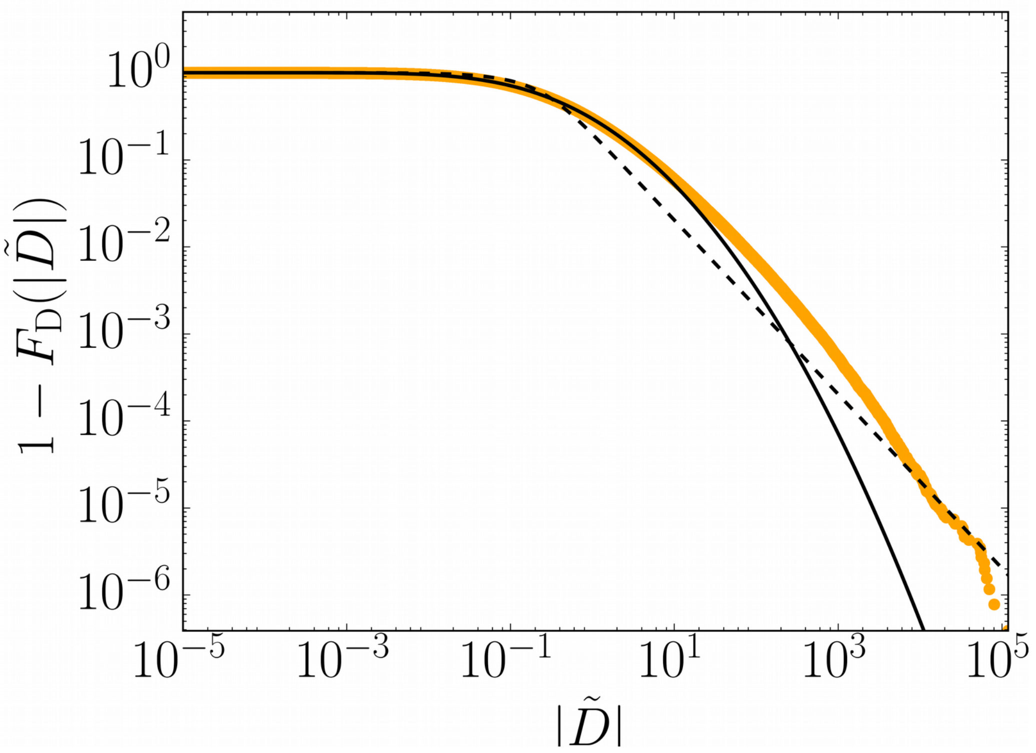
Distribution across the MBL transition



Distribution deep in the MBL phase

$W = 5.5t$





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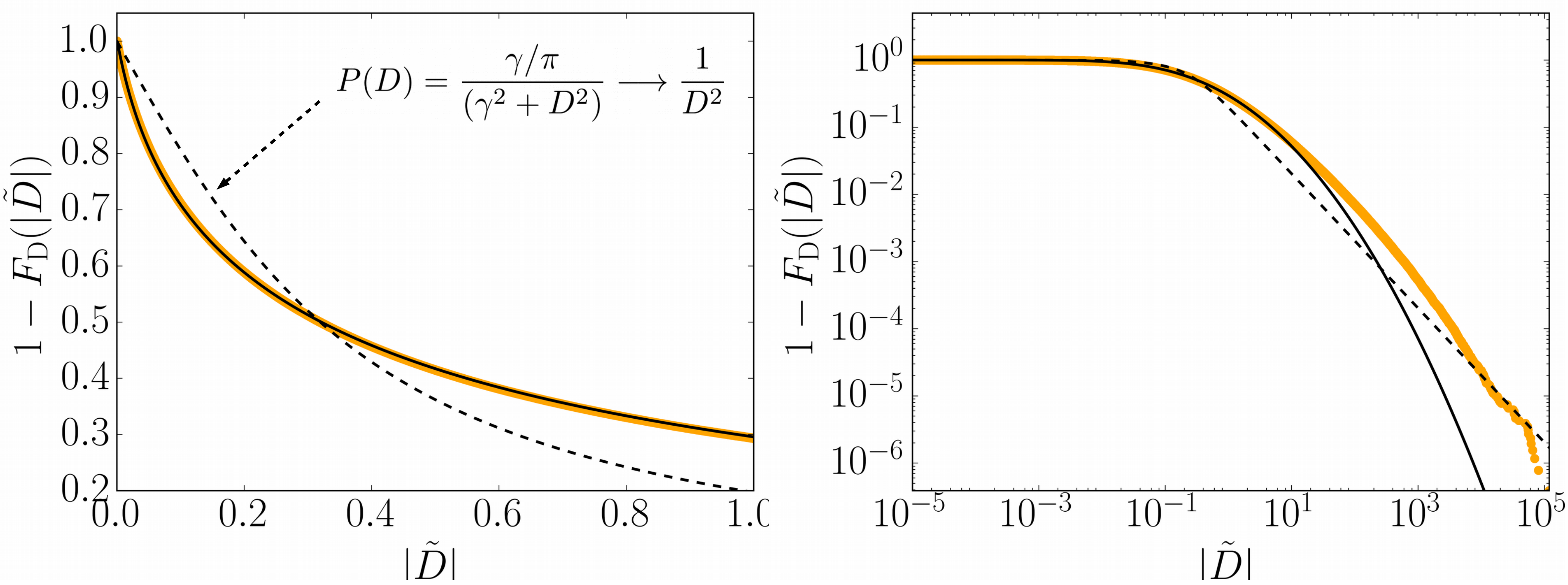

