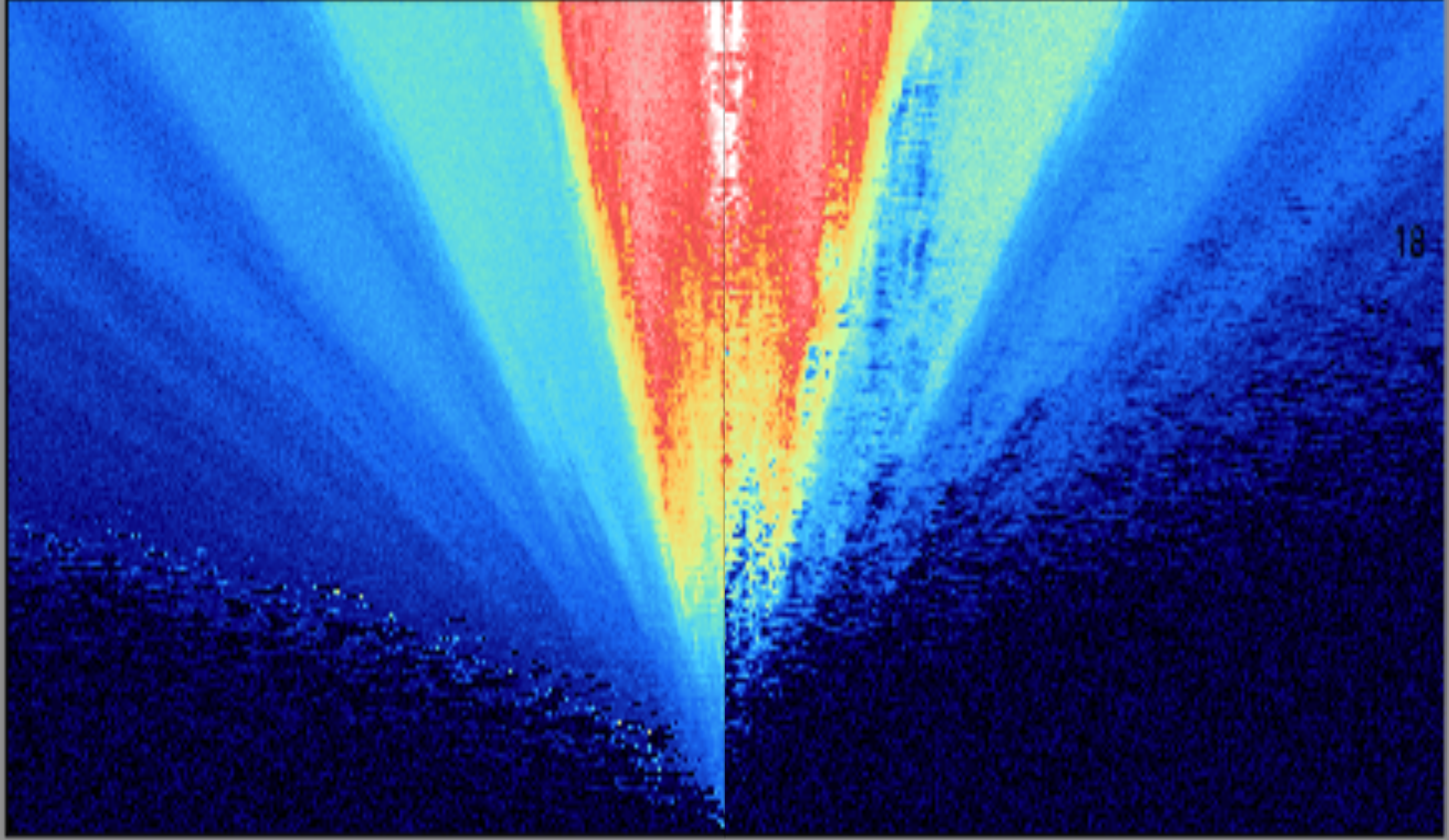


Supercurrent in the quantum Hall regime

F. Amet, C. T. Ke, I. V. Borzenets, J. Wang, K. Watanabe , T. Taniguchi,
R. S. Deacon, M. Yamamoto, Y. Bomze, S. Tarucha, G. Finkelstein

Support:
DOE and ARO

Outline

QH junctions and Andreev bound states

Ballistic Josephson junctions in graphene

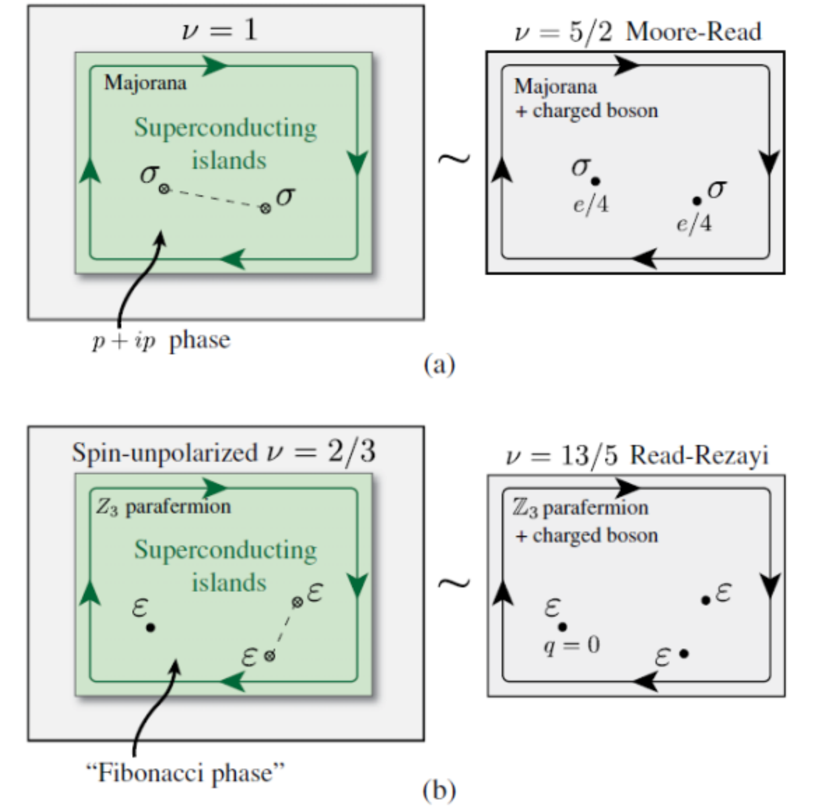
Superconductivity in magnetic field

Microscopic origins and hybrid edge modes

Motivation

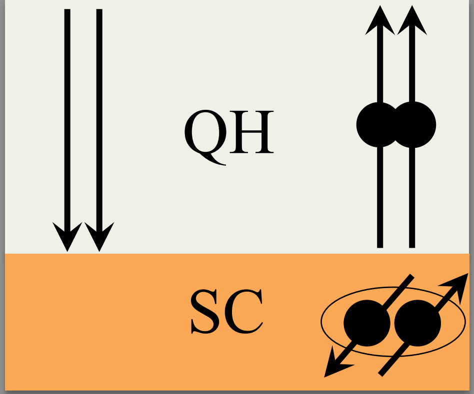
Coupling superconductivity to topological material

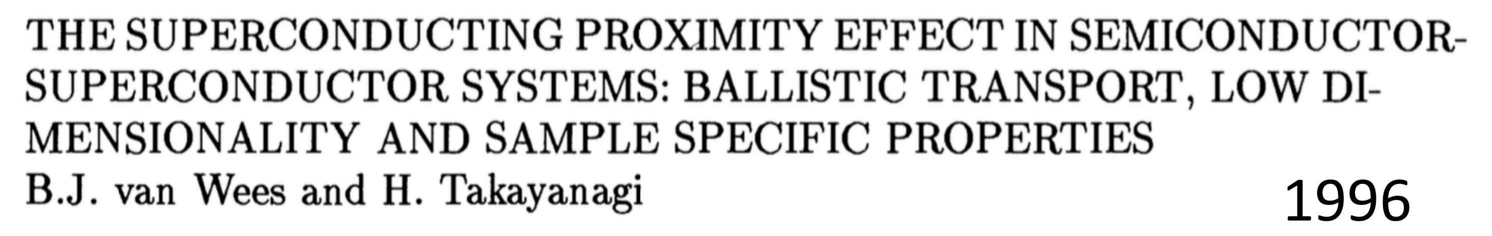
QHE + SC is a natural direction



MONG et al. PHYS. REV. X (2014)

But QHE and SC do not mix well





THE SUPERCONDUCTING PROXIMITY EFFECT IN SEMICONDUCTOR-SUPERCONDUCTOR SYSTEMS: BALLISTIC TRANSPORT, LOW DIMENSIONALITY AND SAMPLE SPECIFIC PROPERTIES

B.J. van Wees and H. Takayanagi

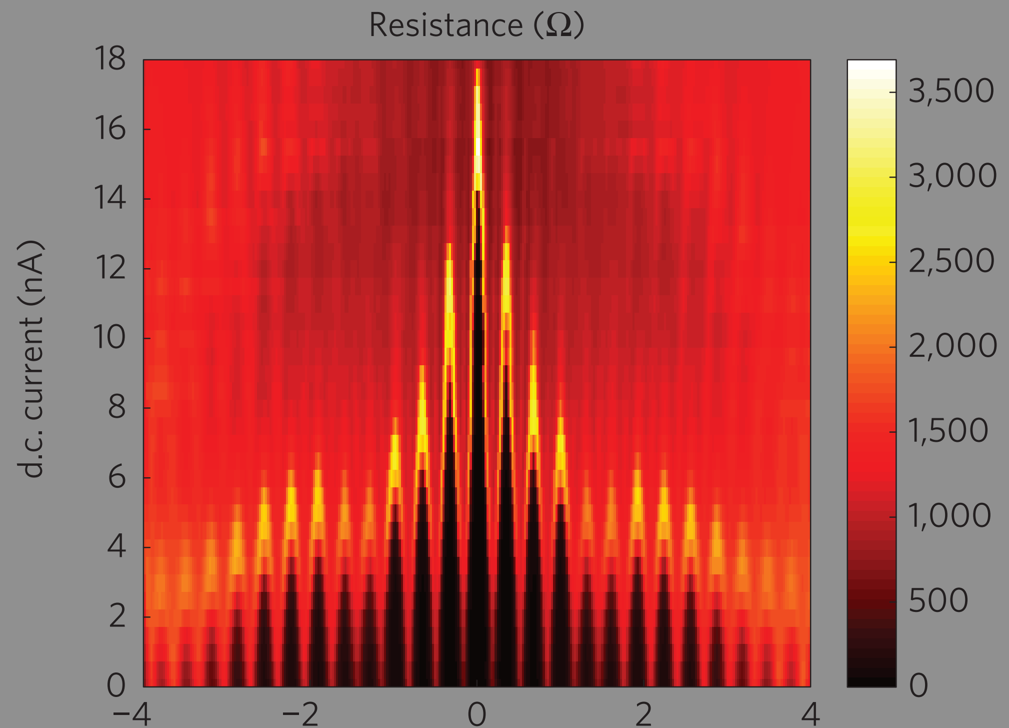
1996

Example: supercurrent in 2D topological insulators

Counterpropagating helical states with opposite spins

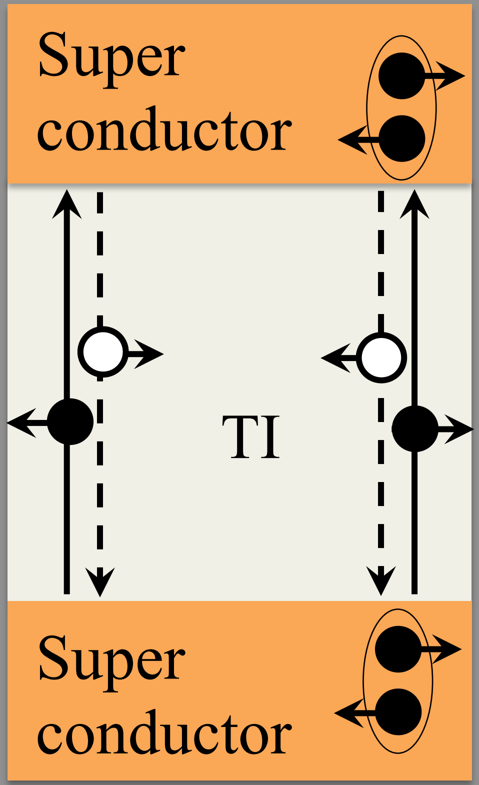
Each edge can form its own Andreev bound states
and can carry its own supercurrent

In magnetic field the two supercurrents interfere,
resulting in a SQUID-like pattern



Hart et al.,

Nature Physics (2014)

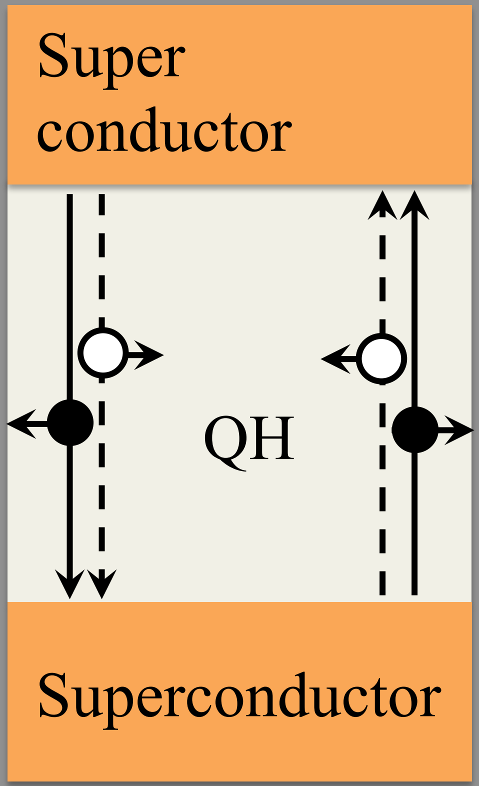


Supercurrent in QH

Chiral states on one edge propagate in the same direction

(both electrons and holes, spin up and down)

How could Andreev bound states be formed?



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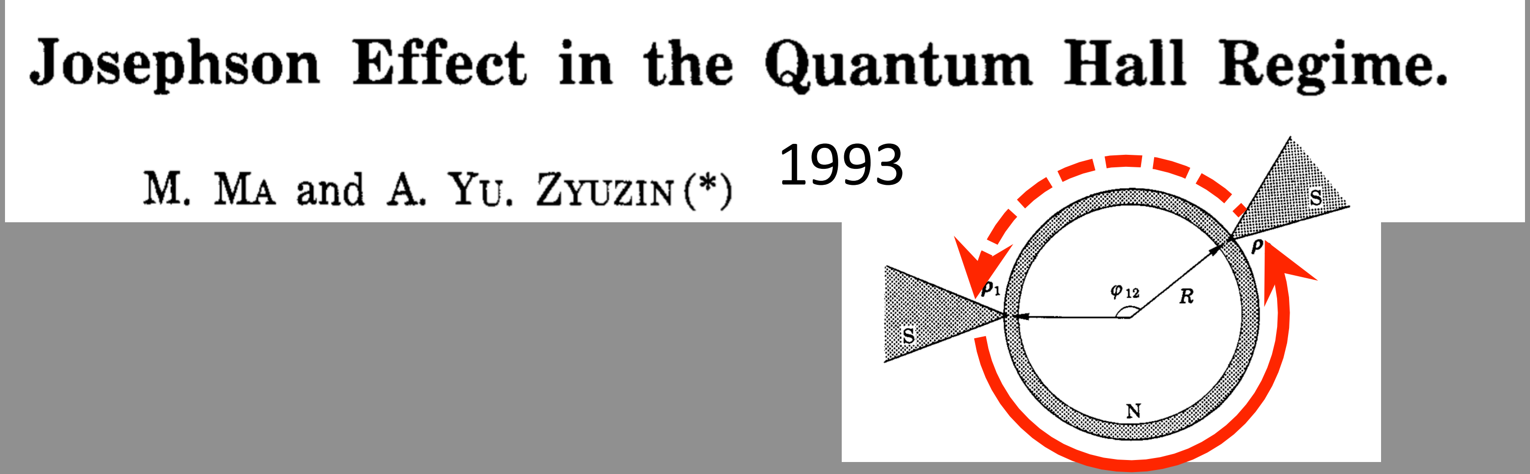
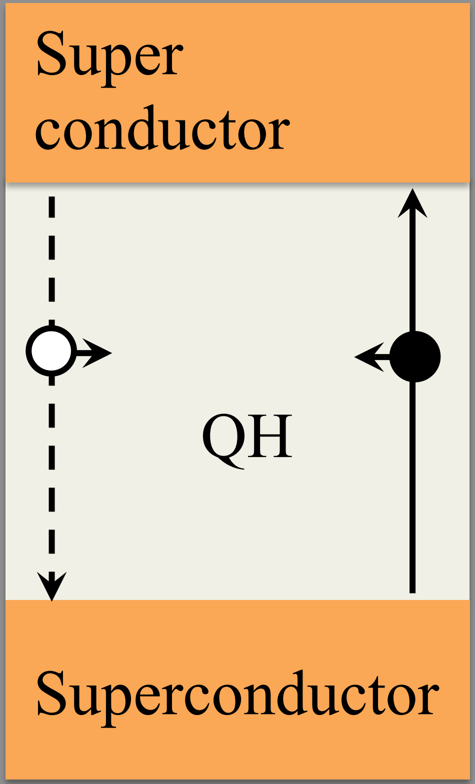


Josephson Effect in the Quantum Hall Regime.

M. Ma and A. Yu. Zyuzin (*)

1993



Super conductor

QH

Superconductor

Outline

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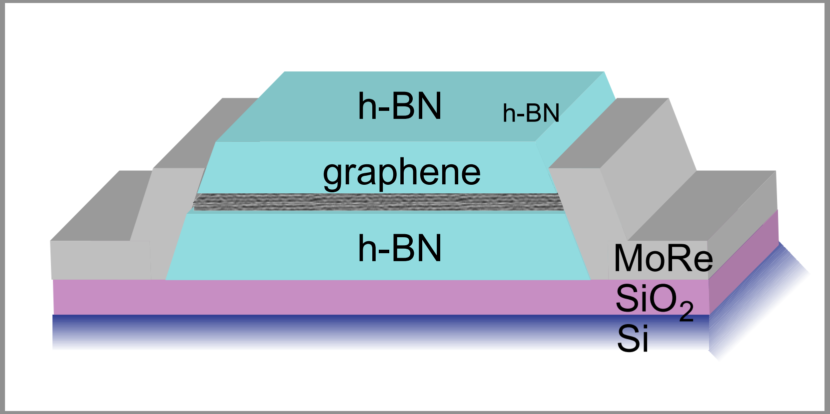
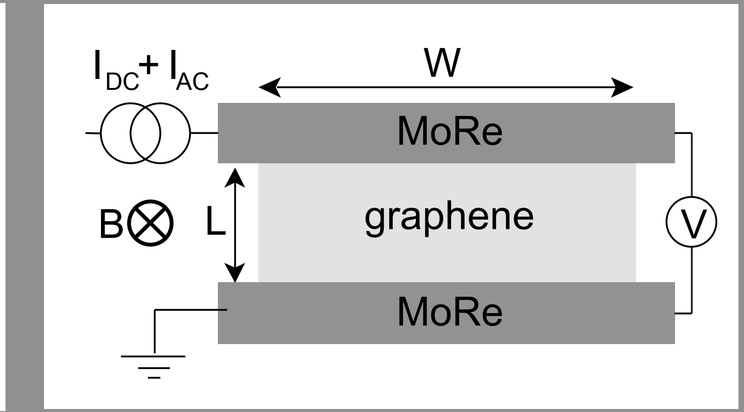
Samples: h-BN/graphene/h-BN stacks

- Exfoliated graphene sandwiched by h-BN with quasi-1D edge contacts
- High mobility, QH regime observed < 1 T
- Molybdenum-Rhenium (MoRe): T_C at 10 K with H_{C2} ≈8 T
- Two-terminal geometry, four-probe measurement
- Base temperature of 35 mK

V. E. Calado et al.,
Nat. Nano. (2015)

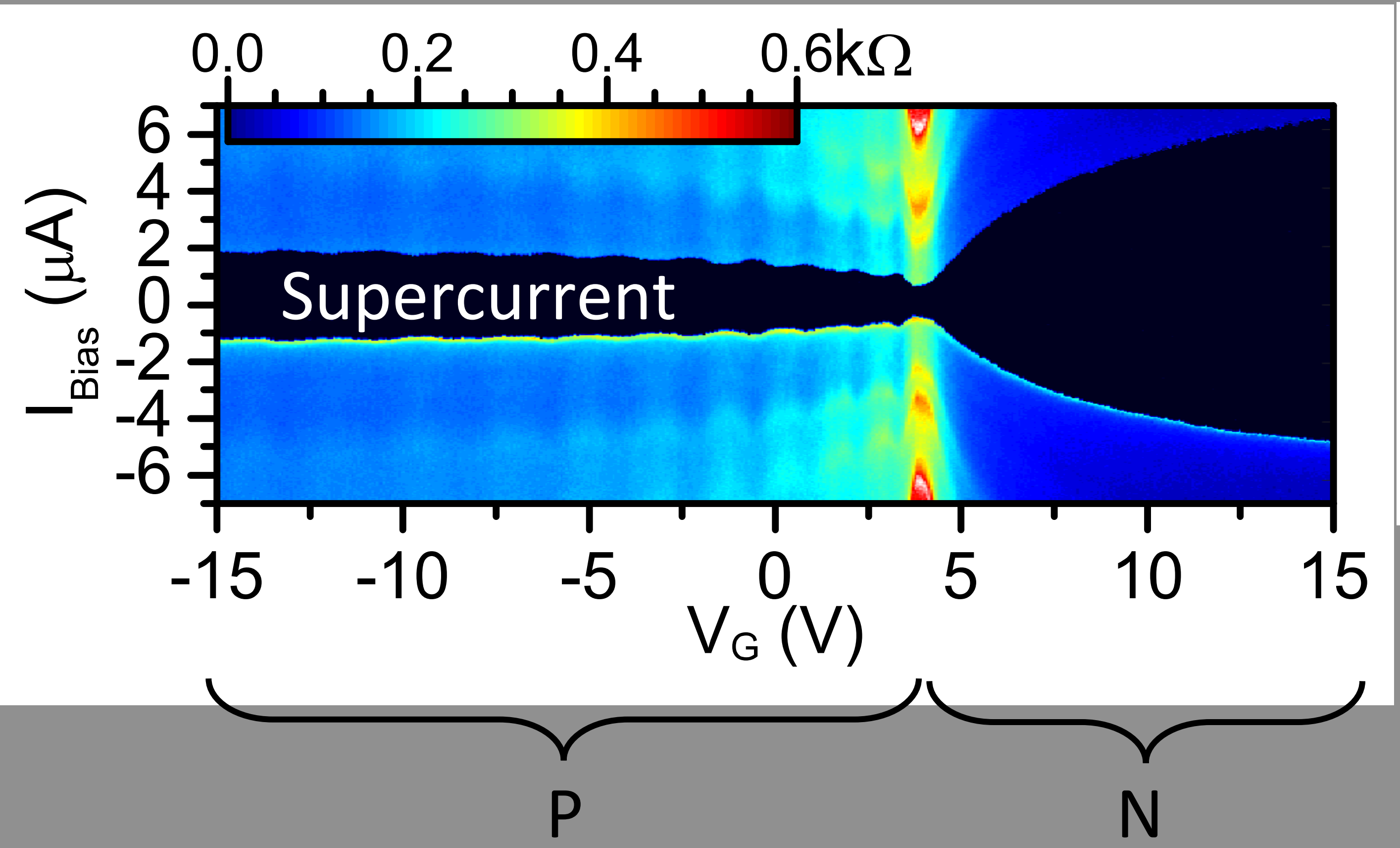




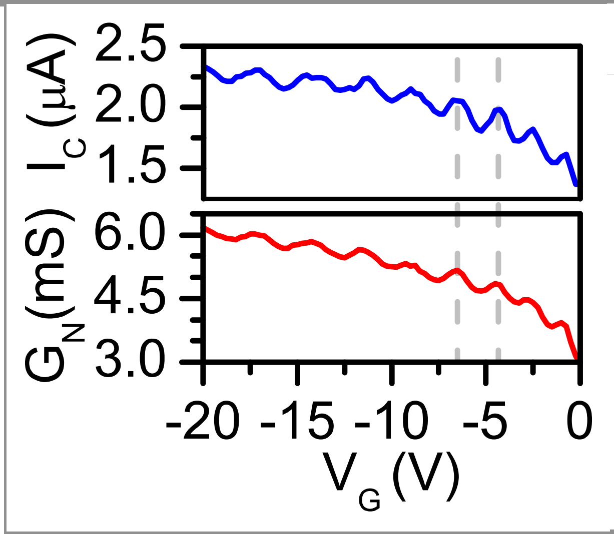
Four junctions studied
in the QHE regime:

	L	W
J1	0.3μm	2.4μm
J2	0.8μm	2.4μm
J3	0.65μm	4.5μm
J4	0.5μm	2.7μm

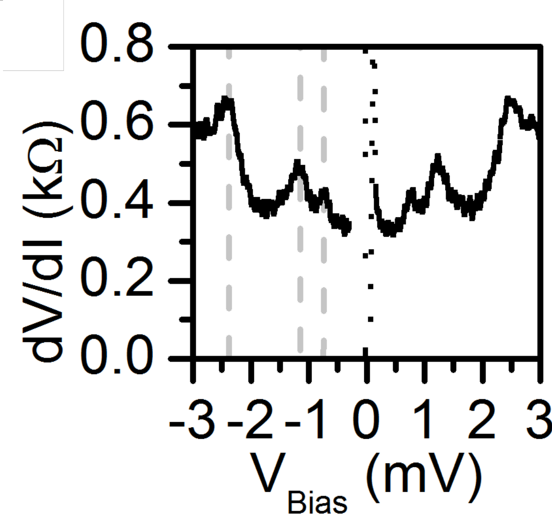
Zero-field properties



Fabry-Perot



MAR

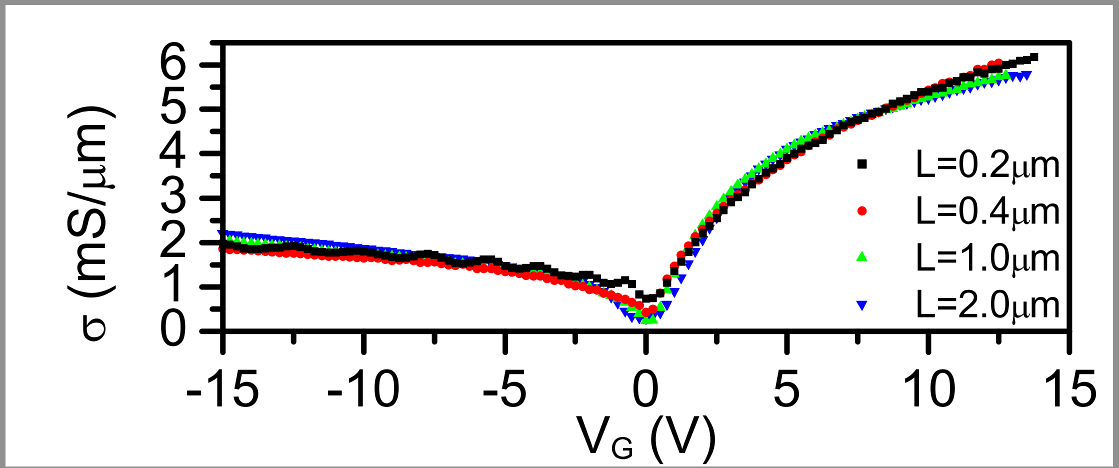


Ballistic Josephson junctions

V. E. Calado et al., Nature Nano. (2015).

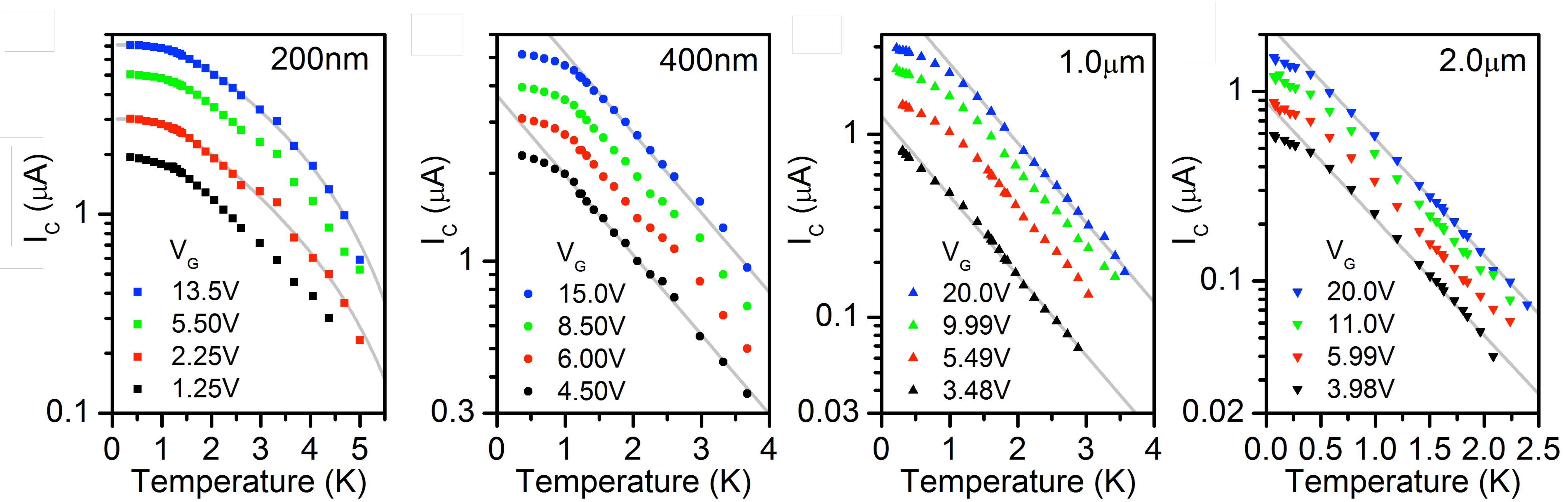
M. T. Allen et al., Nat. Phys. (2016).

M. Ben Shalom et al., Nat. Phys. (2016).



PRL, accepted

Ballistic junctions: from short to long



Short vs. long ballistic junction: L vs. $\xi = \hbar v_F/\Delta$ = 500 nm

Long ballistic junction: $I_C \propto \exp(-kT/\delta E)$, where $\delta E = \hbar v_F/2\pi L$

Kulik (1970)

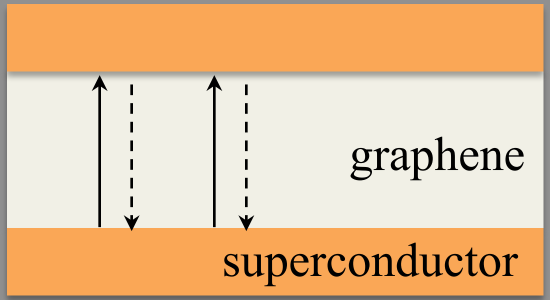
PRL, accepted

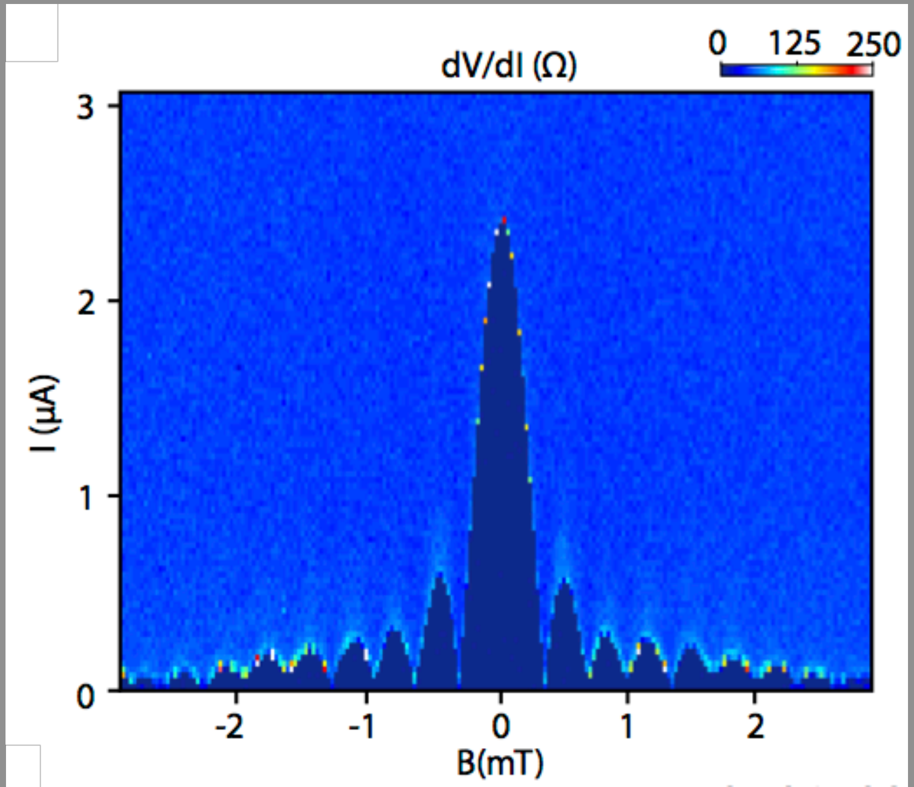
Small B-field: Fraunhofer pattern

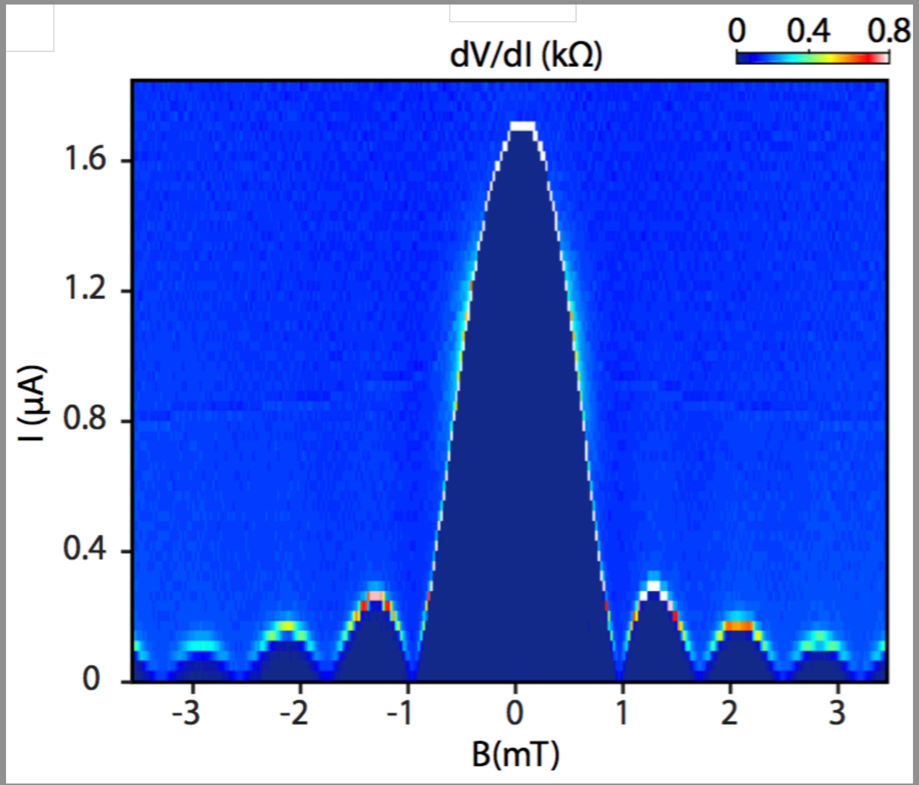
Local phase difference depends on position along the contacts

$I_C(B) \propto |\sin(\pi\Phi/\Phi_0)| / (\pi\Phi/\Phi_0)$

$\Phi = WL \times B \qquad \Phi_0 = h/2e$







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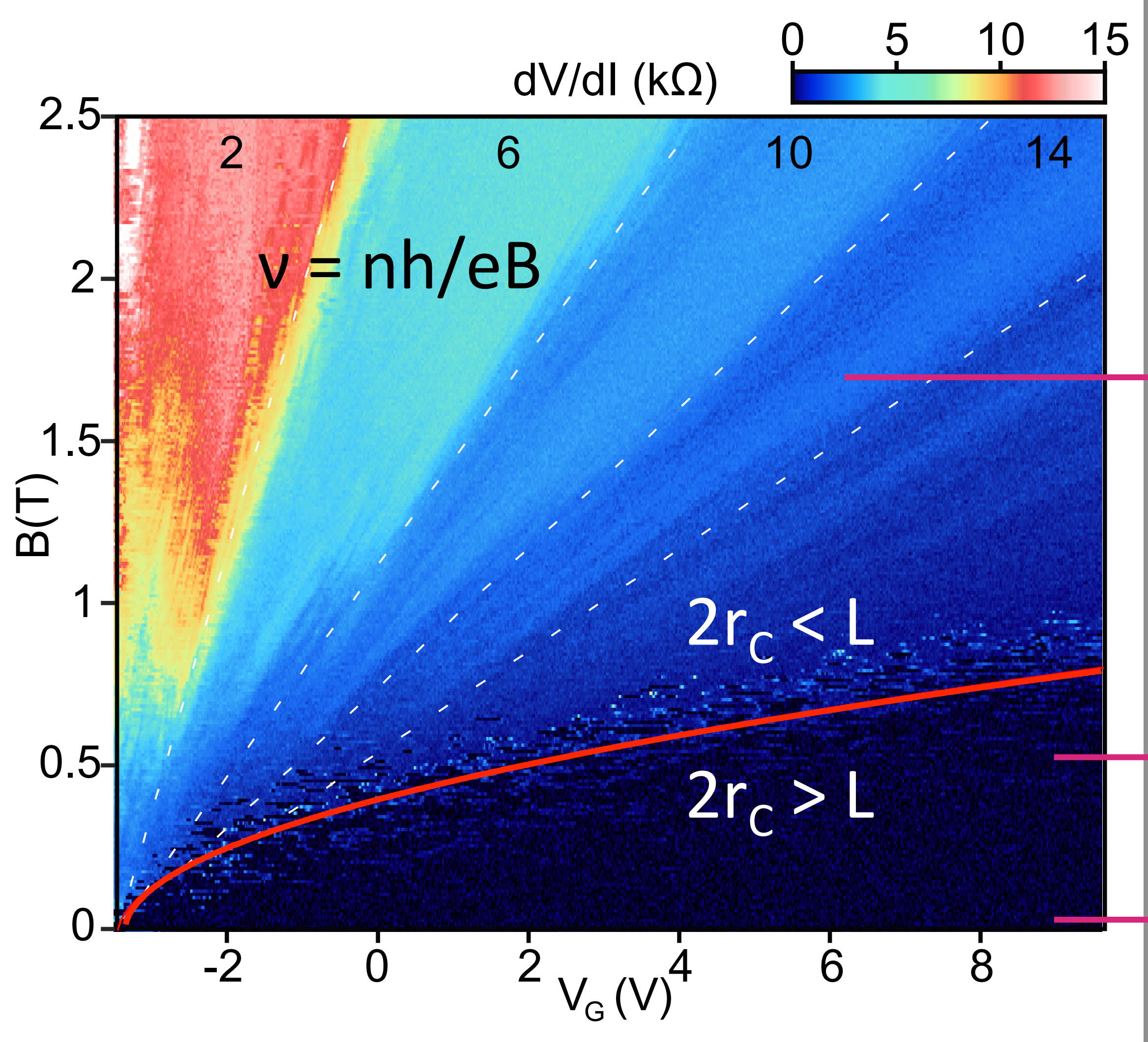
Superconductivity in magnetic field ⇦

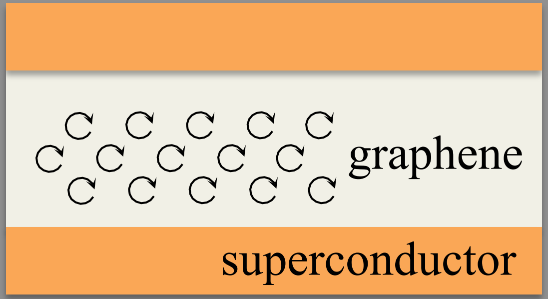
Microscopic origins and hybrid edge modes

Resistance map in magnetic field and quantum Hall



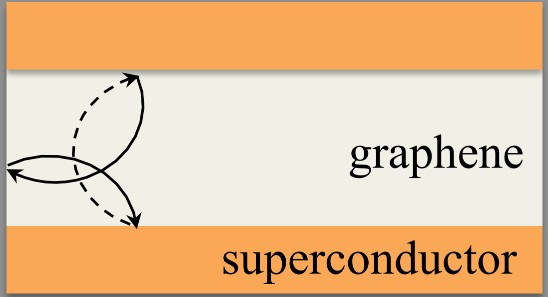
QHE

P. Rickhaus et al.,
Nano Lett. (2012)
K. Komatsu et al.,
Phys. Rev. B (2012)
V. E. Calado et al.,
Nat. Nano. (2015)

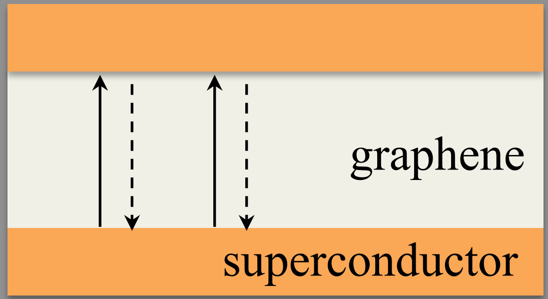


Semiclassical

M. Ben Shalom et al.,
Nat. Phys. (2016)

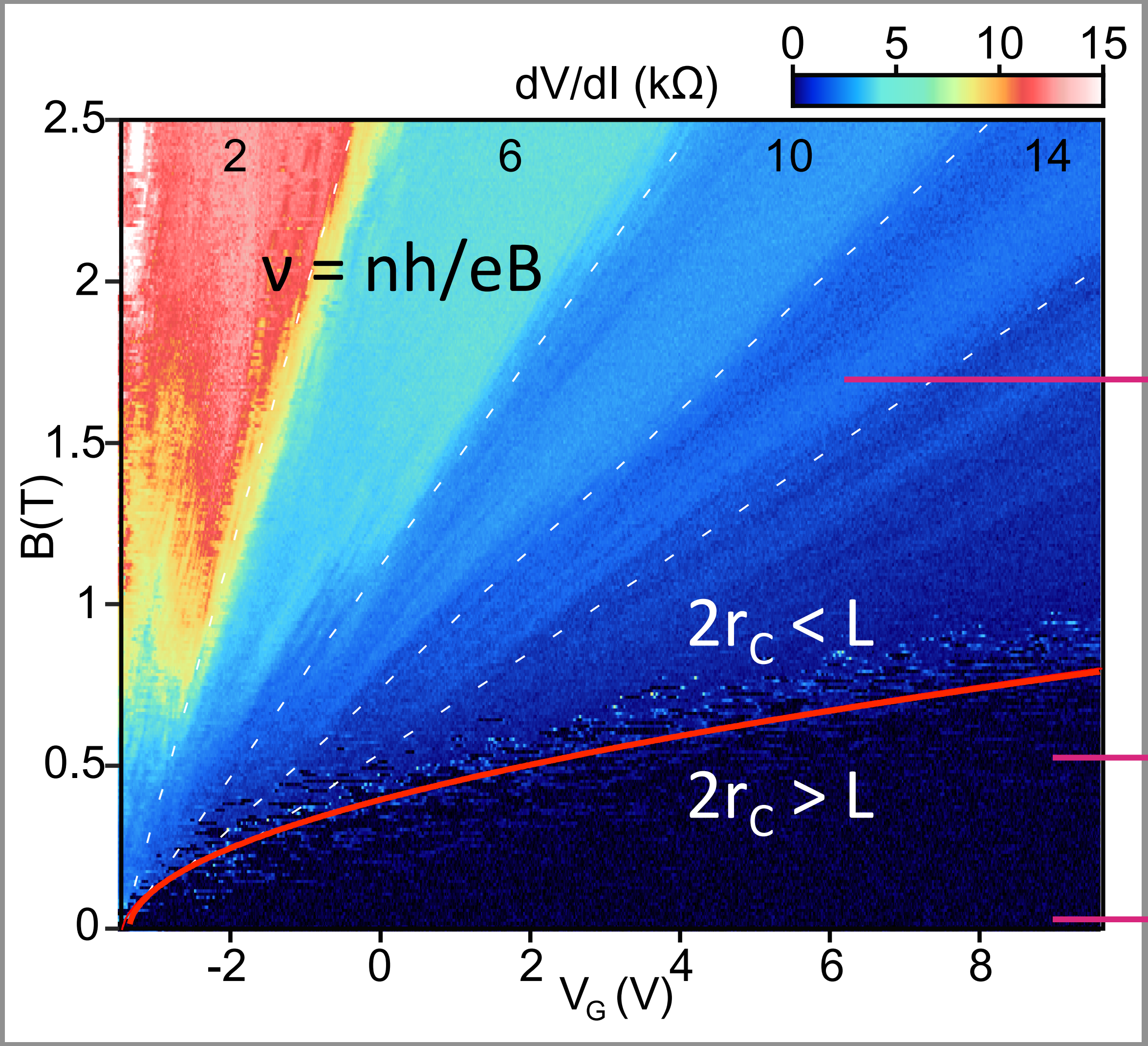


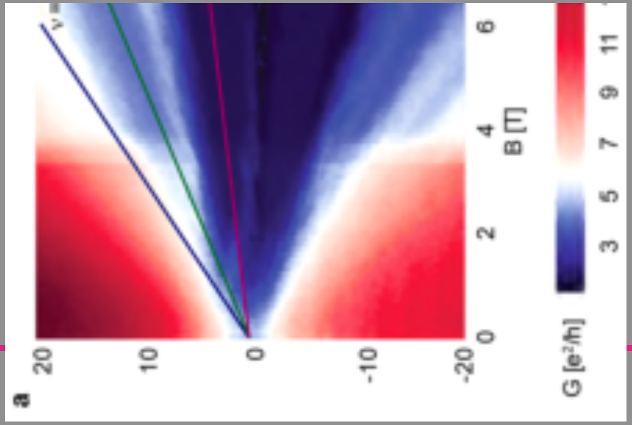
Fraunhofer

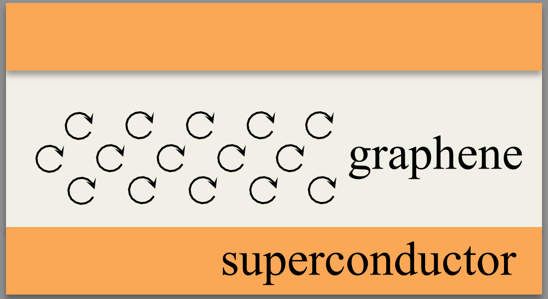


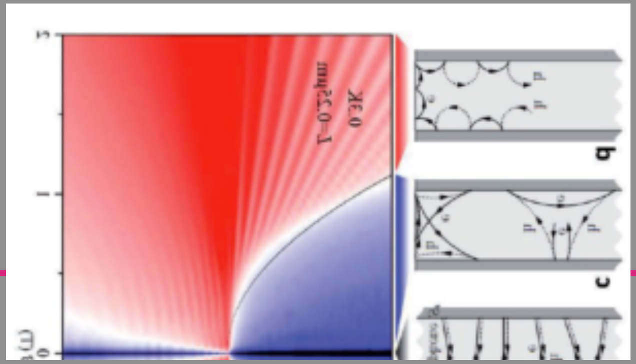
Semiclassical: cyclotron diameter $2r_C > L$
QH regime: cyclotron diameter $2r_C < L$