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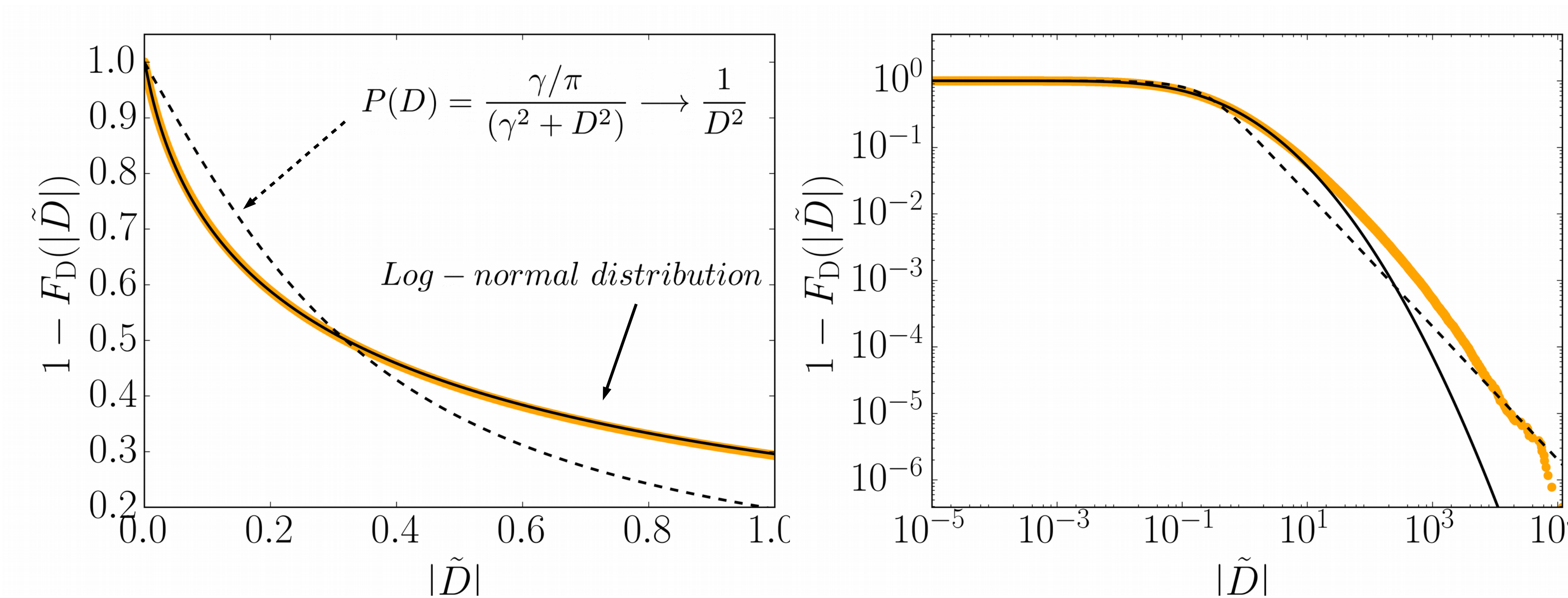

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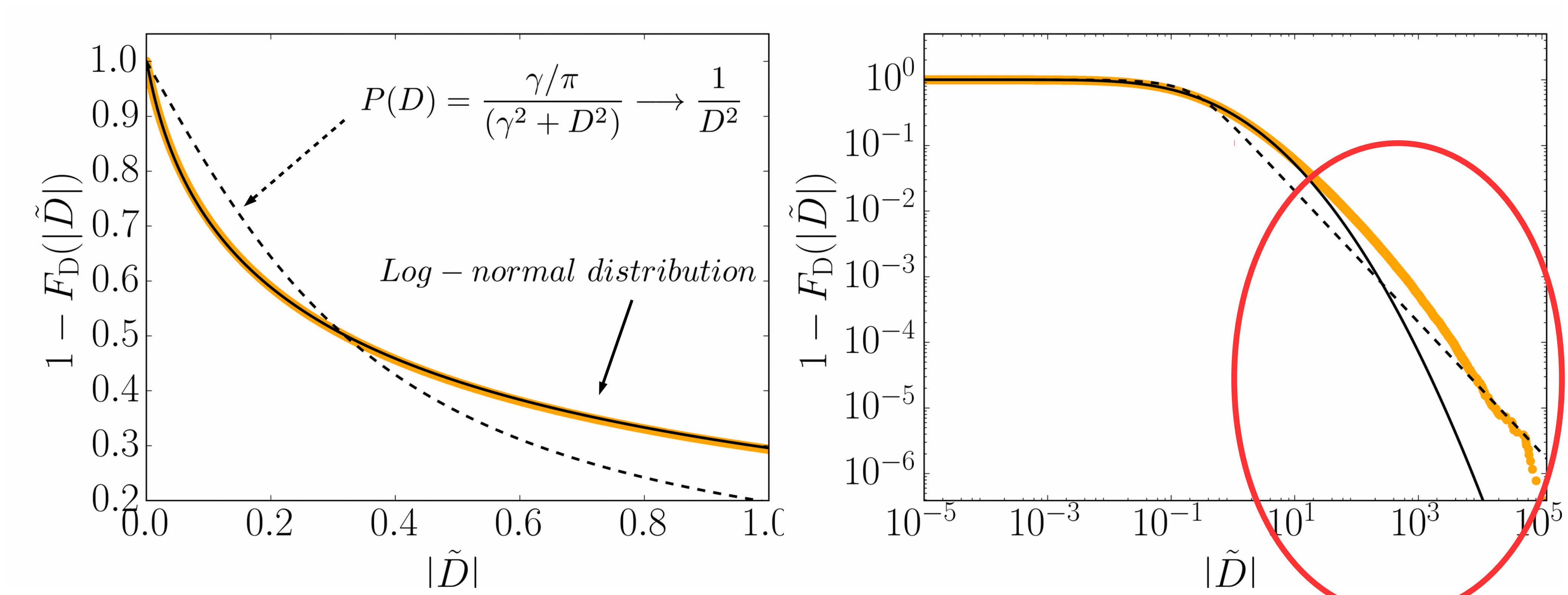
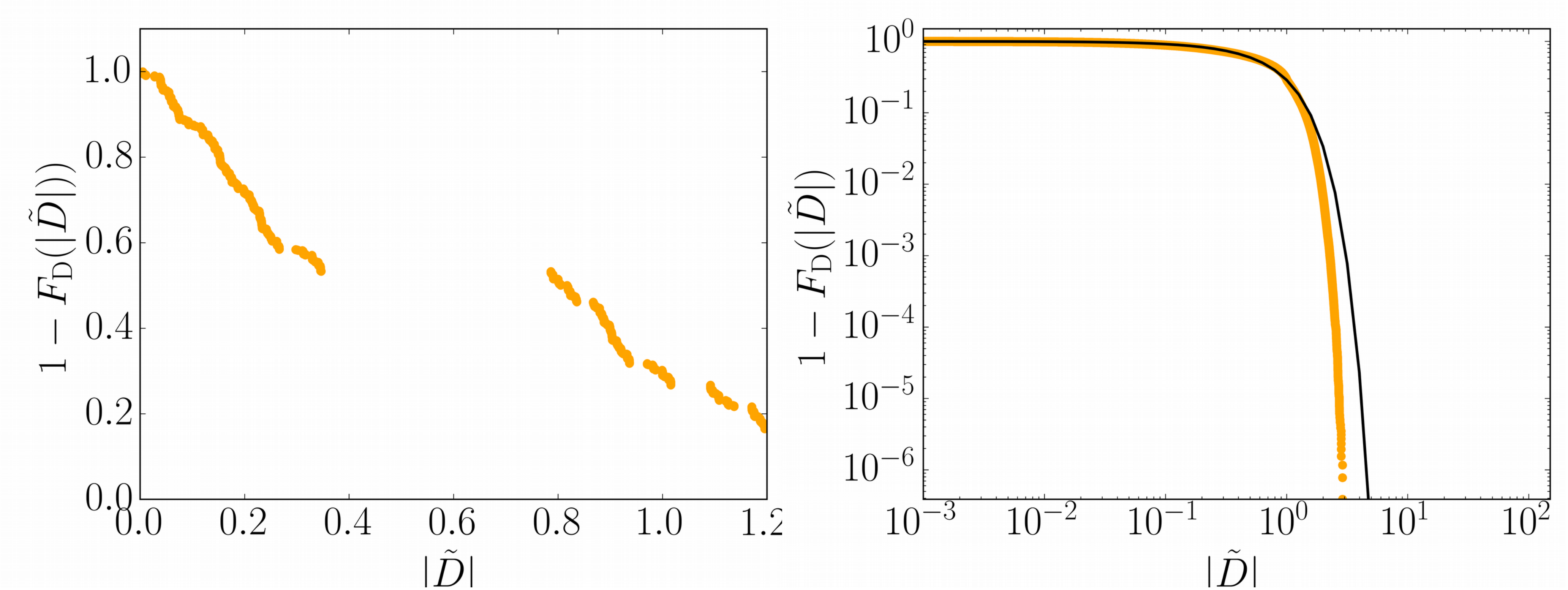