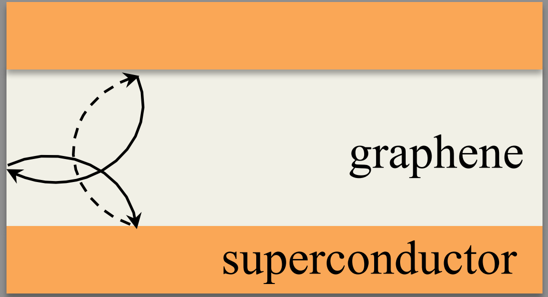
Resistance map in magnetic field and quantum Hall



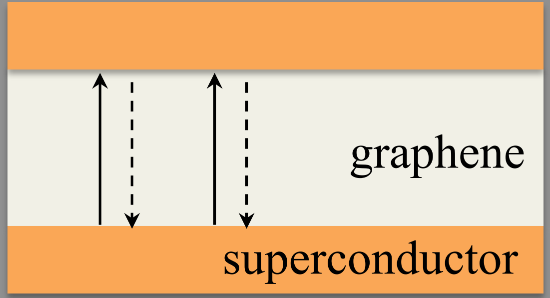








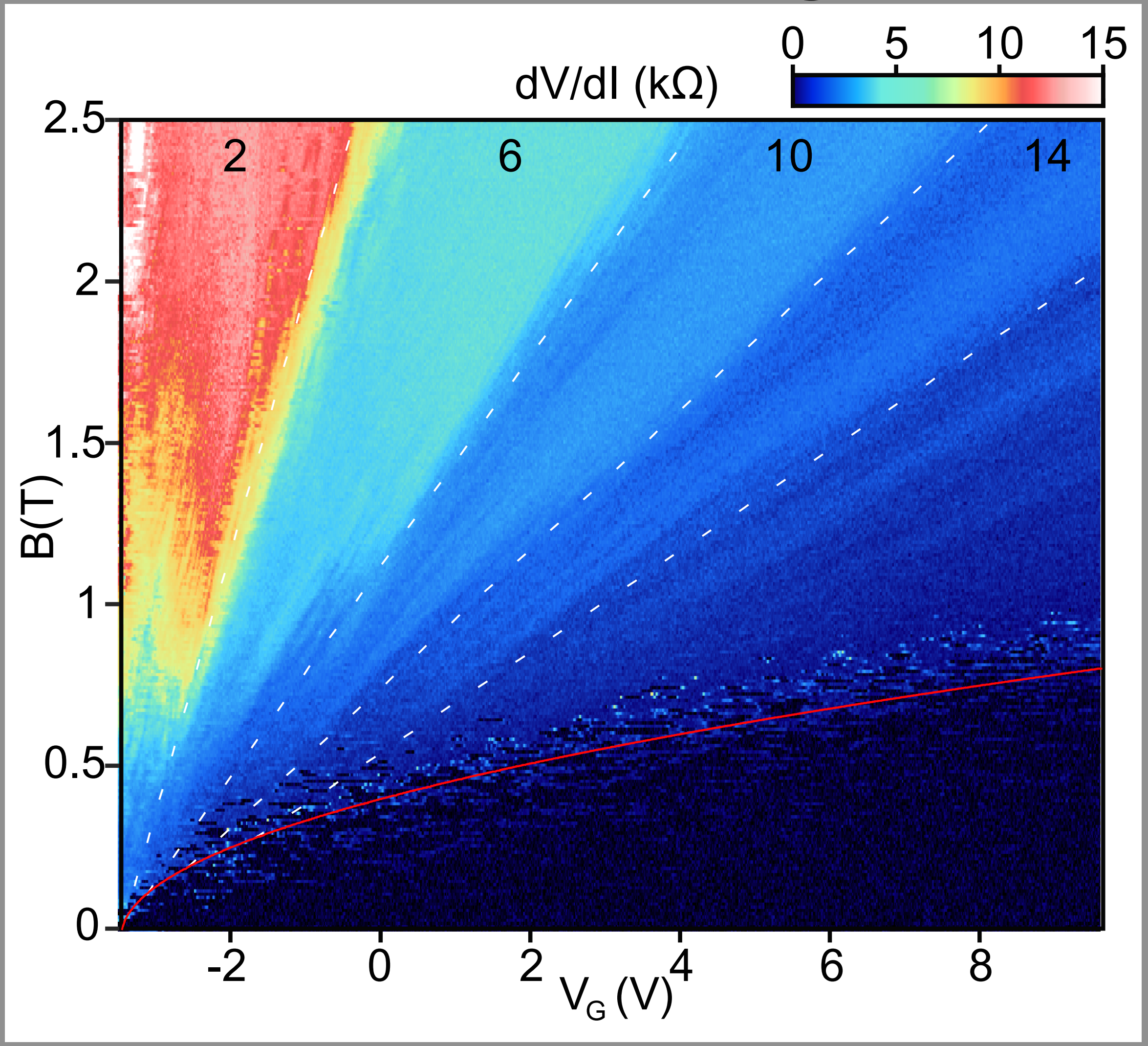
Fraunhofer



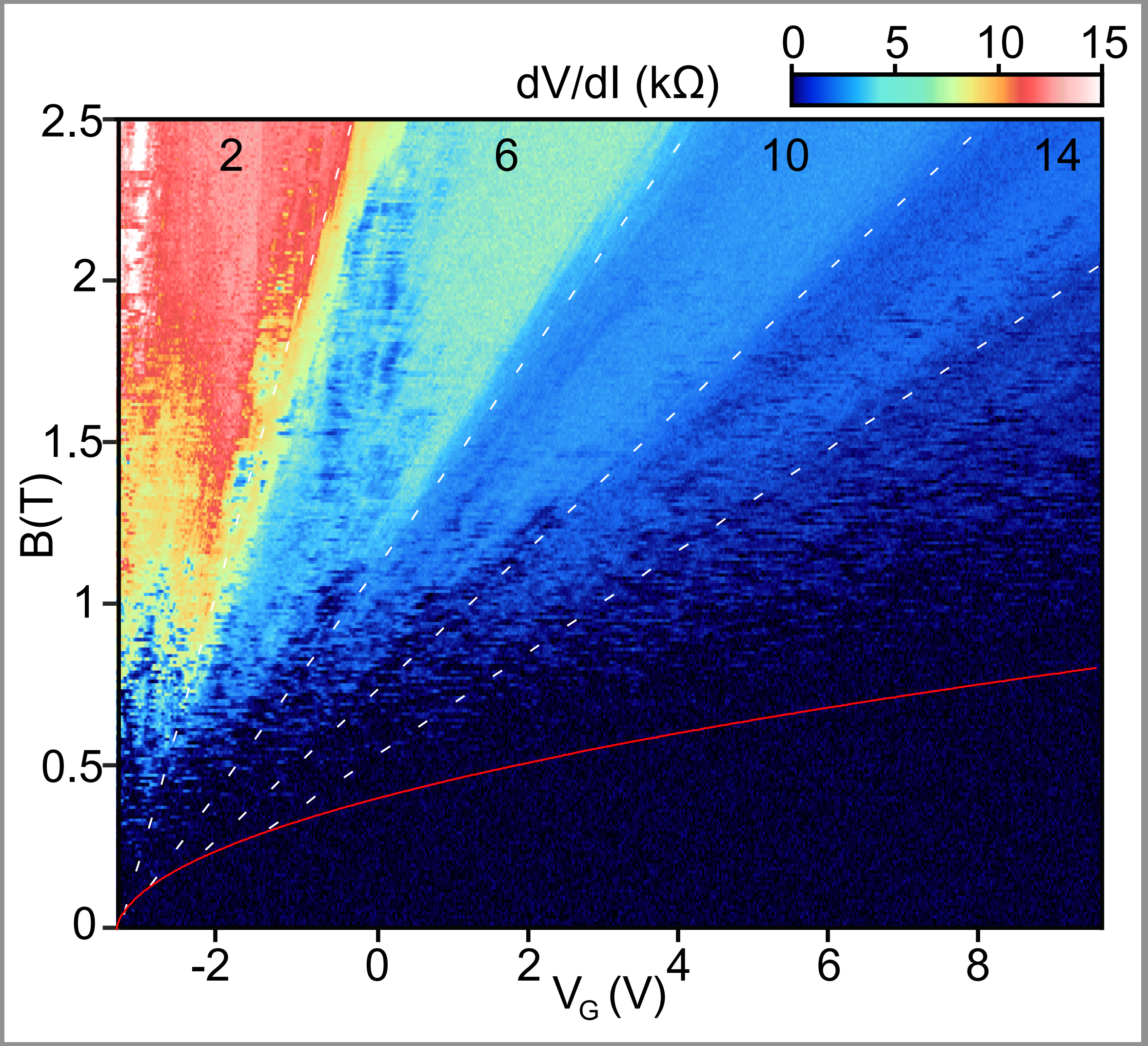
Semiclassical: cyclotron diameter $2r_C > L$

QH regime: cyclotron diameter $2r_C < L$

Quantum Hall regime

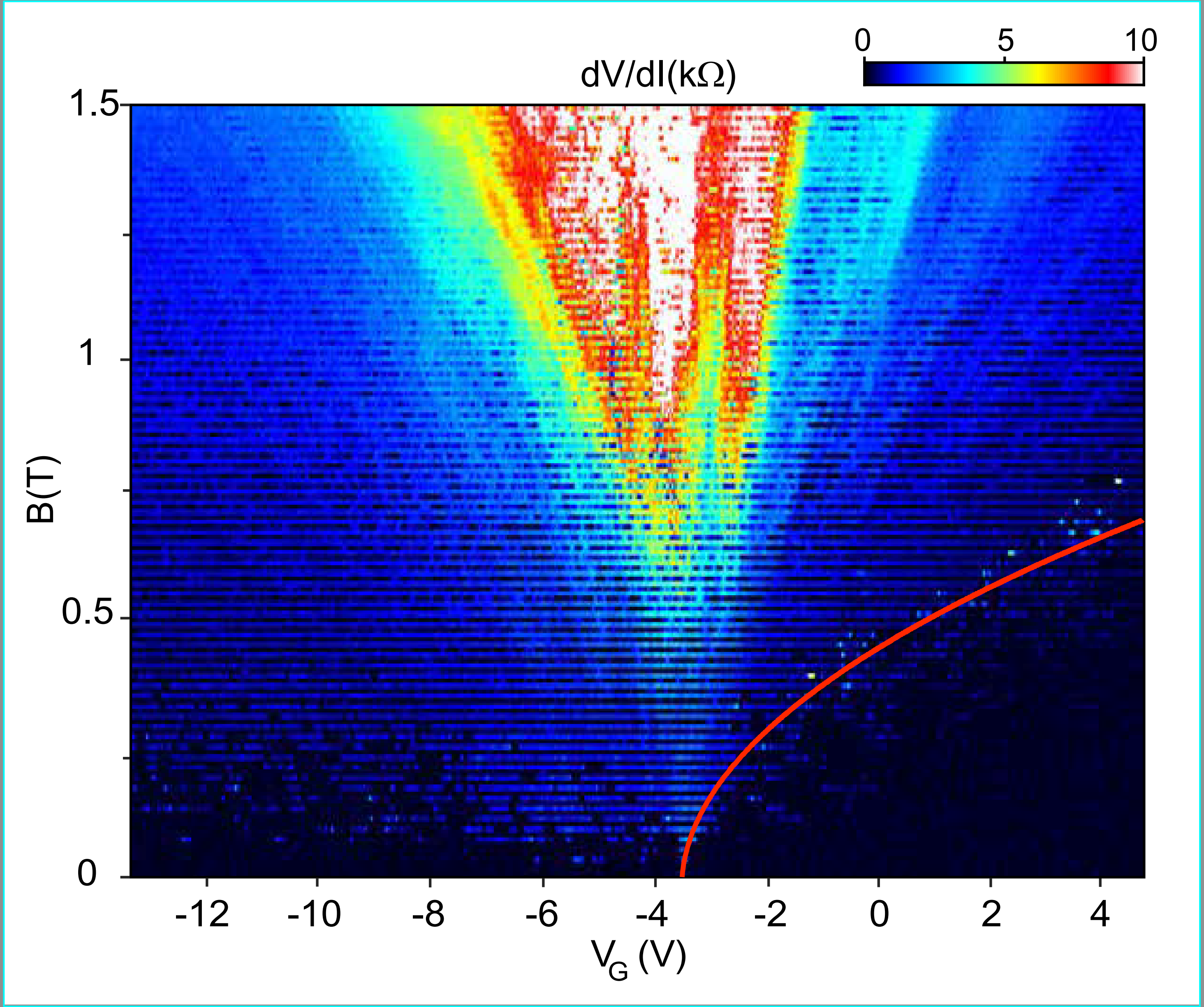


- Left panel: I_{DC}=6nA + I_{AC}=50 pA
- Quantized plateaus



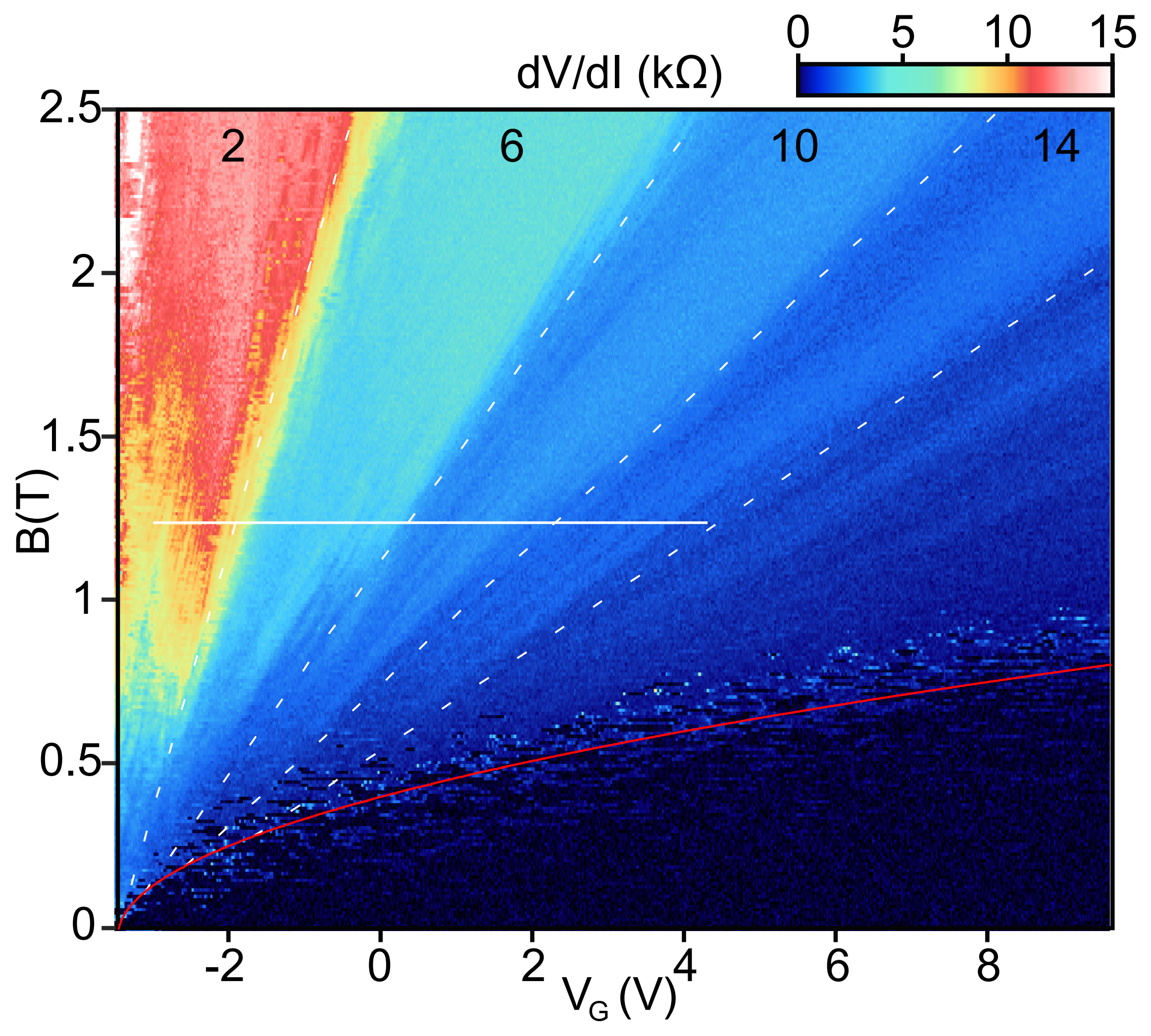
- Right panel: I_{DC}=0nA + I_{AC}=50 pA
- Superconducting pockets on plateaus

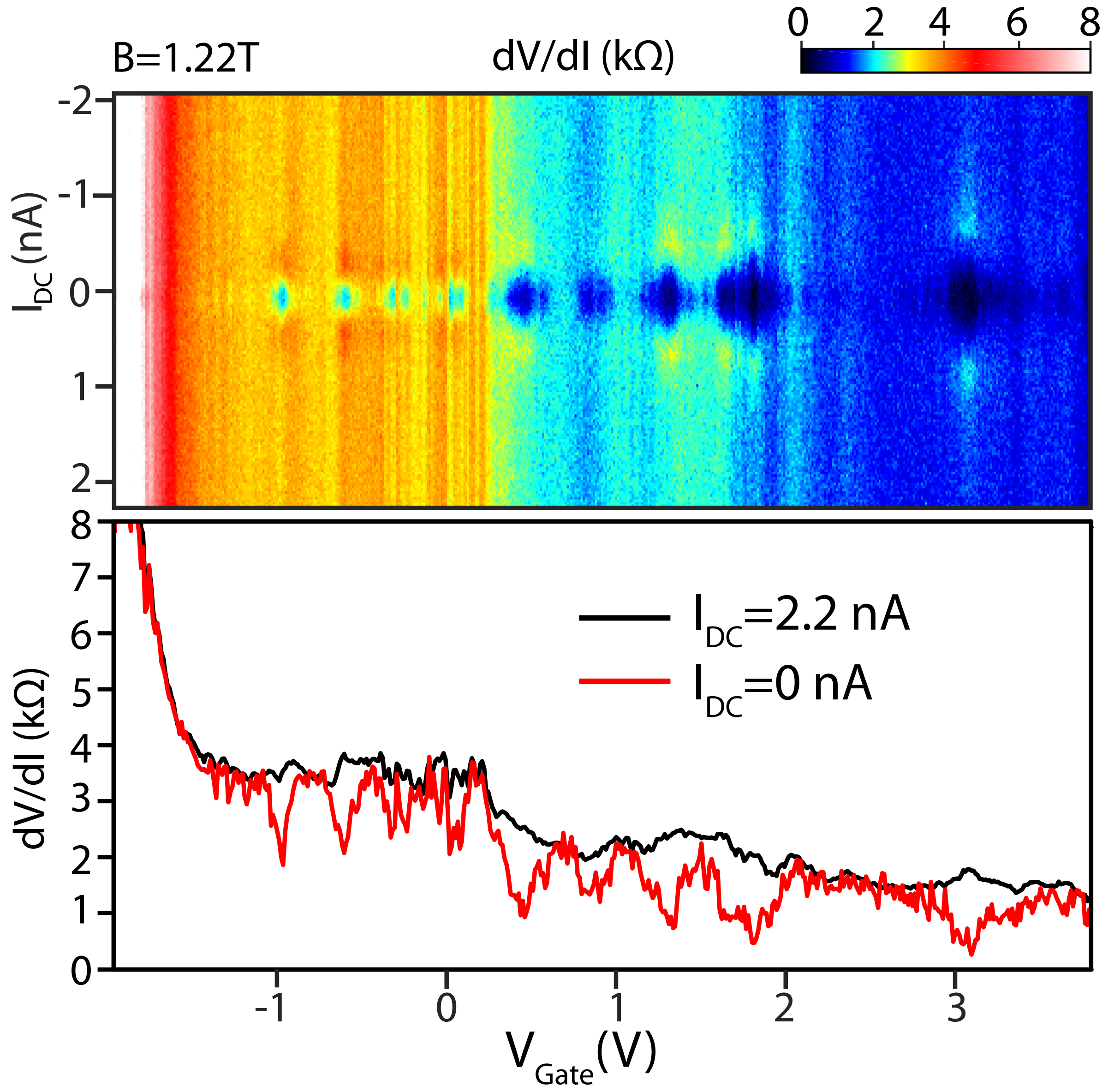
Alternating lines at high and low bias current



- At each B, the gate dependence is recorded at finite (few nA) and vanishing <100 pA current
- Alternating lines of high and low resistance