$$D_n = e^2 \frac{4\pi^2}{L} \left[-\frac{1}{2} \langle \mathcal{T} \rangle + L^2 \sum_{m \neq n} \frac{|\langle \psi_n | \mathcal{I} | \psi_m \rangle|^2}{E_n - E_m} \right]$$

Anderson localization

$W = 2.0t$



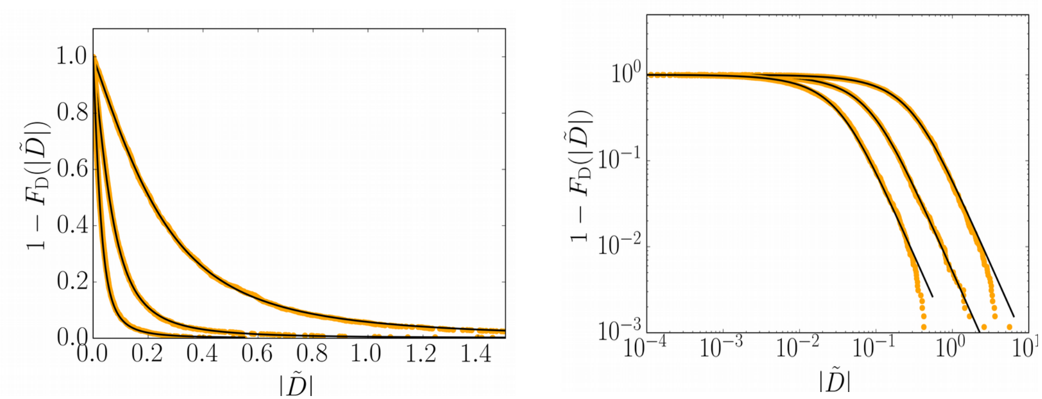
$$D_n = \sum_n d_n$$

d_n : *single particle curvatures*

~ 16 for MB state

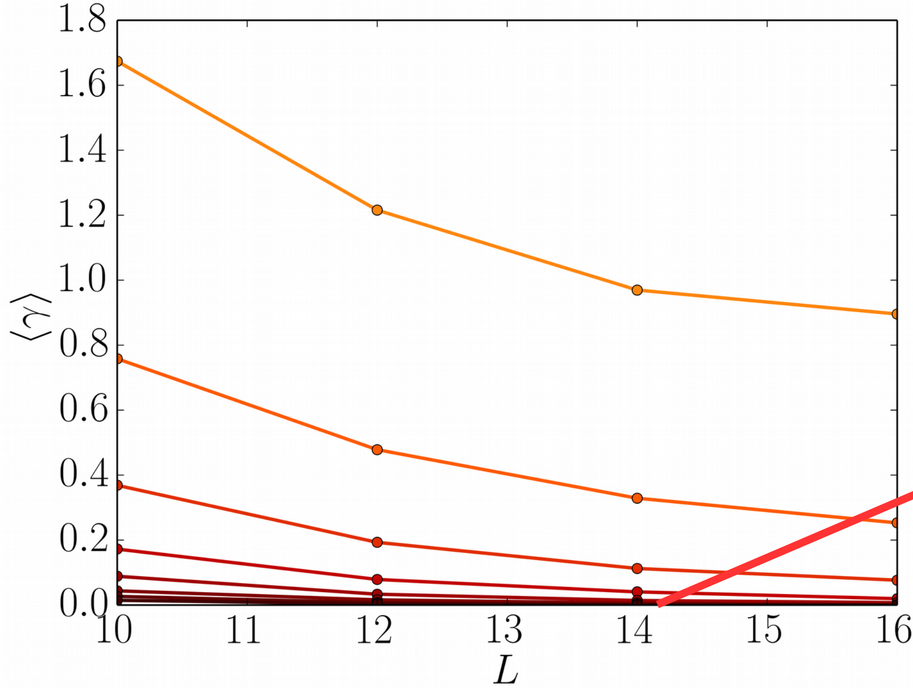
A scaling order parameter for the MBL transition

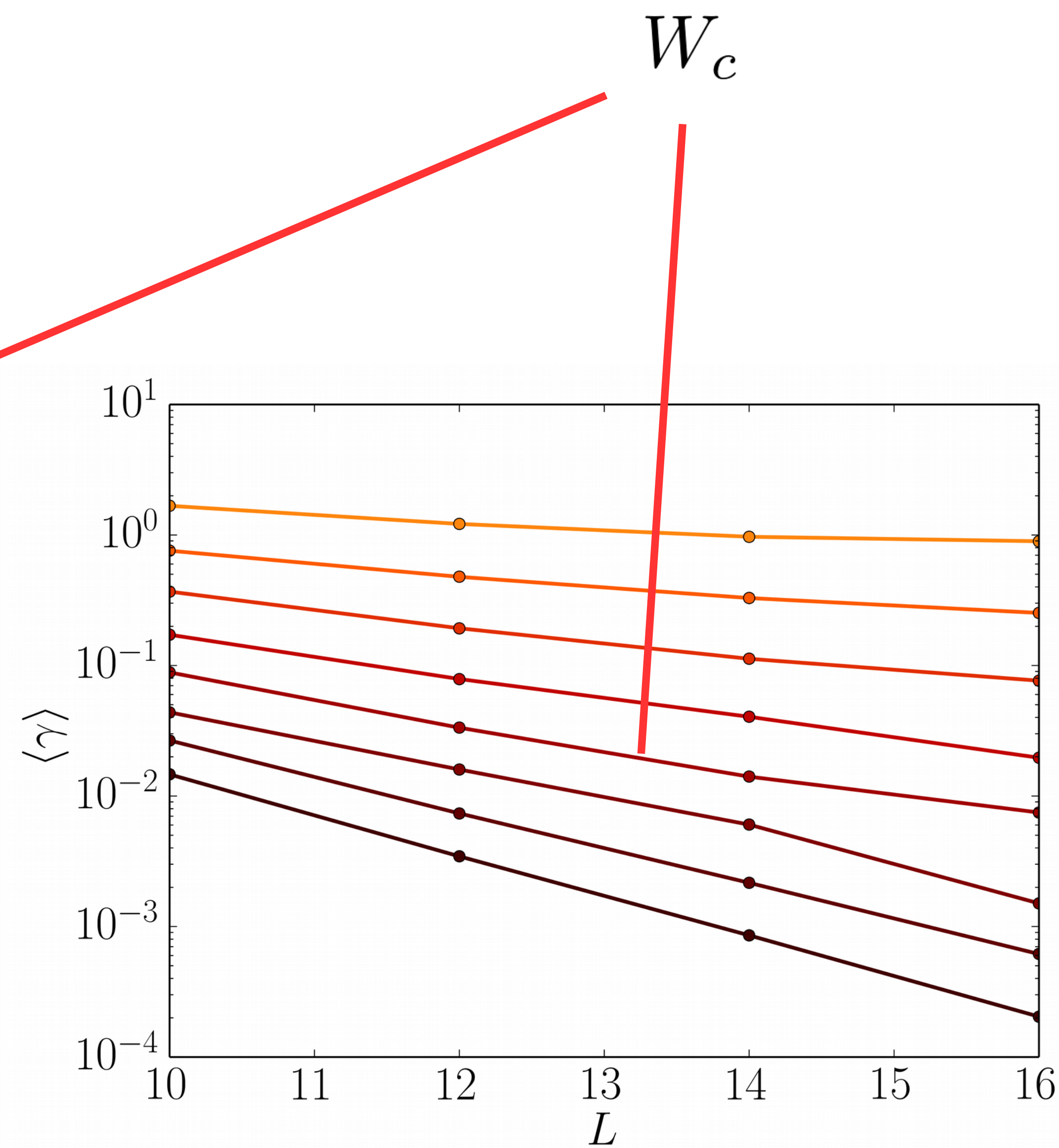
A scaling order parameter for the MBL transition