Pockets of supercurrent on top of QH plateaus

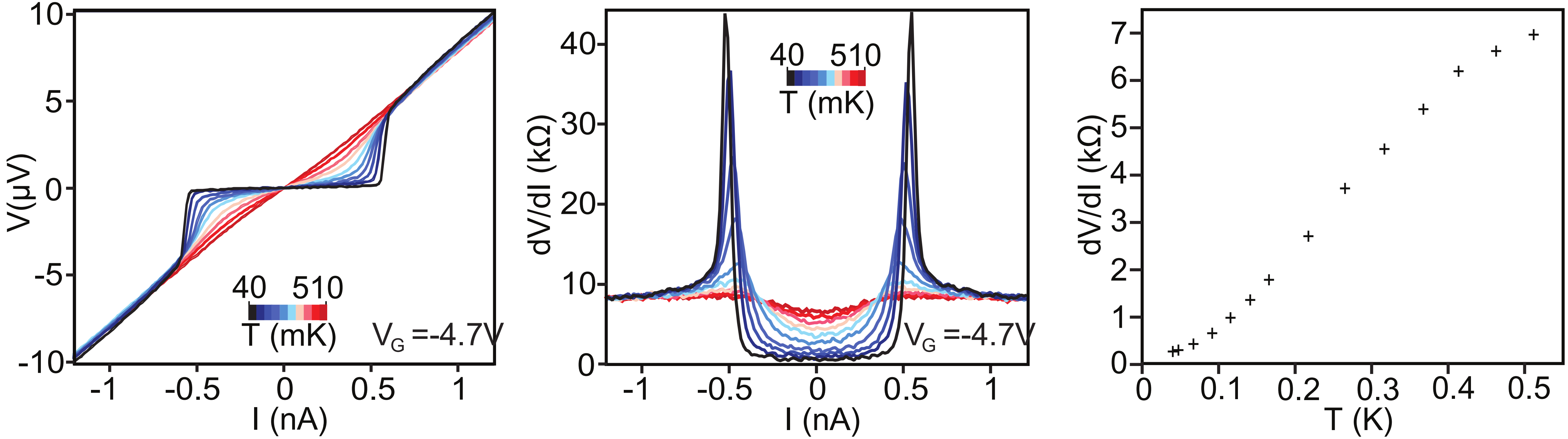




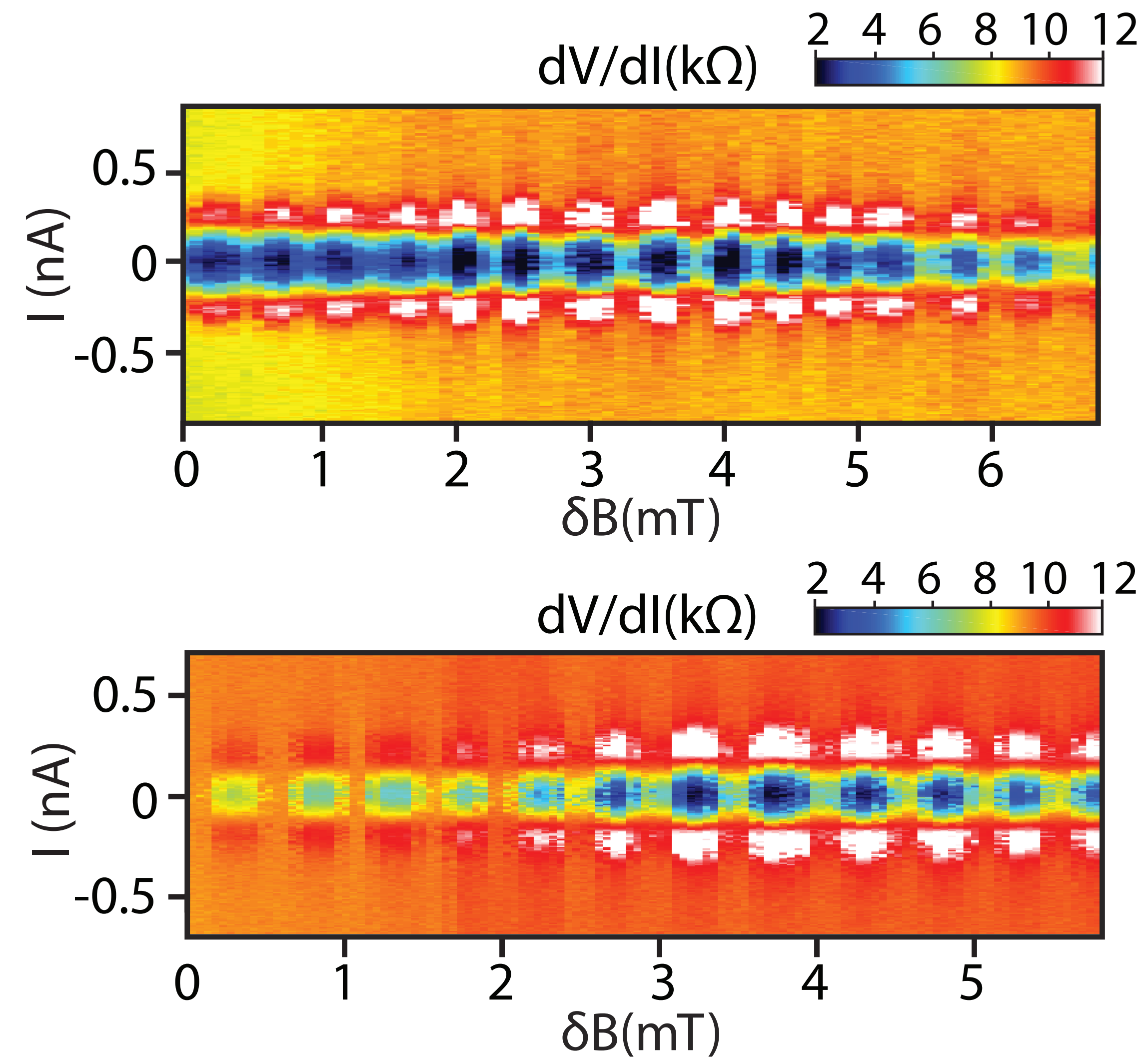
Superconductivity in the quantum Hall regime

- I-V curve: superconducting branch
- dV/dI – phase diffusion

V_G= - 4.7 V
B = 1 T
T = 40 - 500 mK

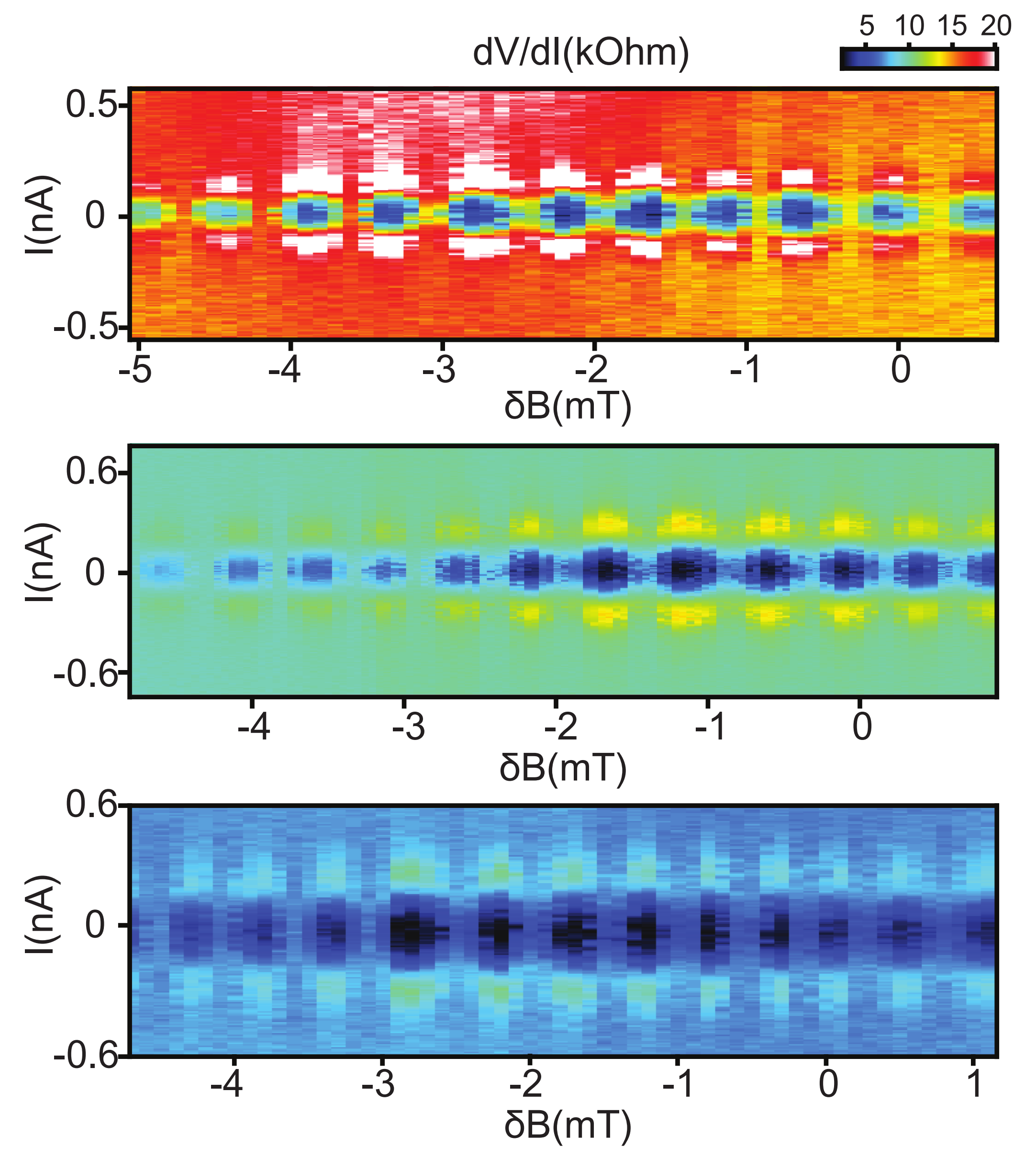


Magnetic interference in QH regime



- V_G=-5.1 and -2.6 V, filling factors +/- 2
- A small field δB applied on top of 1T
- SQUID-like pattern indicates edge channel transport
- All interference patterns show the same periodicity as the Fraunhofer pattern around zero field.

Interference at different filling factors

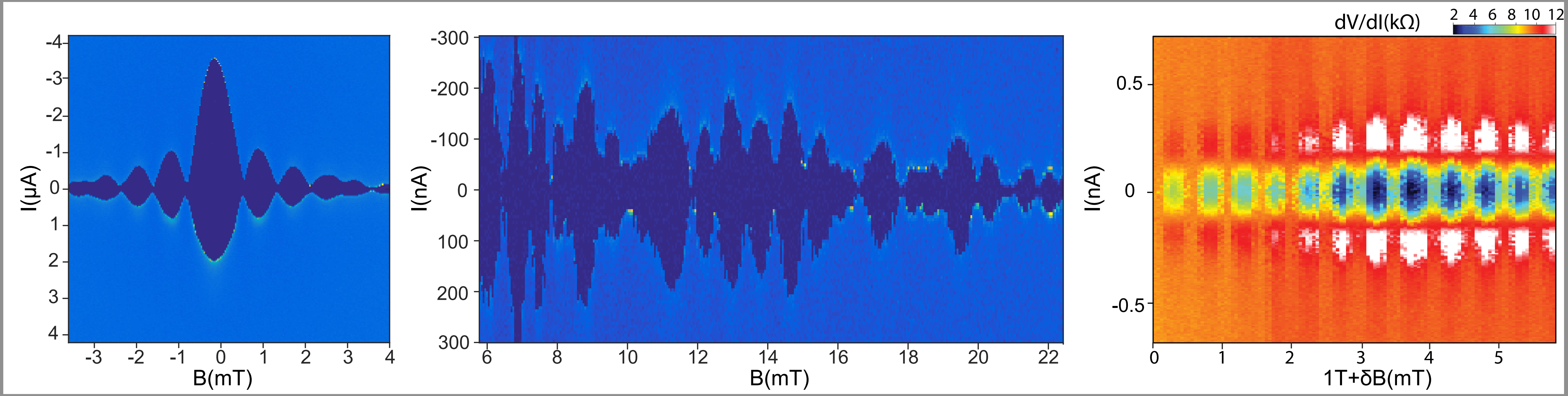


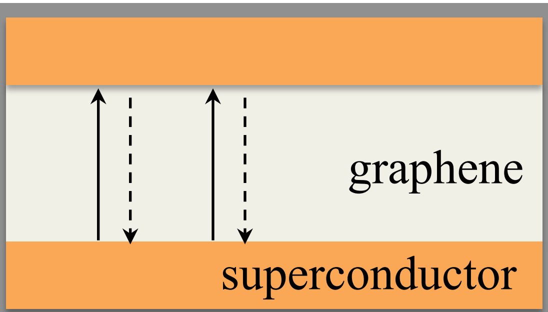
- ν = 2, 6 and 10 around 1 T
- SQUID-like pattern indicates edge channel transport
- All interference patterns show the same periodicity as the Fraunhofer pattern around zero field.

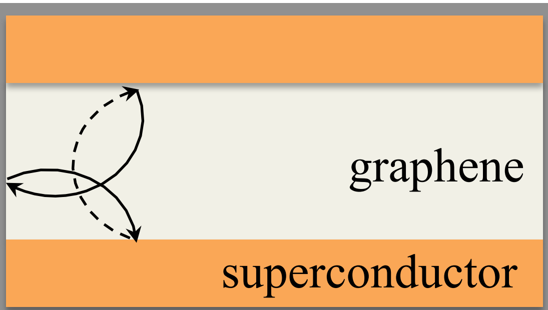
Interference pattern – distribution of current flow

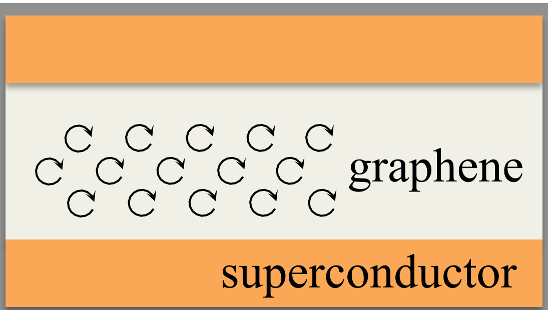
- B ~ 0 T, regular Fraunhofer pattern due to uniform current flow
- The random interference starts at a few mT – semiclassical trajectories
- The pattern eventually becomes regular in the QH regime

Fraunhofer Semiclassical QHE









Outline

QH junctions and Andreev bound states

Ballistic Josephson junctions in graphene

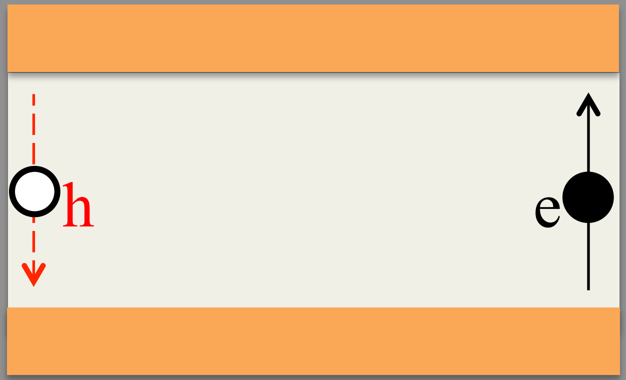
Superconductivity in magnetic field

Microscopic origins and hybrid edge modes ⇦

Supercurrent at the QH edge states

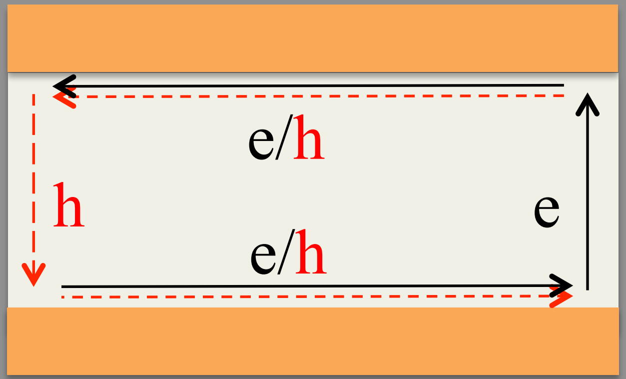
To complete the Andreev bound state, an electron has to be converted to a hole and return back

The electron and hole move in the same direction – the returning hole is on the opposite side – several microns away



Supercurrent at the QH edge states

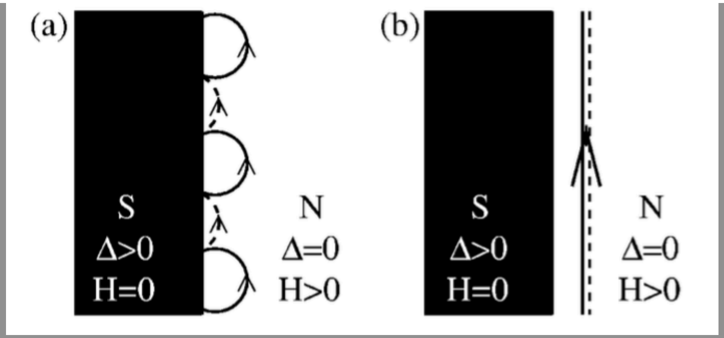
To complete the Andreev bound state, an electron
has to be converted to a hole and return back
The electron and hole move in the same direction –
the returning hole is on the opposite side
– several microns away

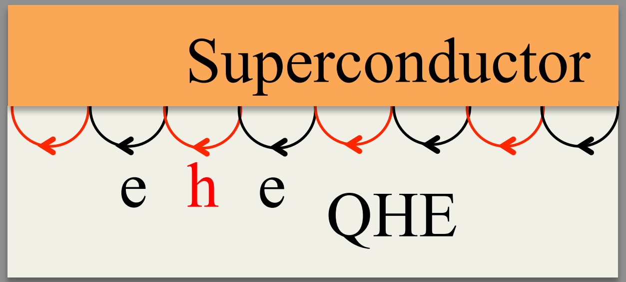


The opposite edges are connected by the hybrid e/h mode running along SC

Andreev Reflection in Strong Magnetic Fields

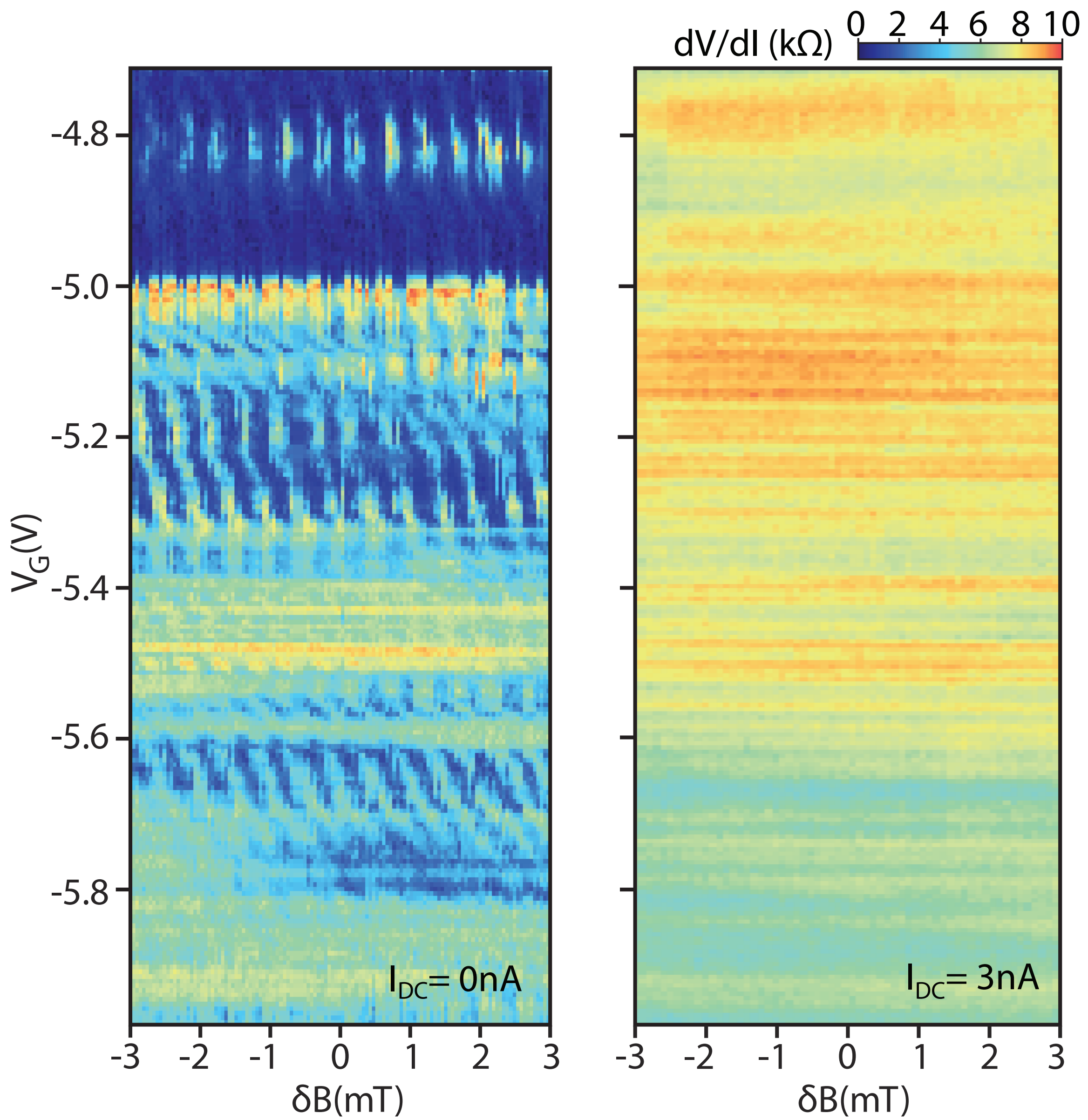
H. Hoppe, U. Zülicke, and Gerd Schön 1999





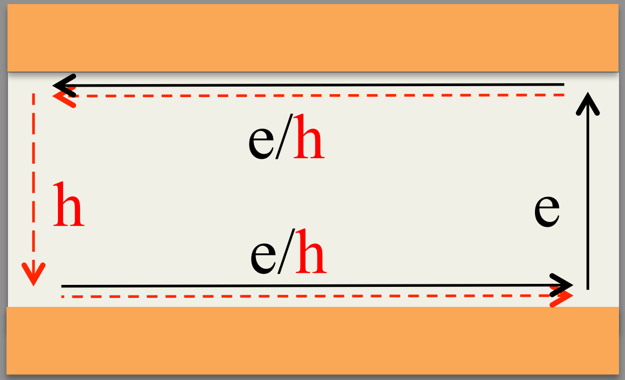
Ostaay, Akhmerov, Beenakker (2011)

Resistance vs. B and gate

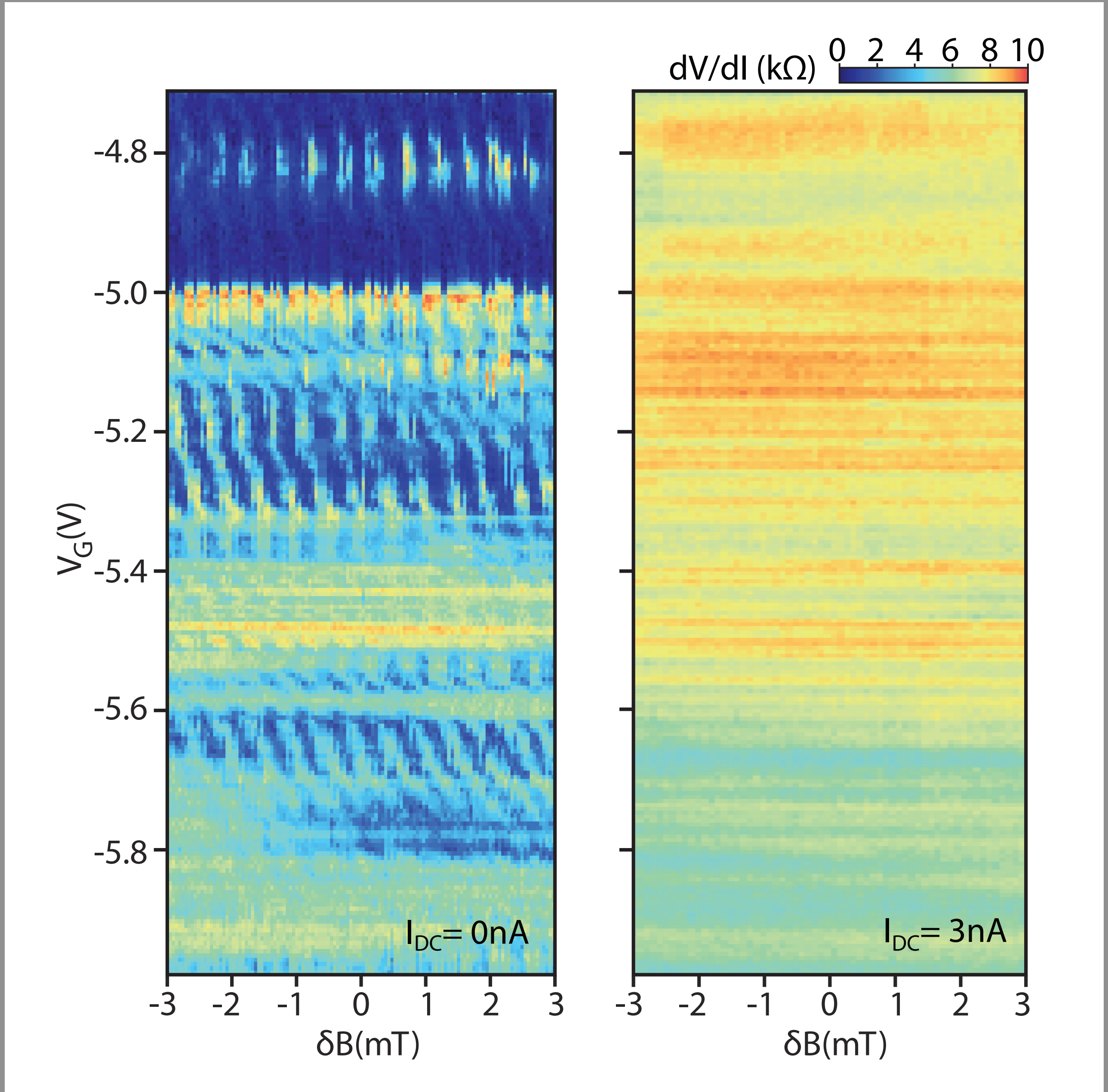


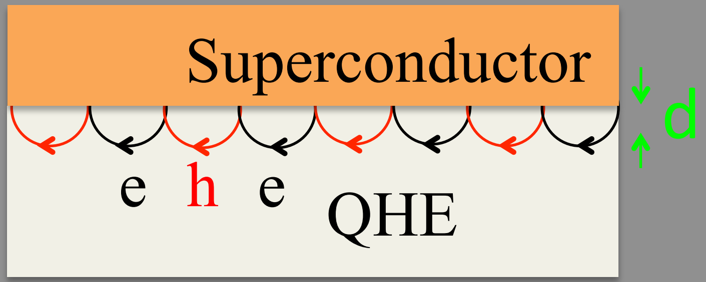
- The same periodicity at different gate voltages
- The phase depends on gate – (stripes are not vertical) – not a simple SQUID !

Consistent with this scenario:



Resistance vs. B and gate





$\Delta\phi = 2\pi BW\,(L-2d)\,/\Phi_0 =$

$2\pi BWL/\Phi_0 + Wd/l_0^2$

Geometric flux $2\pi\ \Phi/\Phi_0$

$+ Wd/l_0^2$

d depends on Vgate

– phase depends on Vgate

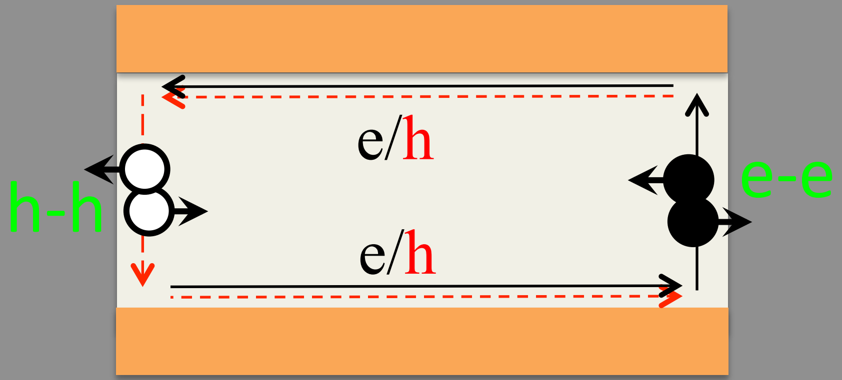
Question: periodicity

Predicted: h/e

Measured: h/2e

~~Charge poisoning~~

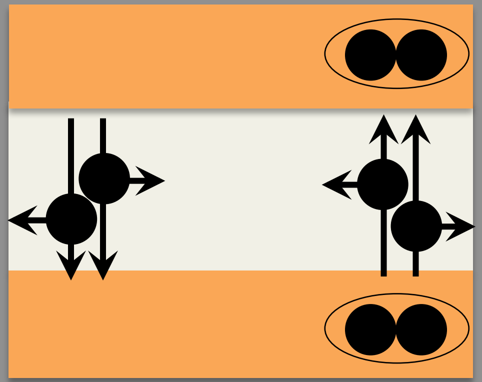
Coulomb interactions between Andreev states?



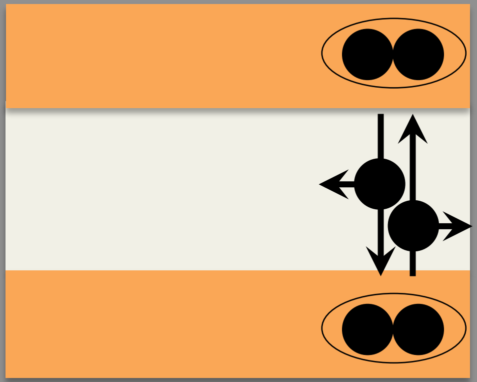
Averaging of a lower harmonic to zero?

Anomalous current phase relation?

Supercurrent flows along each edge?



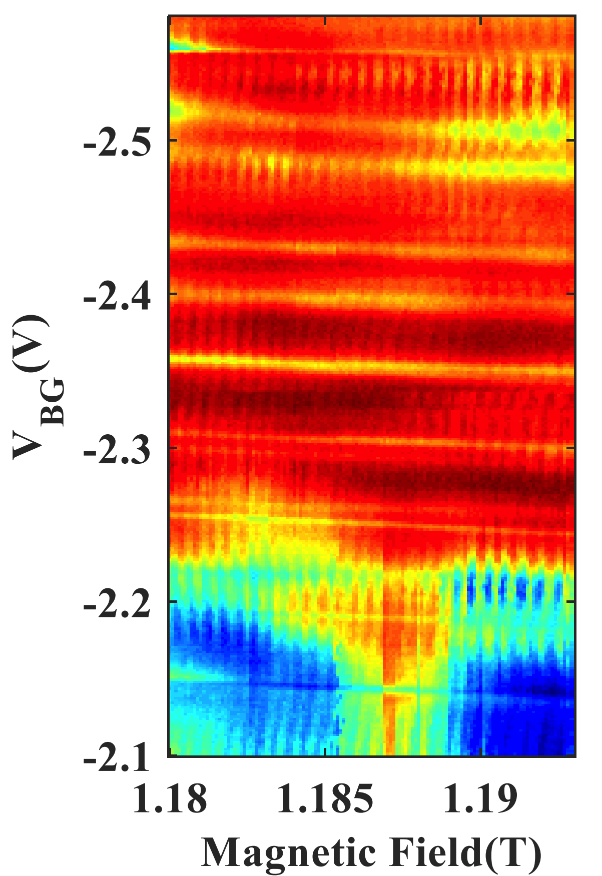
Supercurrent flowing along each edge?

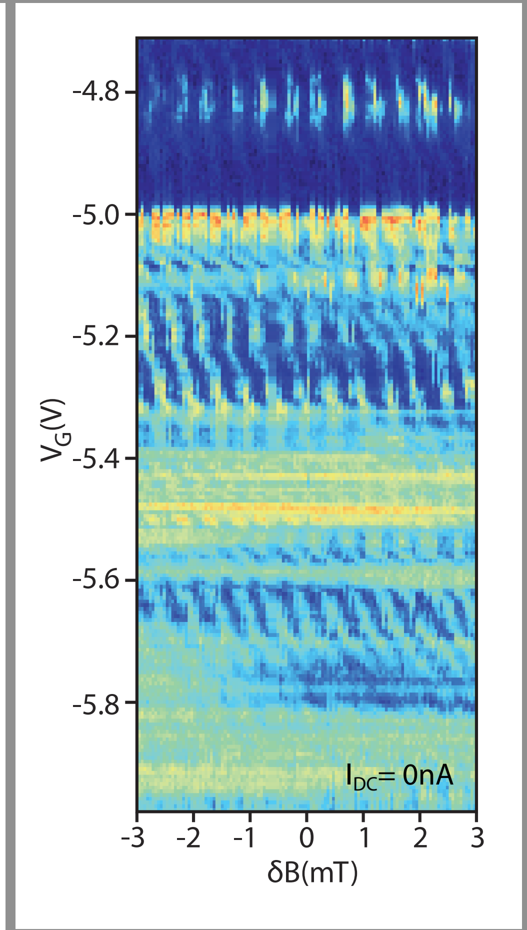


Requires counter-propagating edge states, which may be possible due to density build-up near edges.

This contradicts:

- plateau quantization
- supercurrent existence across the whole plateau
- SC stronger for L=800 nm, W=2.5µm then for L=650 nm, W=4.5µm
- phase-gate dependence having the same sign of the slope for electrons and holes





Summary

- Supercurrent in the QH regime
- Magnetic field periodicity very close to that

of the Fraunhofer pattern

- Phase dependence of Vgate is consistent with

the existence of hybrid e/h modes running along SC.

These modes couple counterpropagating electrons and hole edge states on the opposite sides of the sample

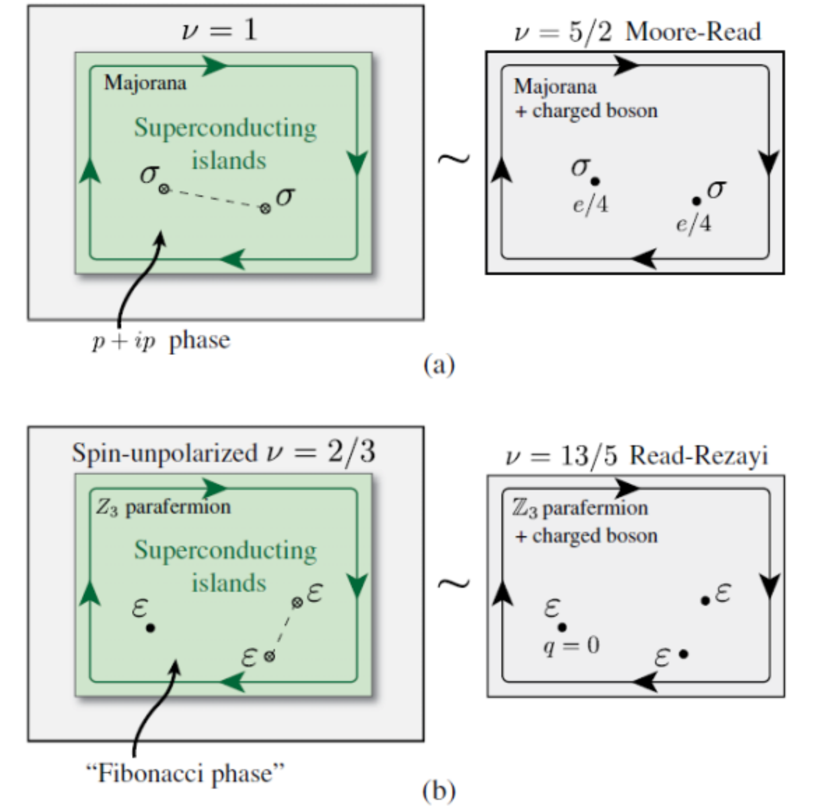
François Amet and Chung Ting Ke

Ivan Borzenets and Yura Bomze

Motivation: superconductivity + QHE

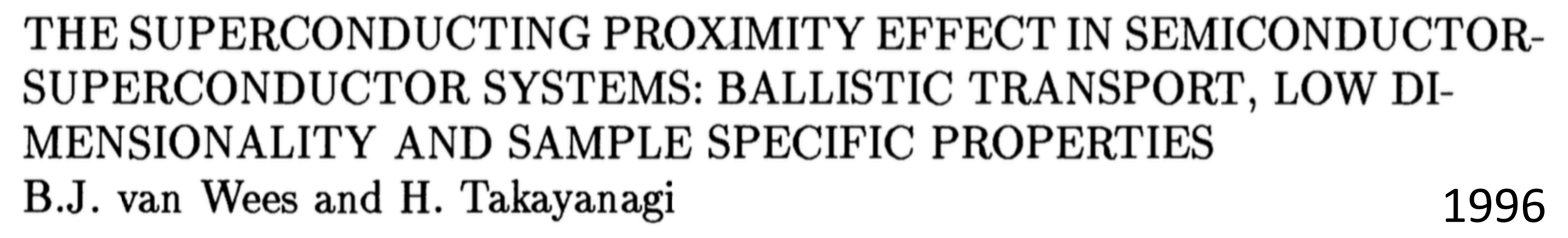
Coupling superconductivity to topological states

QHE + SC is a natural direction



MONG et al. PHYS. REV. X (2014)

But QHE and SC do not mix well



THE SUPERCONDUCTING PROXIMITY EFFECT IN SEMICONDUCTOR-SUPERCONDUCTOR SYSTEMS: BALLISTIC TRANSPORT, LOW DIMENSIONALITY AND SAMPLE SPECIFIC PROPERTIES

B.J. van Wees and H. Takayanagi

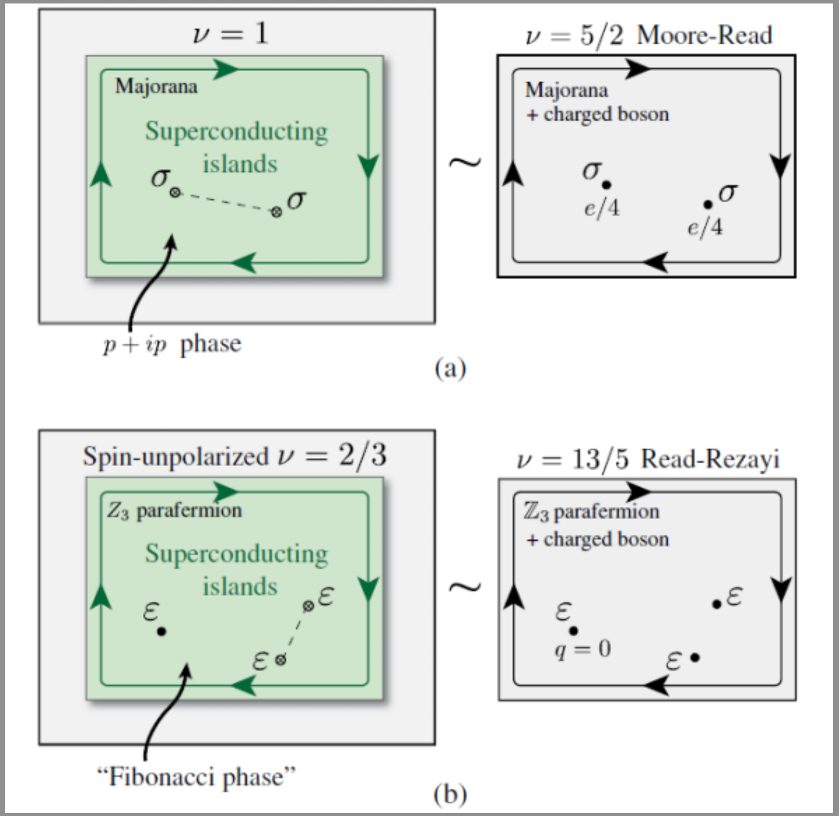
1996

Motivation: superconductivity + QHE

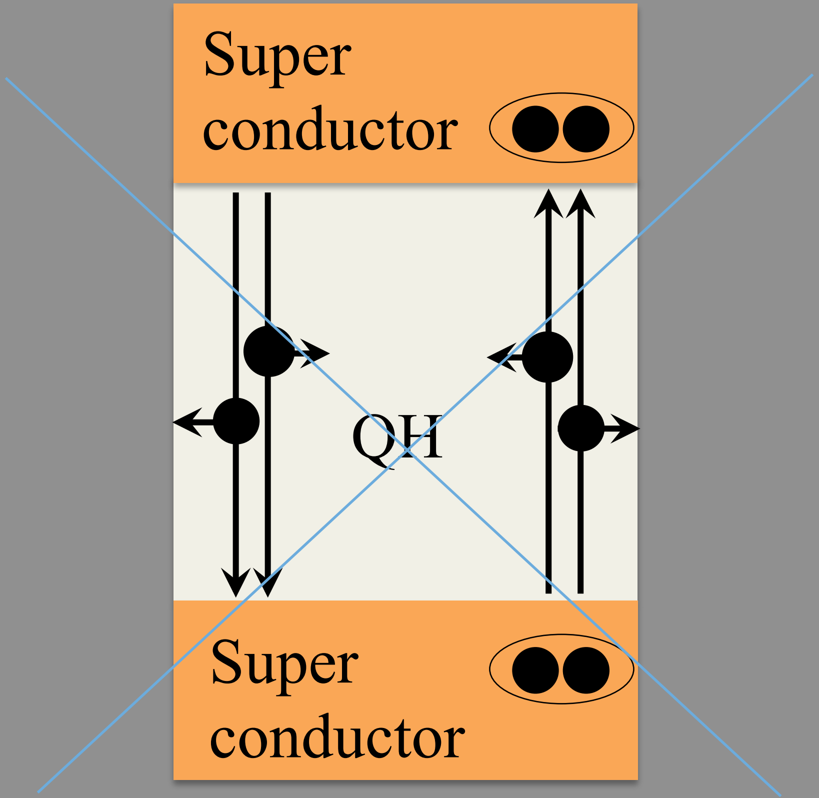
Coupling superconductivity to topological states

QHE + SC is a natural direction

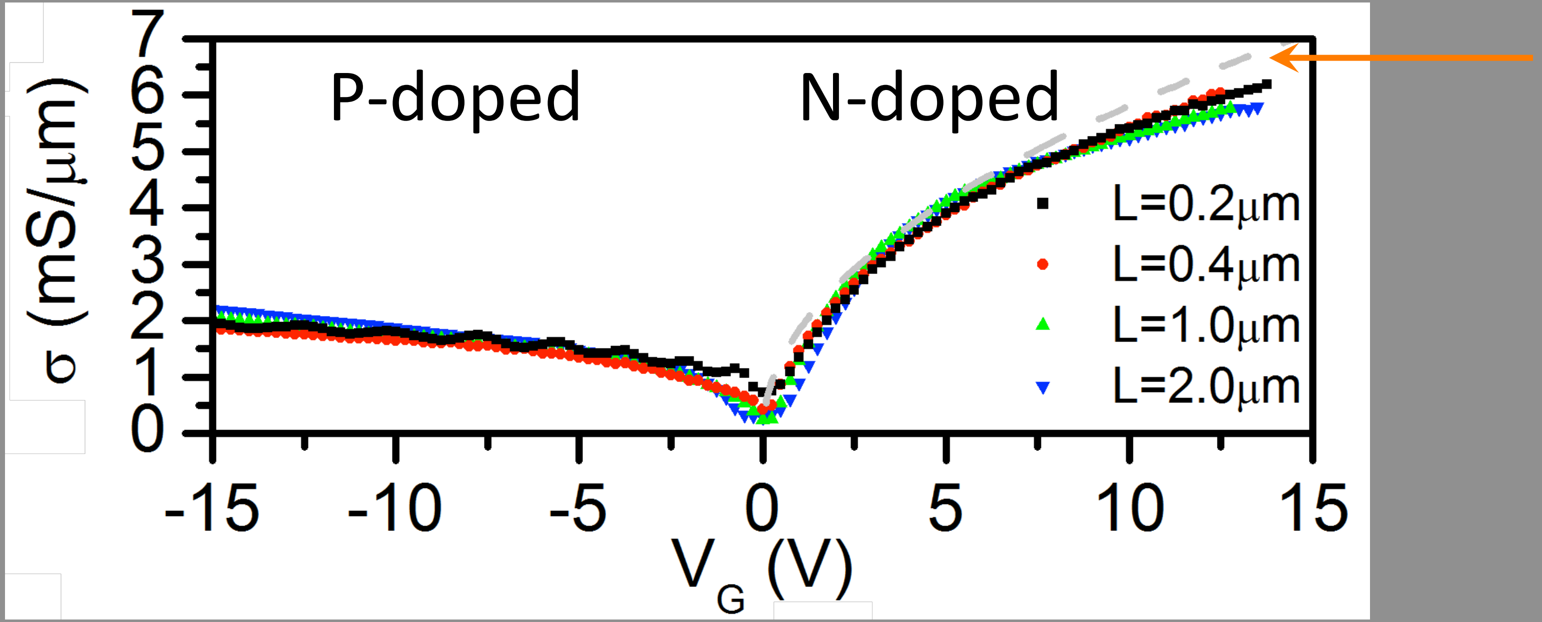
But QHE and SC do not mix well



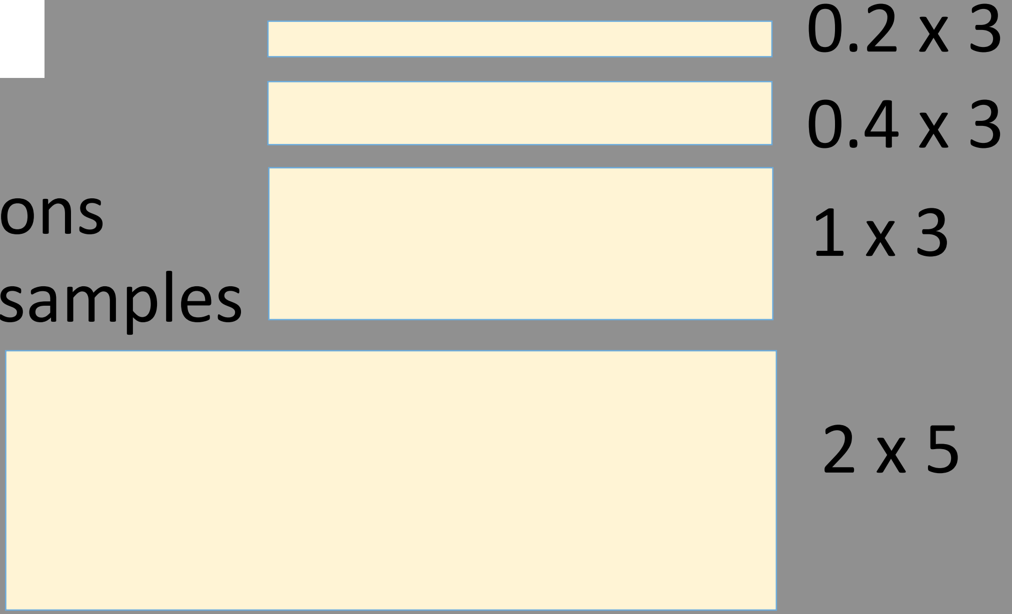
MONG et al. PHYS. REV. X (2014)