$$F_{\mathrm{D,RMT}}(D) = \frac{|D|}{\sqrt{\gamma^2 + D^2}} \longrightarrow 1 - \frac{\gamma^2}{2D^2}$$

Width scaling with systems size





Exponential decay for W>Wc

$$\gamma \simeq L \left\langle \left| \frac{\partial E_n}{\partial \Phi} \right| \right\rangle$$

An order parameter for the transition

The dimensionless "Thouless" conductance …

$$\sigma = \frac{\langle \gamma \rangle}{L\Delta}$$

… nicely detects the MBL transition

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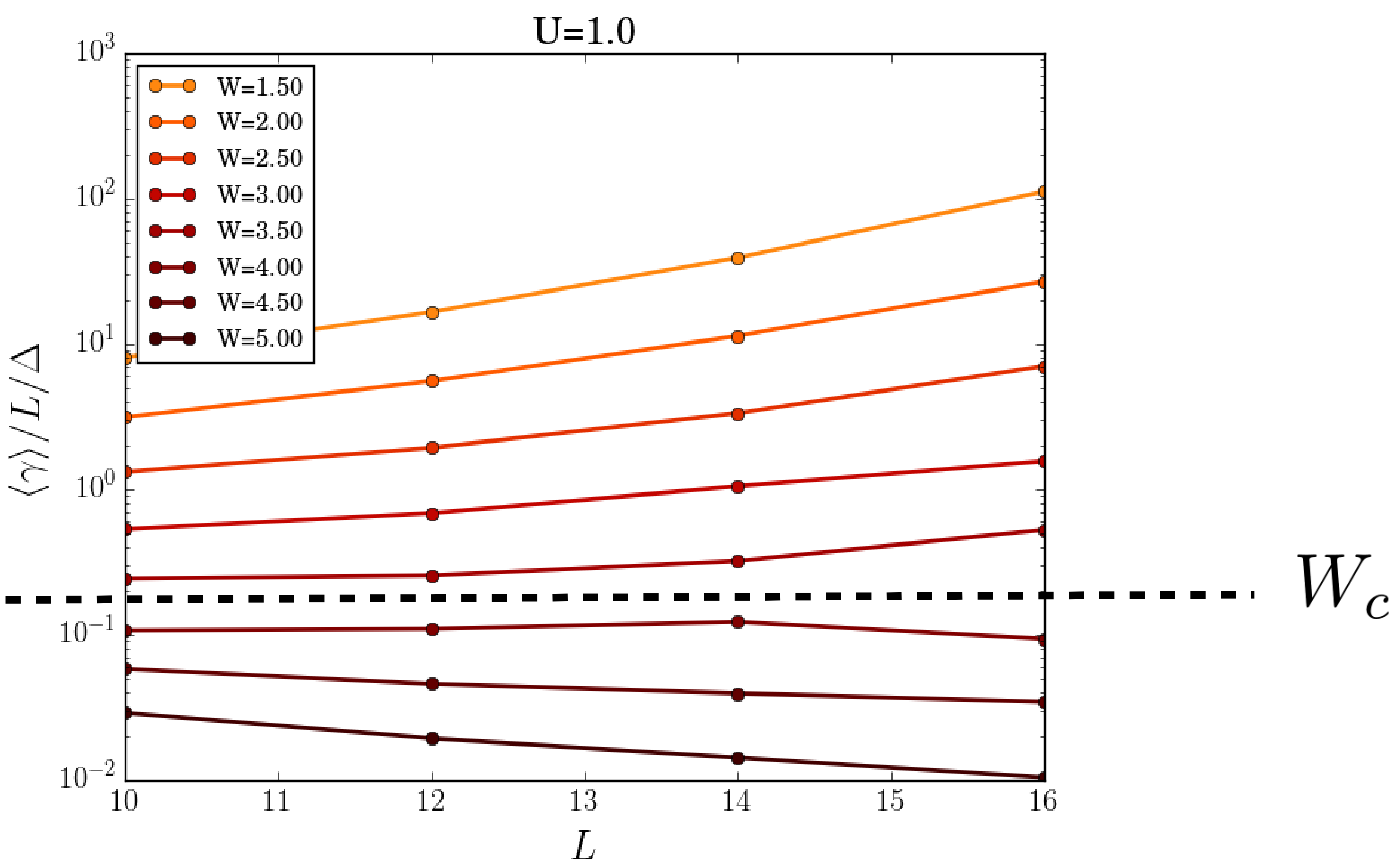
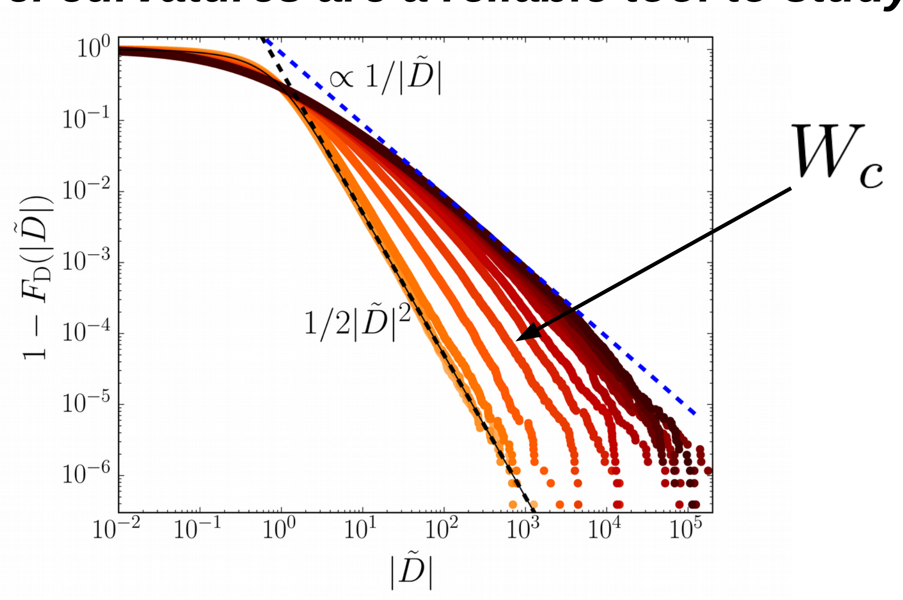
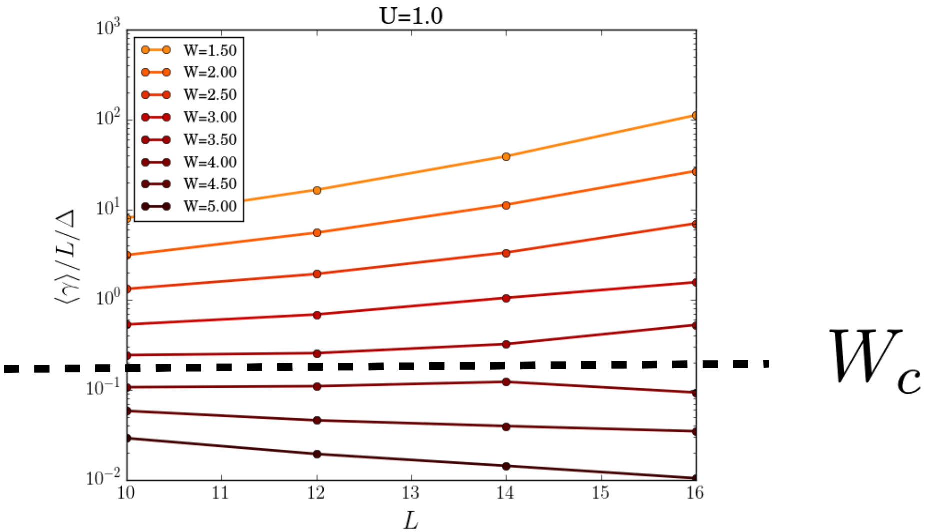


Conclusions

Level curvatures are a reliable tool to study transport in equilibrium MBL systems





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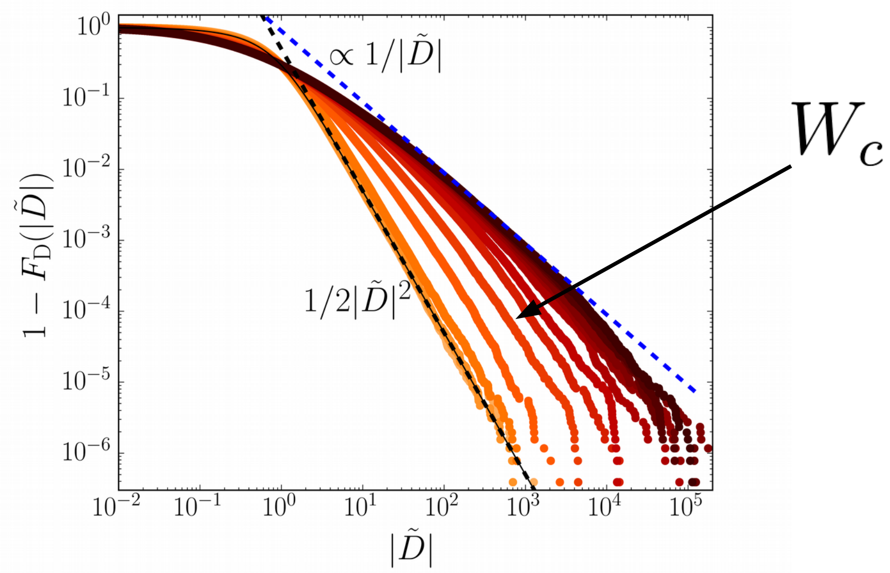

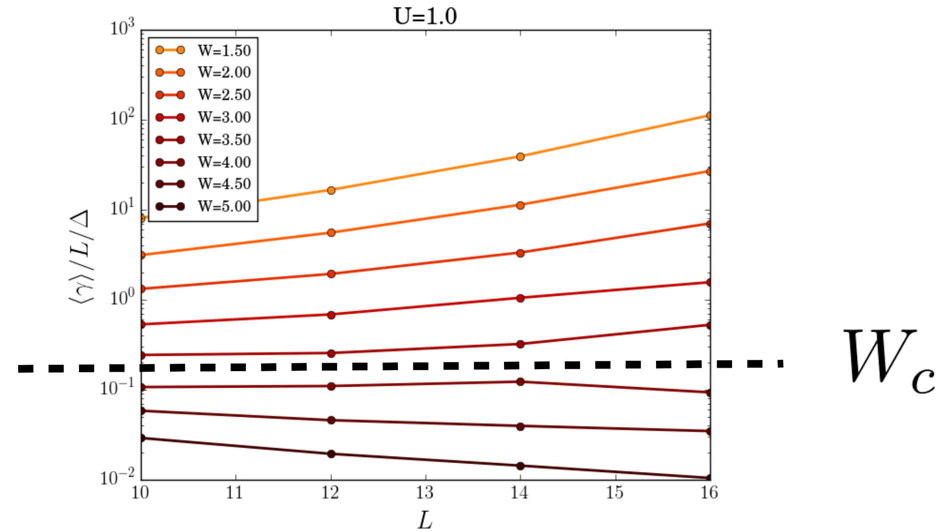

- ***Bigger system sizes and analytical tools are needed***
- ***Interaction increase the localization length of Many-Body systems***
- ***Study mobility edge and the transition***

Thank you for your attention !!!