Ballistic junctions – normal state conductance



Fully transparent ballistic limit



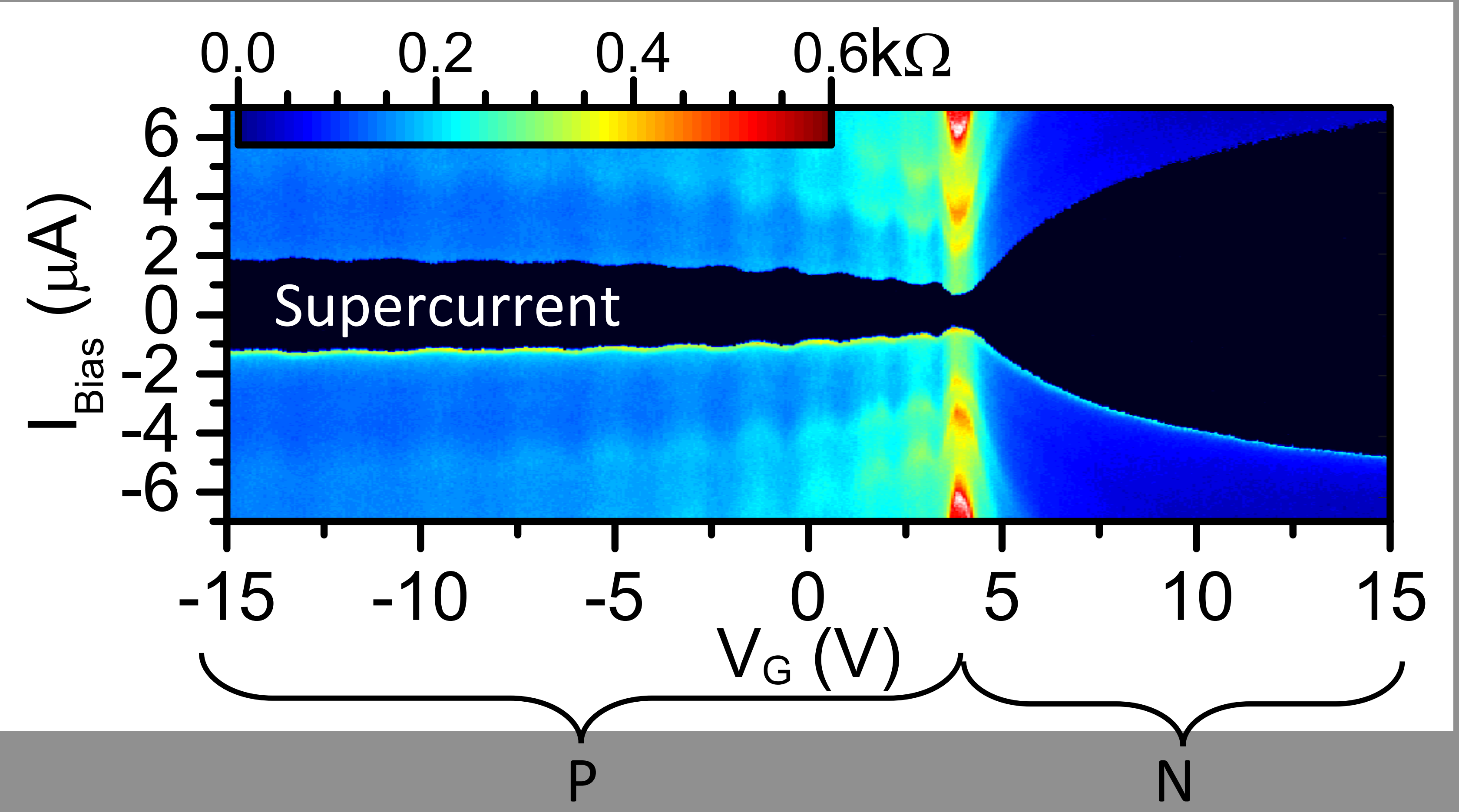
G_N/W does not depend on L – ballistic junctions

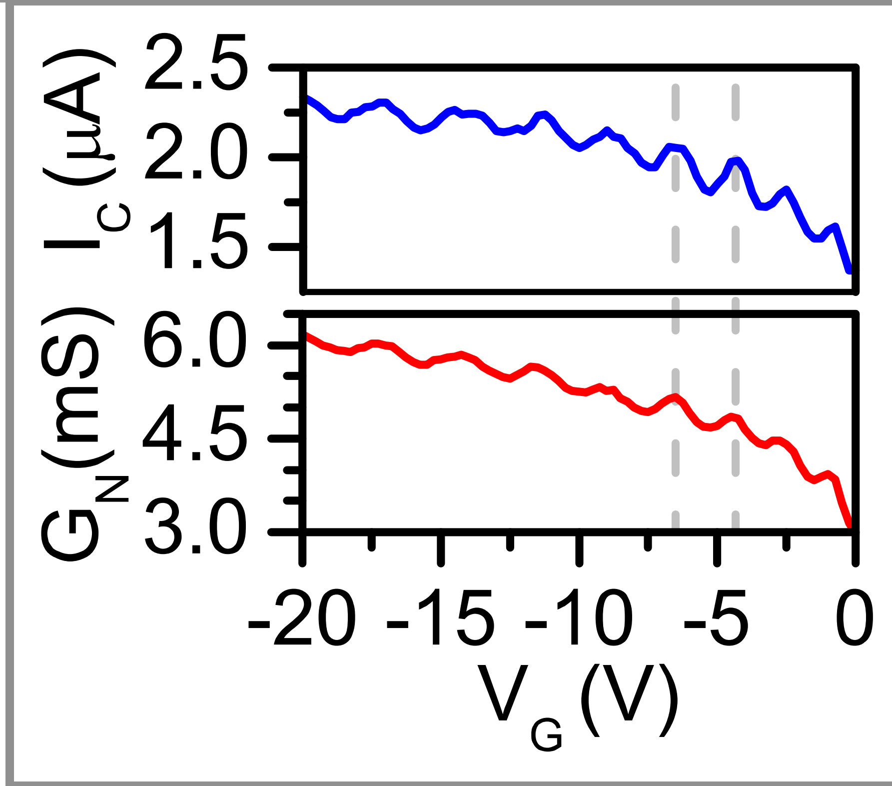
Contact transparency is consistent between samples

N-type transparency approaching 1

PN junctions at negative Vgate

Zero-field properties – ballistic junctions





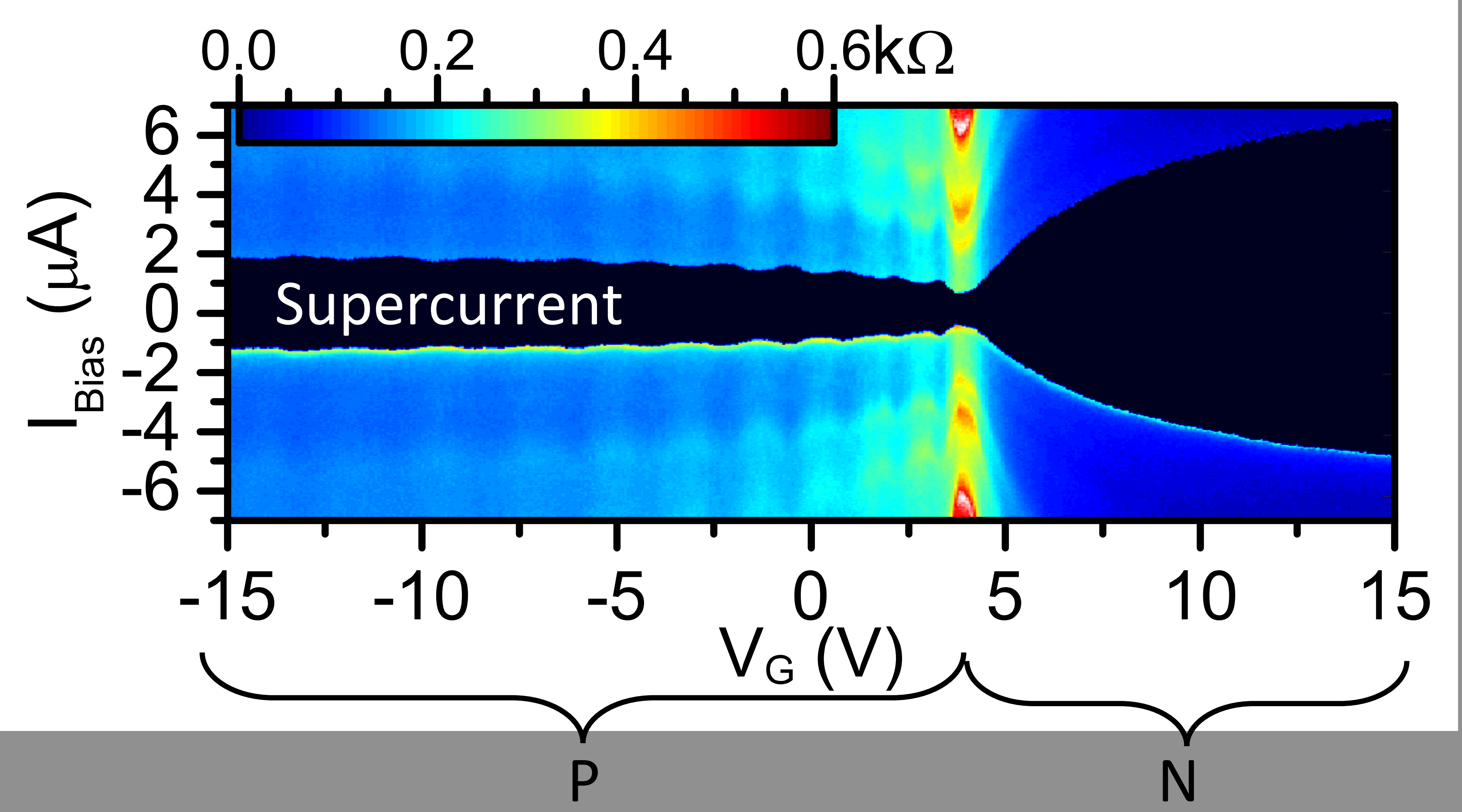
Fabry-Perot

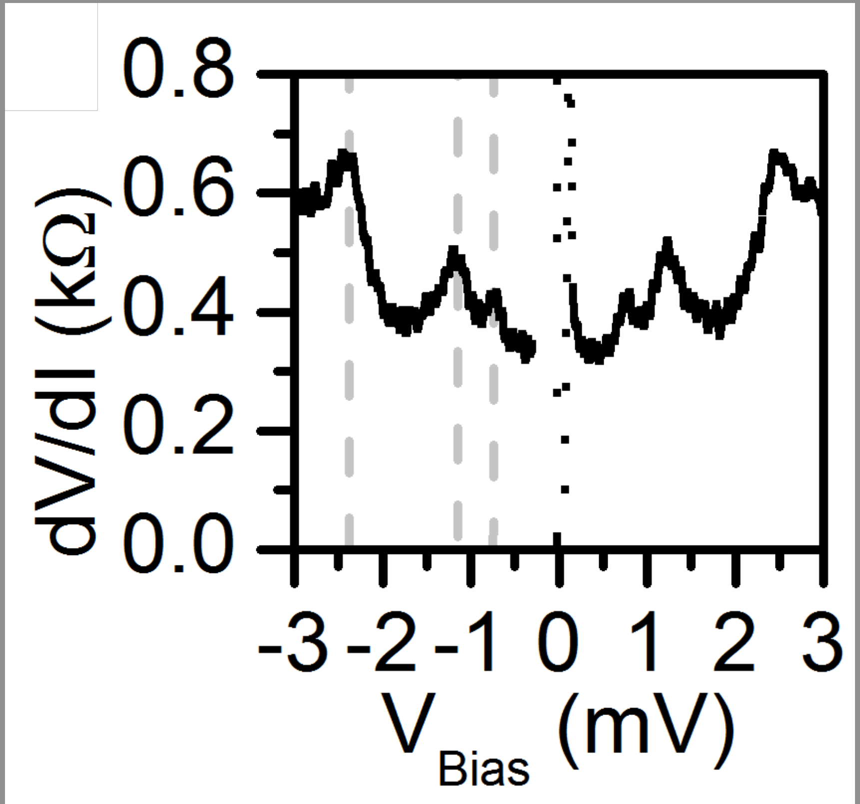
V. E. Calado et al., Nature Nano. (2015).

M. T. Allen et al., Nat. Phys. (2016).

M. Ben Shalom et al., Nat. Phys. (2016).

Zero-field properties – ballistic junctions



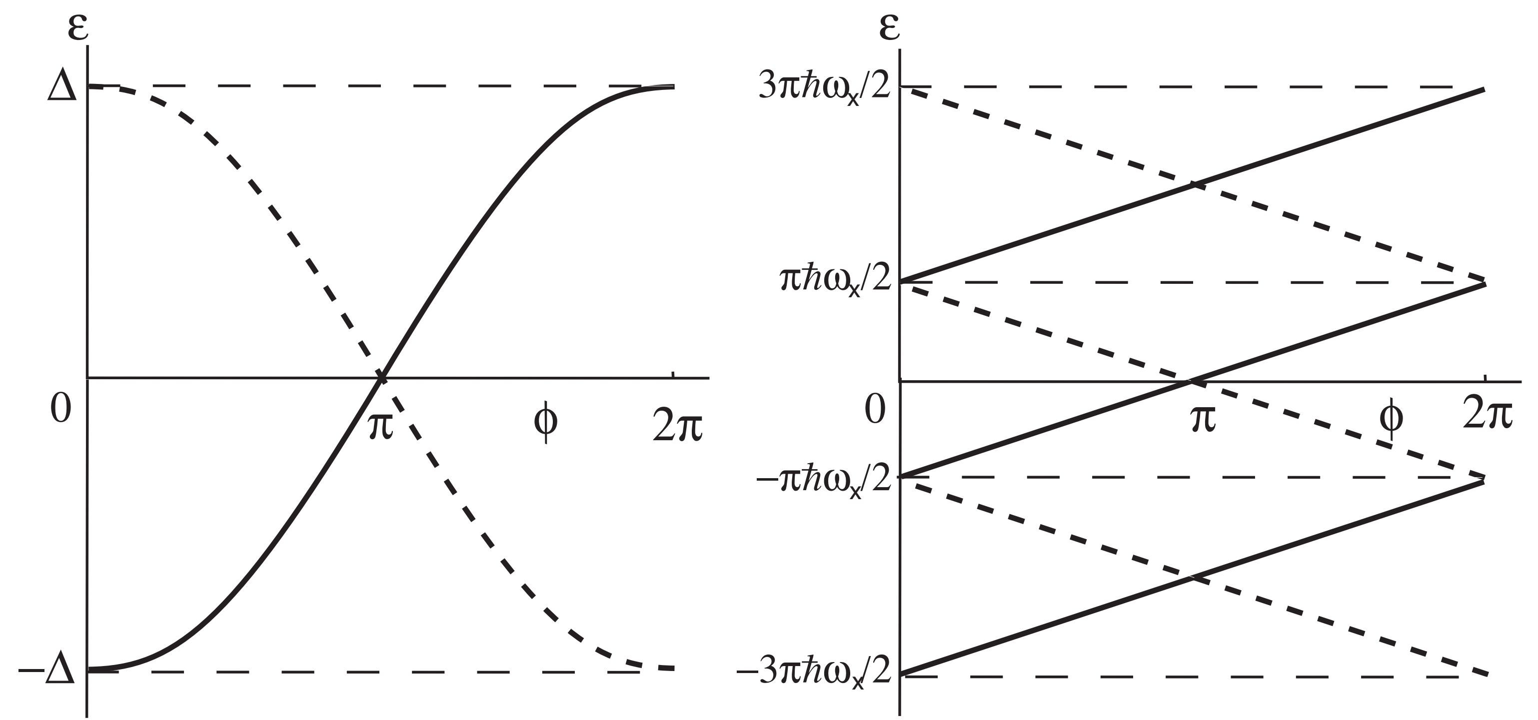


V. E. Calado et al., Nature Nano. (2015).
M. T. Allen et al., Nat. Phys. (2016).
M. Ben Shalom et al., Nat. Phys. (2016).

Multiple Andreev Reflections (MAR)

$eV = 2\Delta/n$

Ballistic junctions: from short to long

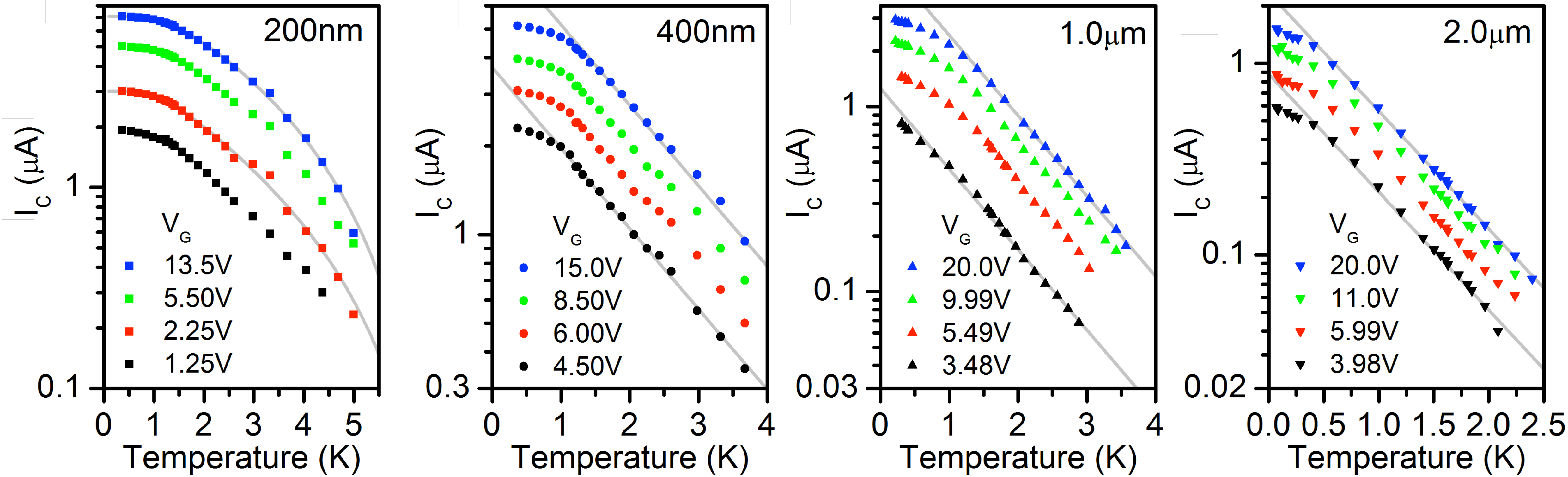


Short vs. long ballistic junction: L vs. $\xi = \hbar v_F/\Delta$ = 500 nm

Long ballistic junction: $I_C \propto \exp(-kT/\delta E)$, where $\delta E = \hbar v_F/2\pi L$

Kulik (1970)

Ballistic junctions: from short to long



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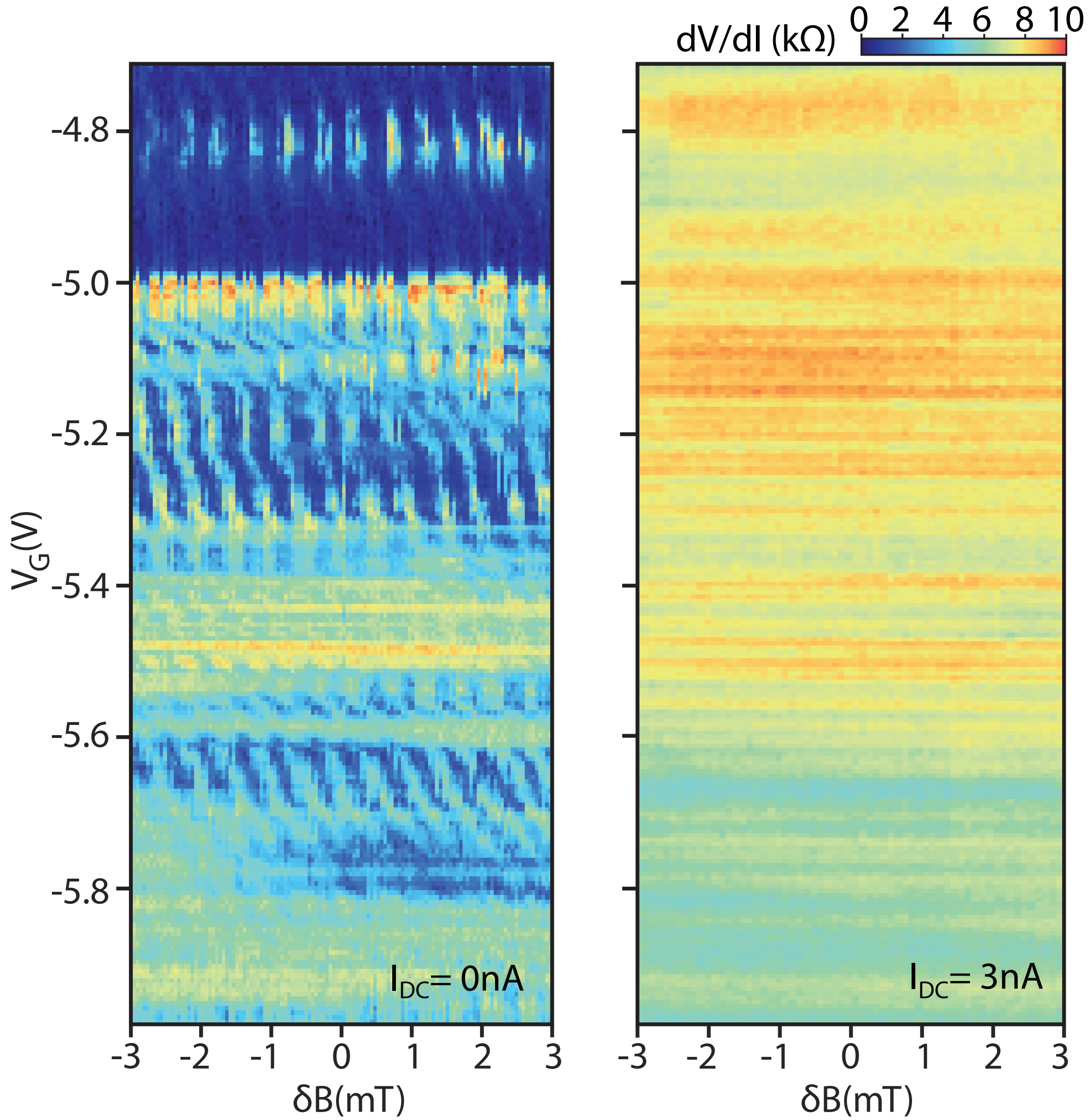
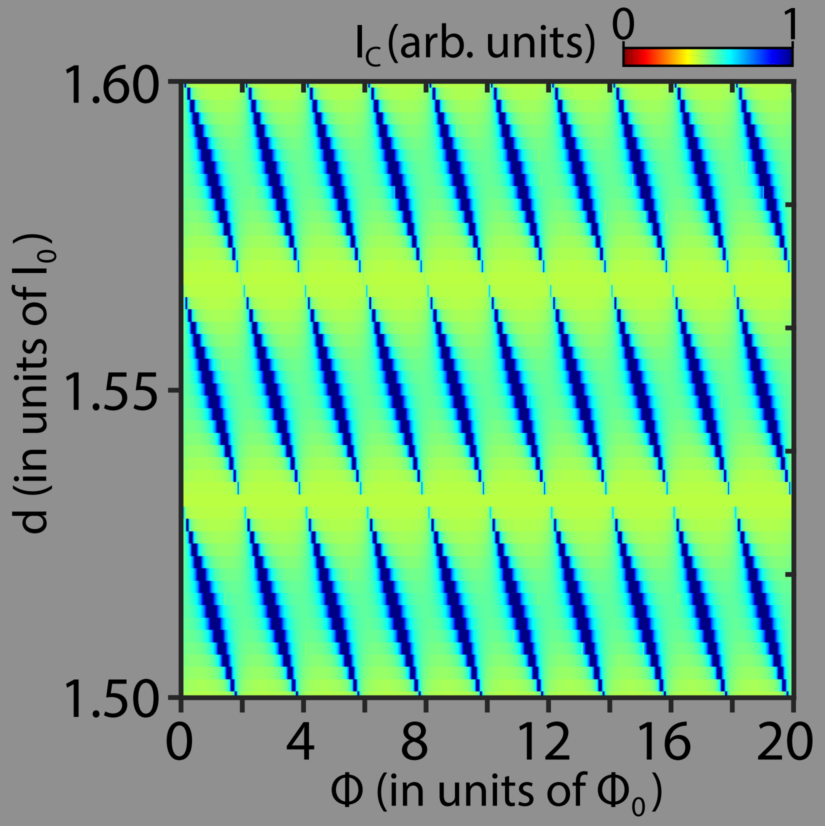
Kulik (1970)

Resistance vs. B and gate





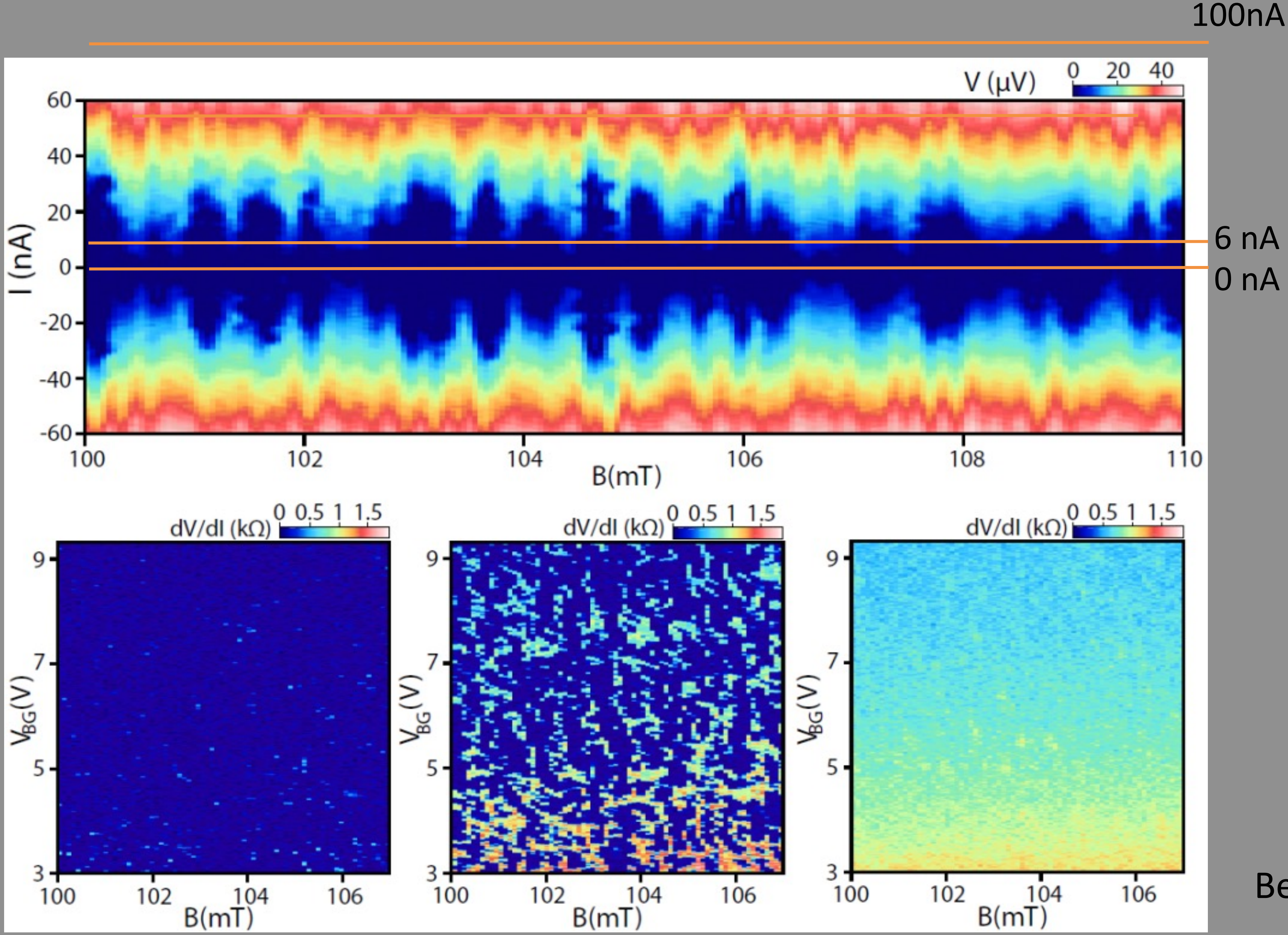
I_C oscillates with Wd/l_0^2

Mesoscopic fluctuations in d

– I_C at fixed B is patchy in Vgate

Ostaay, Akhmerov, Beenakker (2011)

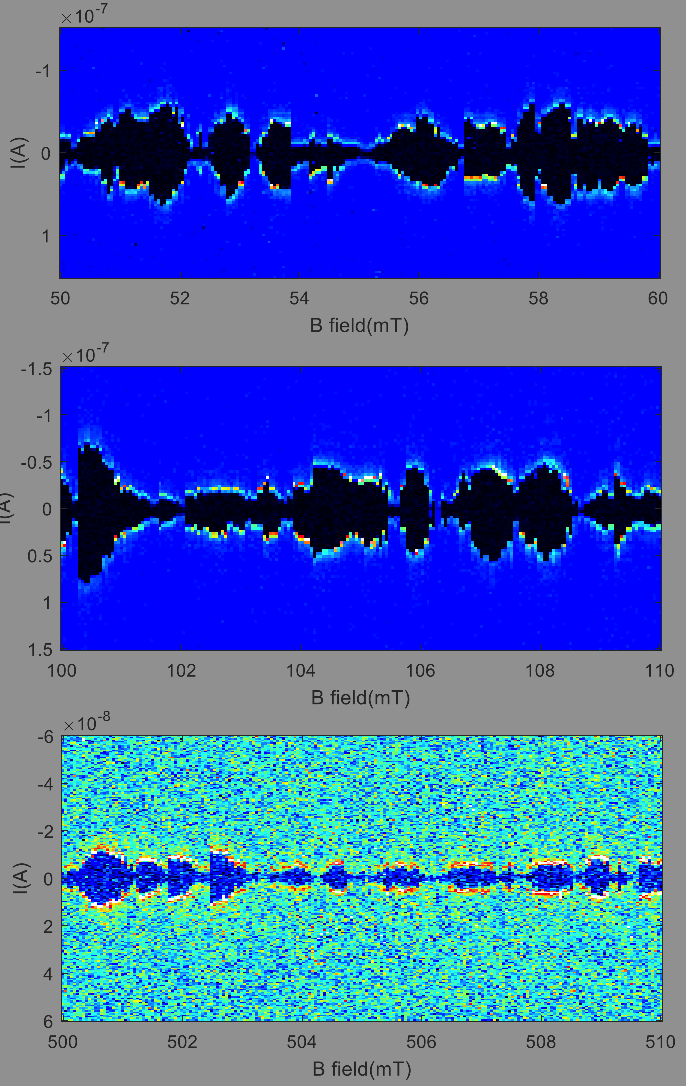
Semiclassical regime ($2r_C>L$)

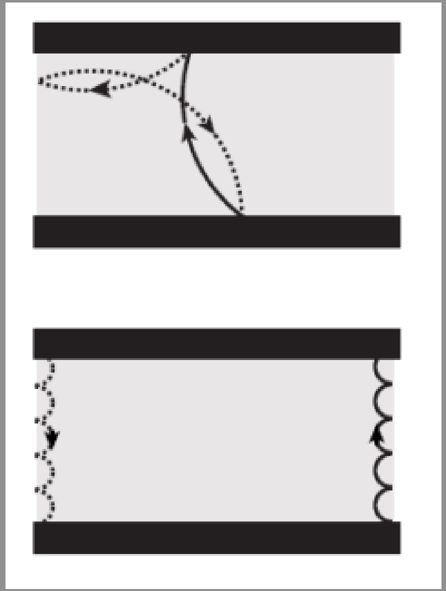


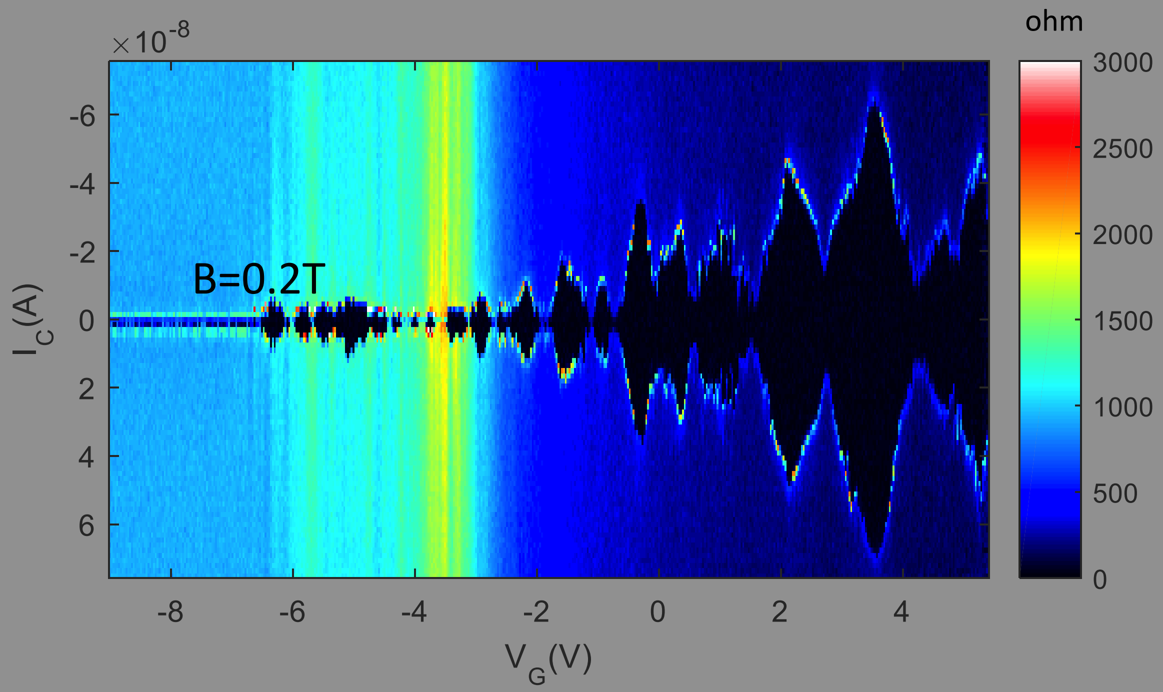
- Random trajectories create irregular interference patterns
- Sample stays superconducting through the entire semiclassical regime
- Maps are biased at 0, 6 & 100 nA.

Ben Shalom et al., Nat. Phys. (2015).

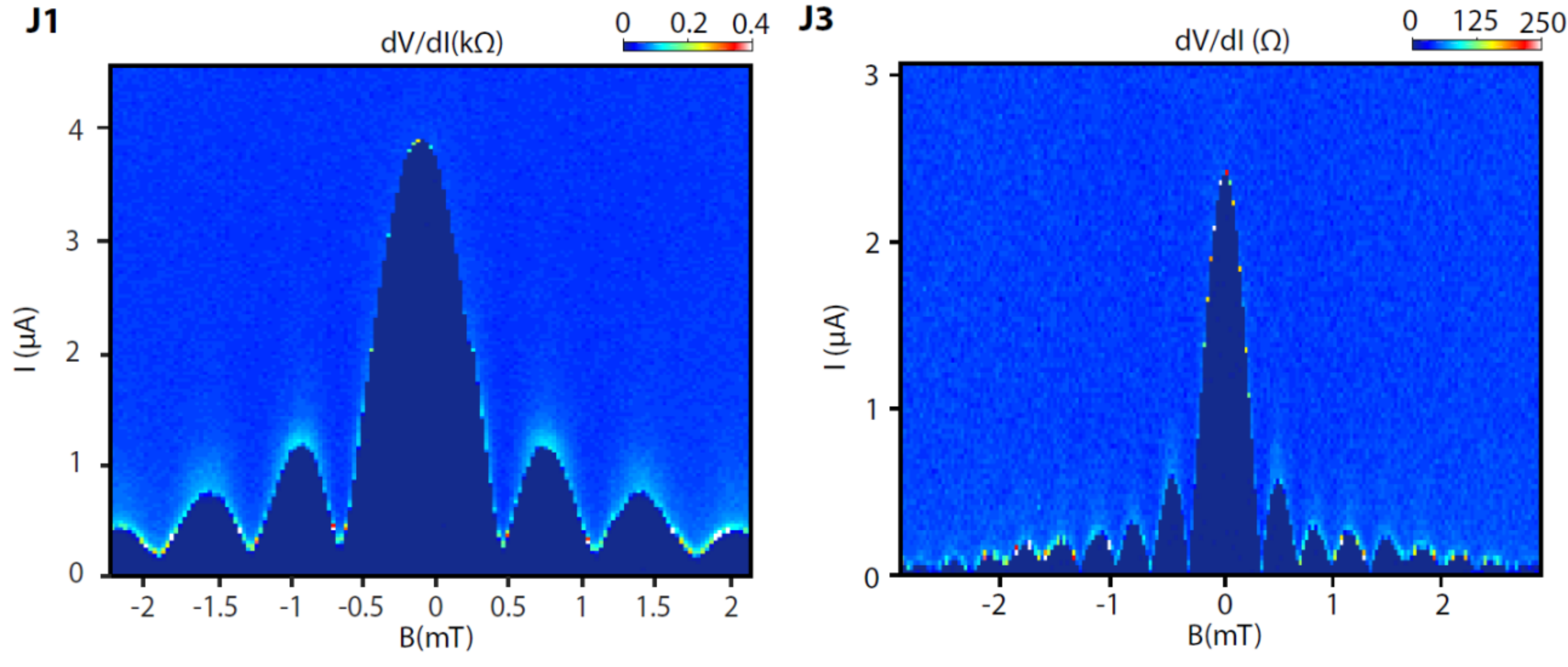
Intermediate range of B field





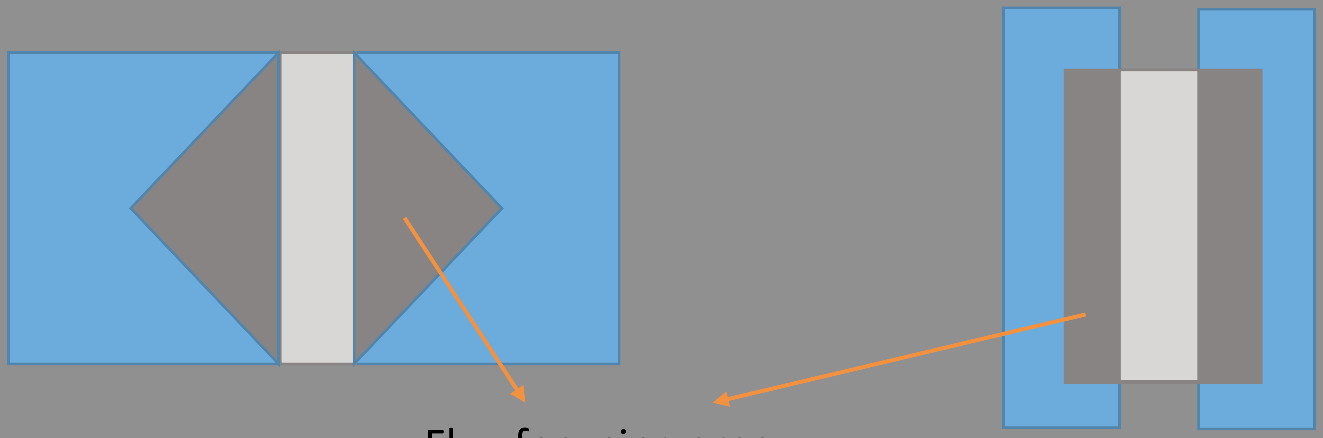


Fraunhofer around zero field



- $\Phi = BS,\ \Phi 0 = h/2e$
- The magnetic focusing area is considered

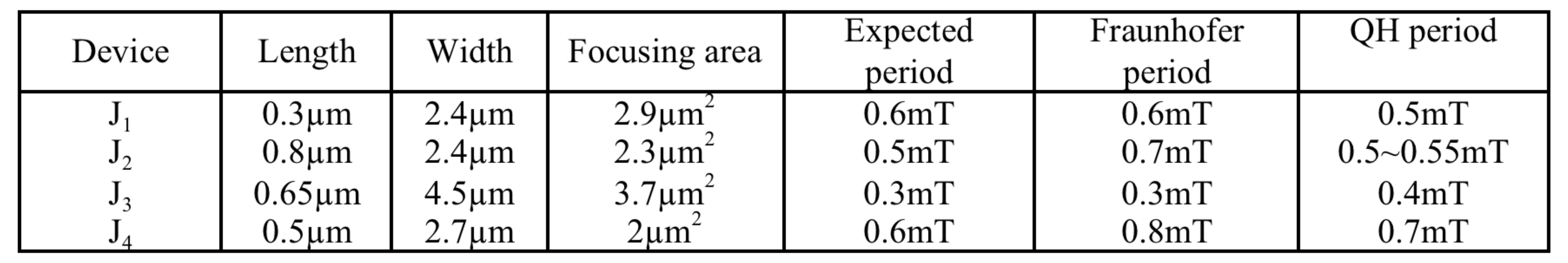
	Expected Periodicity
J1	0.6mT
J3	0.3mT



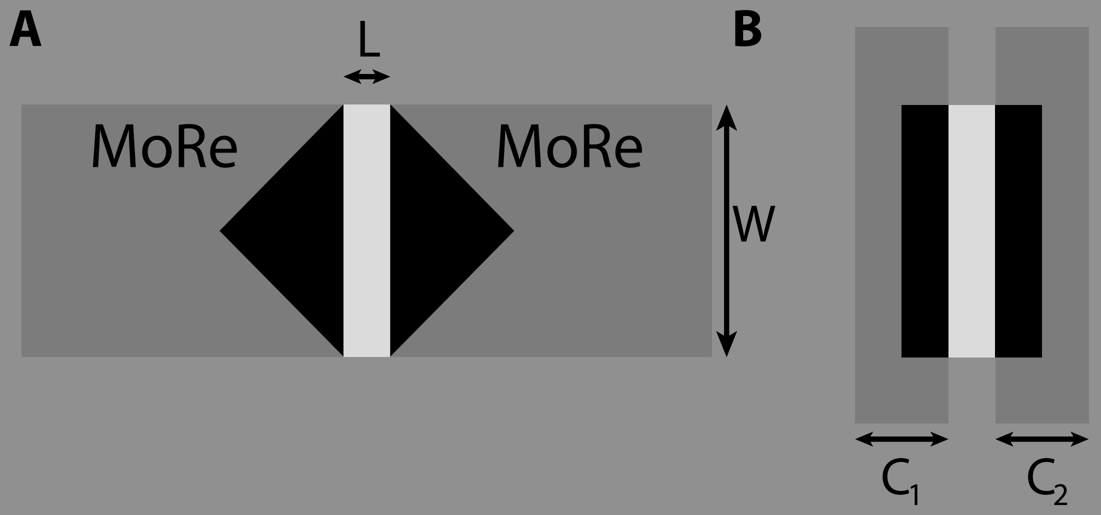
Flux focusing area

S. Hart et al Nat. Phys.
10 638-643(2014)
P. A. Rosenthal et al A.P.L. **59**(1991)

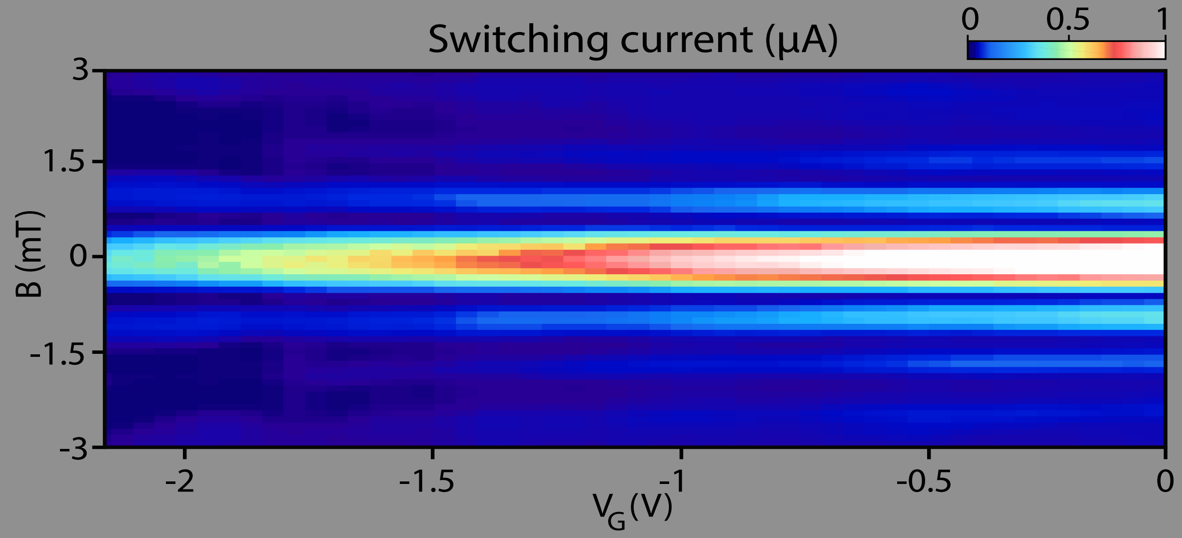
Magnetic focusing area



Device	Length	Width	Focusing area	Expected period	Fraunhofer period	QH period
J_1	0.3μm	2.4μm	2.9μm^2	0.6mT	0.6mT	0.5mT
J_2	0.8μm	2.4μm	2.3μm^2	0.5mT	0.7mT	0.5~0.55mT
J_3	0.65μm	4.5μm	3.7μm^2	0.3mT	0.3mT	0.4mT
J_4	0.5μm	2.7μm	2μm^2	0.6mT	0.8mT	0.7mT

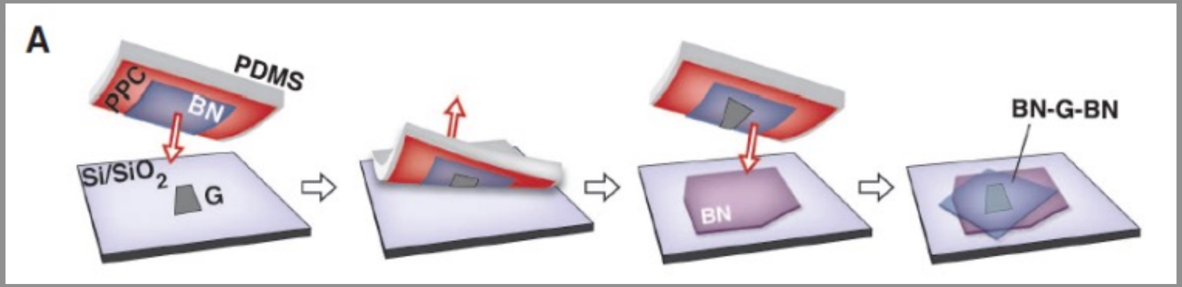


Fraunhofer vs. gate

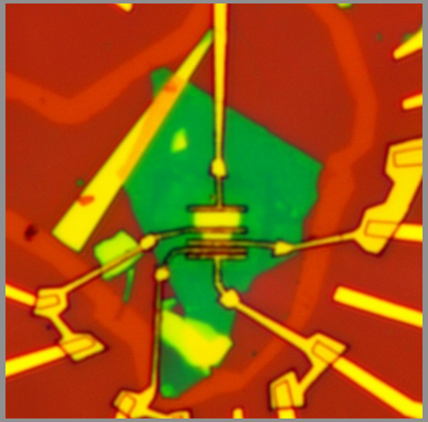


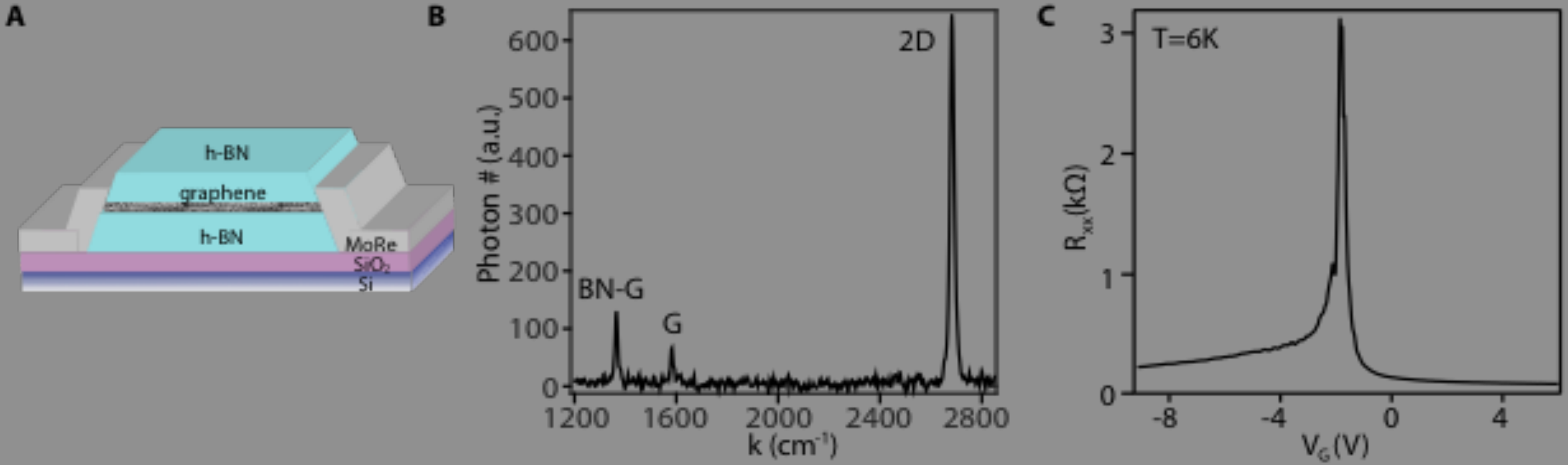
Sample Fabrication

- Standard pick up method :
- RIE define the mesa
- Ebeam litho define MoRe contacts
- Sputtering MoRe



L. Wang et al. Science **342** 614-7 (2013)



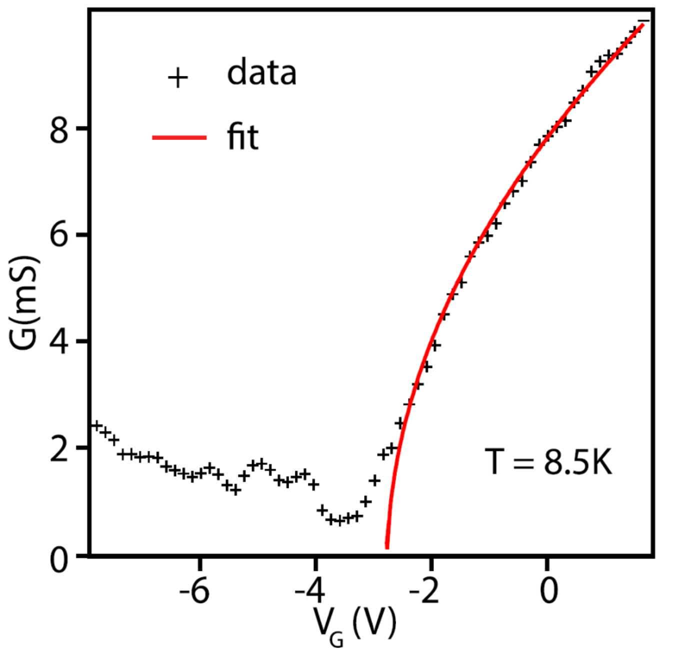


Sample characterization

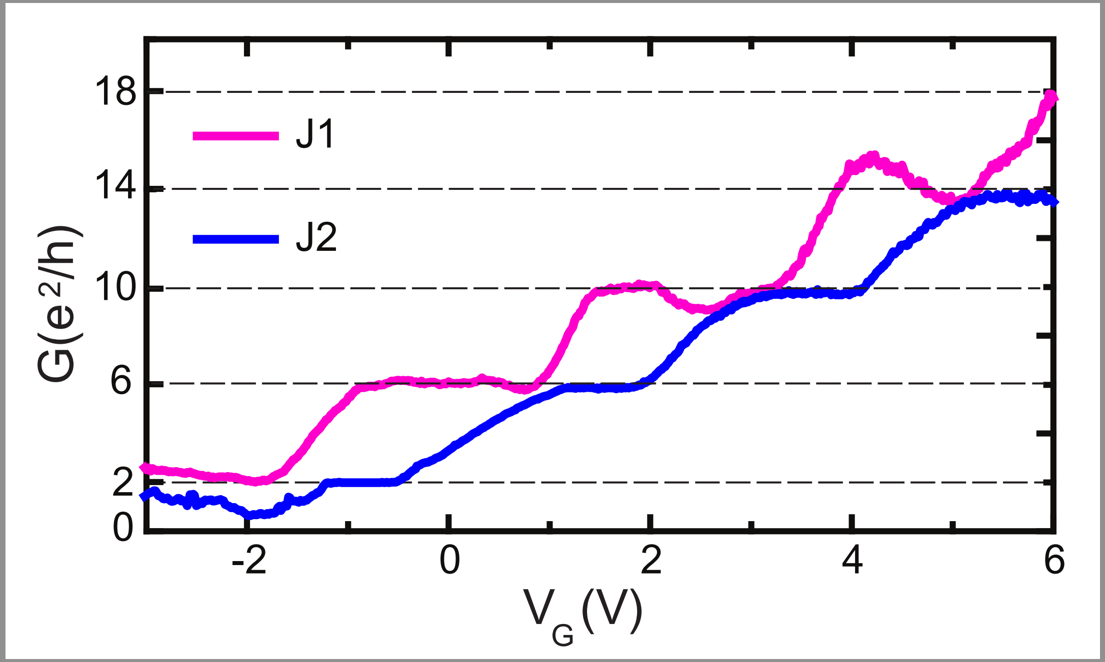
$G(V_G)=T_1T_2G_Q$ where $G_Q=N*e^2/h$

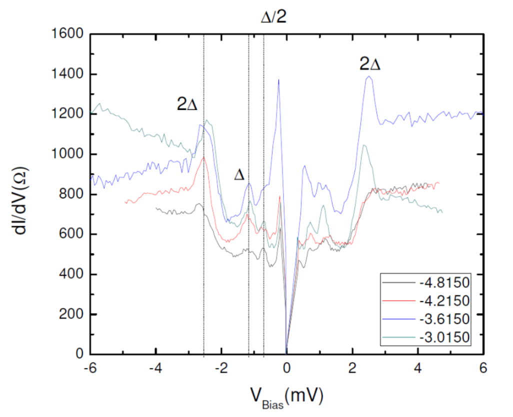
T_N= 0.95 per interface

T_P=0.45 to 0.7

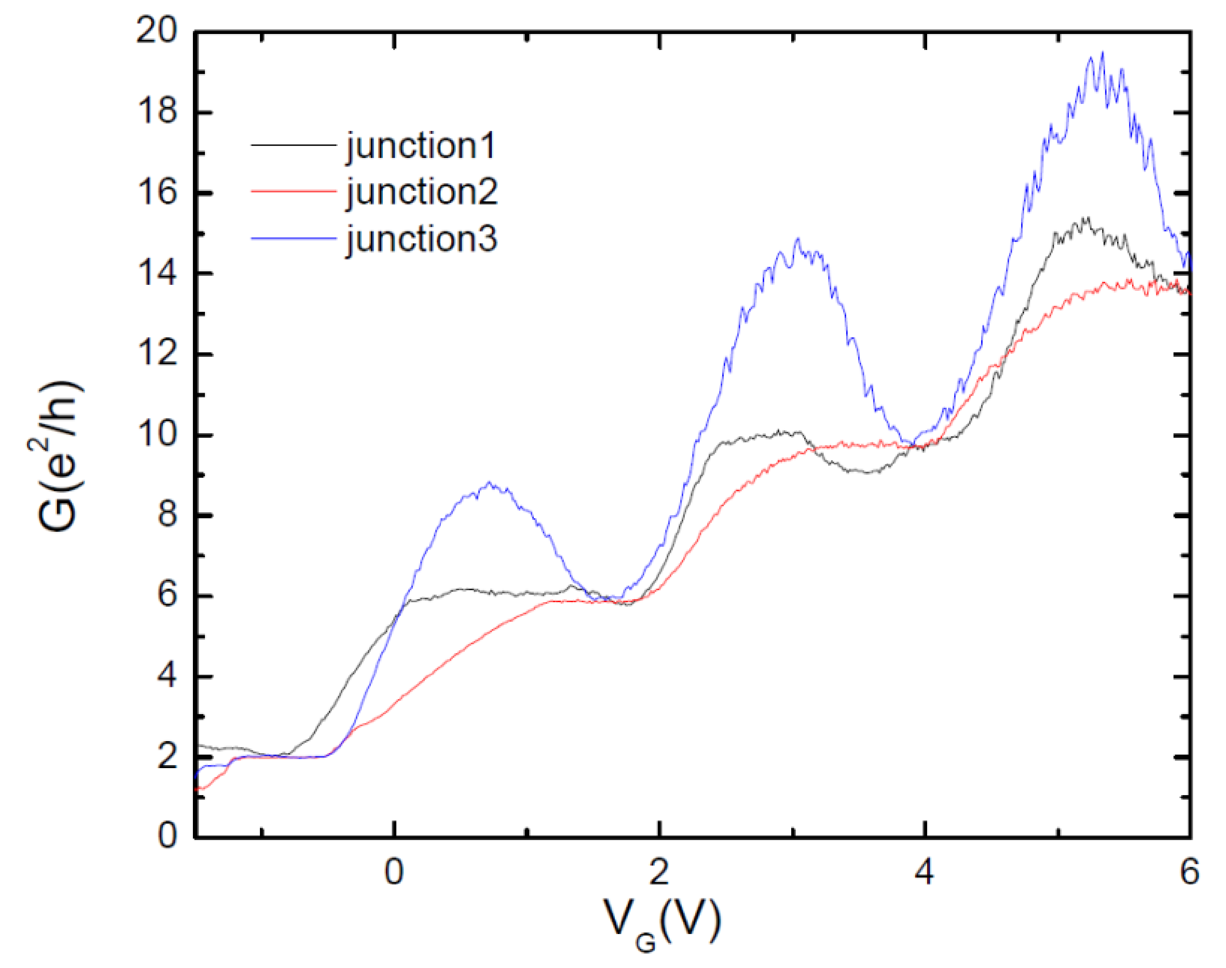


Quantum Hall at 1.5 T, 3.5 K





QHE quantization at 4 K

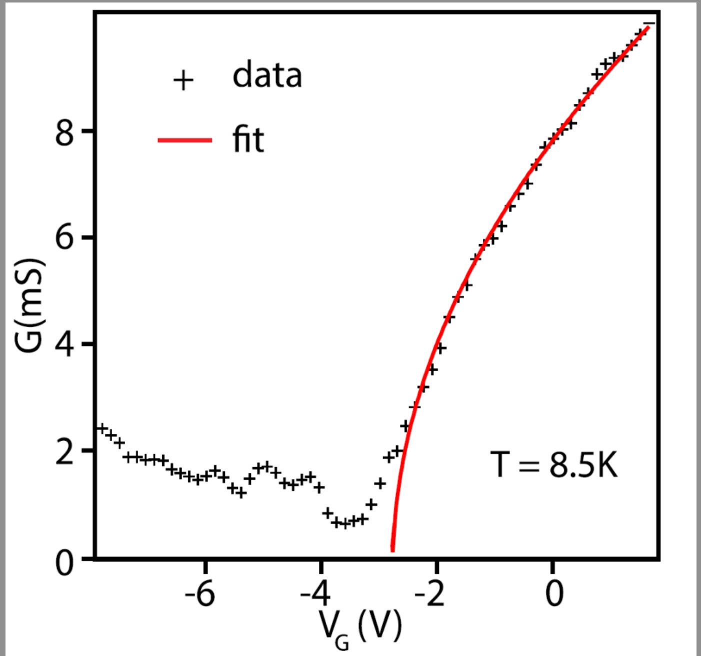


- Well quantized plateaus at 4 K under 1T
- Due to the geometry, i.e. aspect ratio, J3 has unusual quantum plateau.

Transparency of SC contacts

- From quantum limit

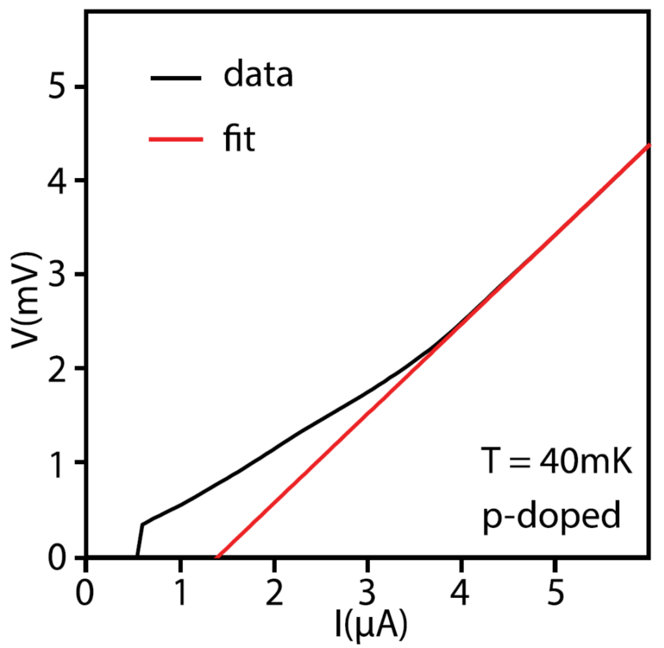
$G(V_G)=T_1T_2G_Q$ where $G_Q=N*e^2/h$
T_N= 0.95 per interface
T_P=0.45 to 0.7

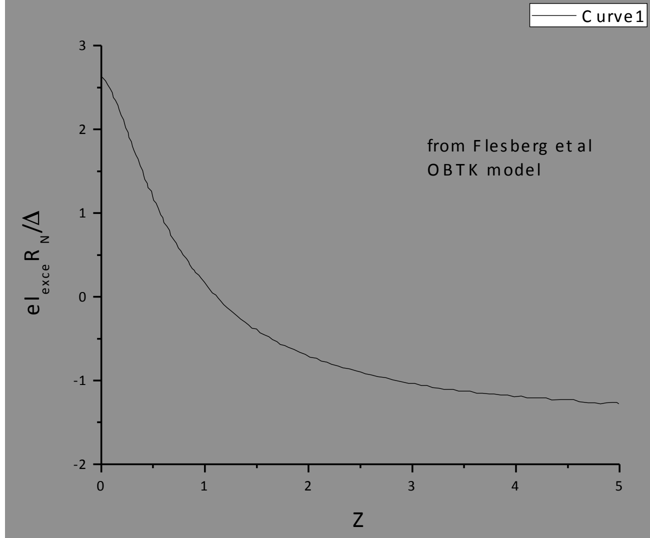


- From OBTK model:

K. Flensberg et al. PRB 38 8707 (1988)

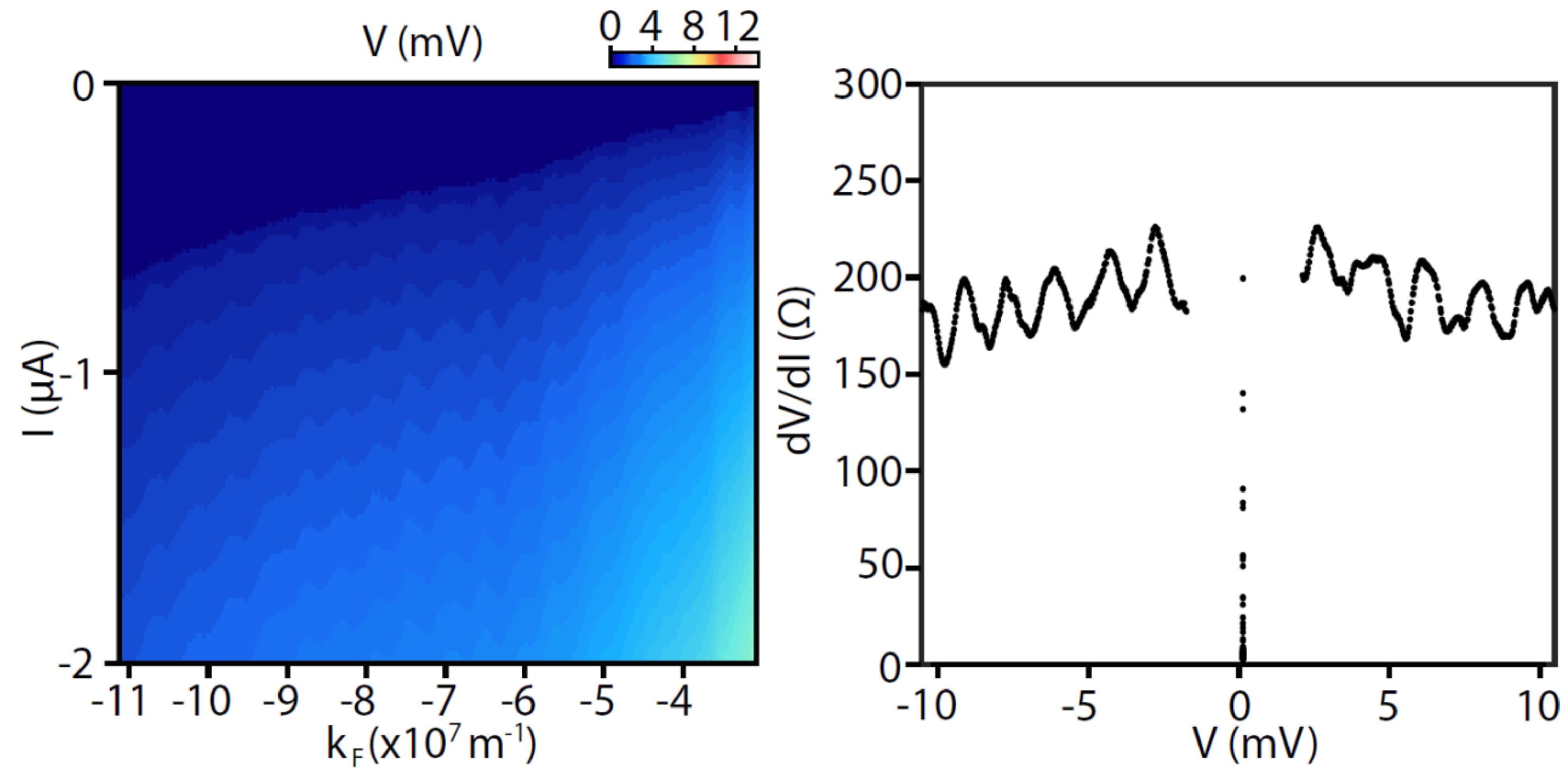
T_P=0.75 per interface
From excess current I_{excess}=1.45 μA
With the ratio of $eI_{excess}R_N/\Delta$



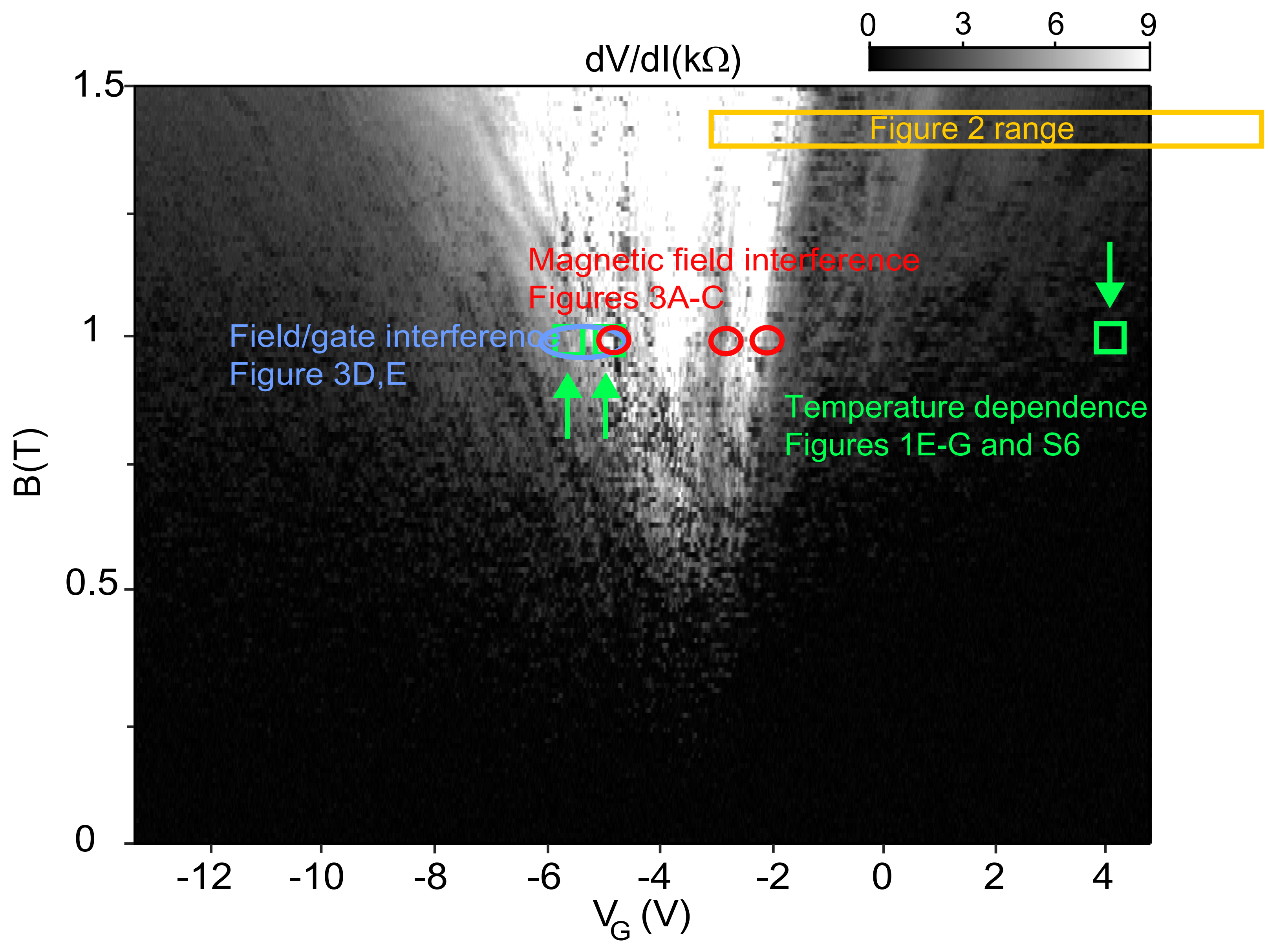


Fabry-Perot interferometry

- From $Lk_F = n\pi$, the trajectory length is estimated as 500 nm, which is close to the junction length of 650 nm for J_3; difference due to PN junctions

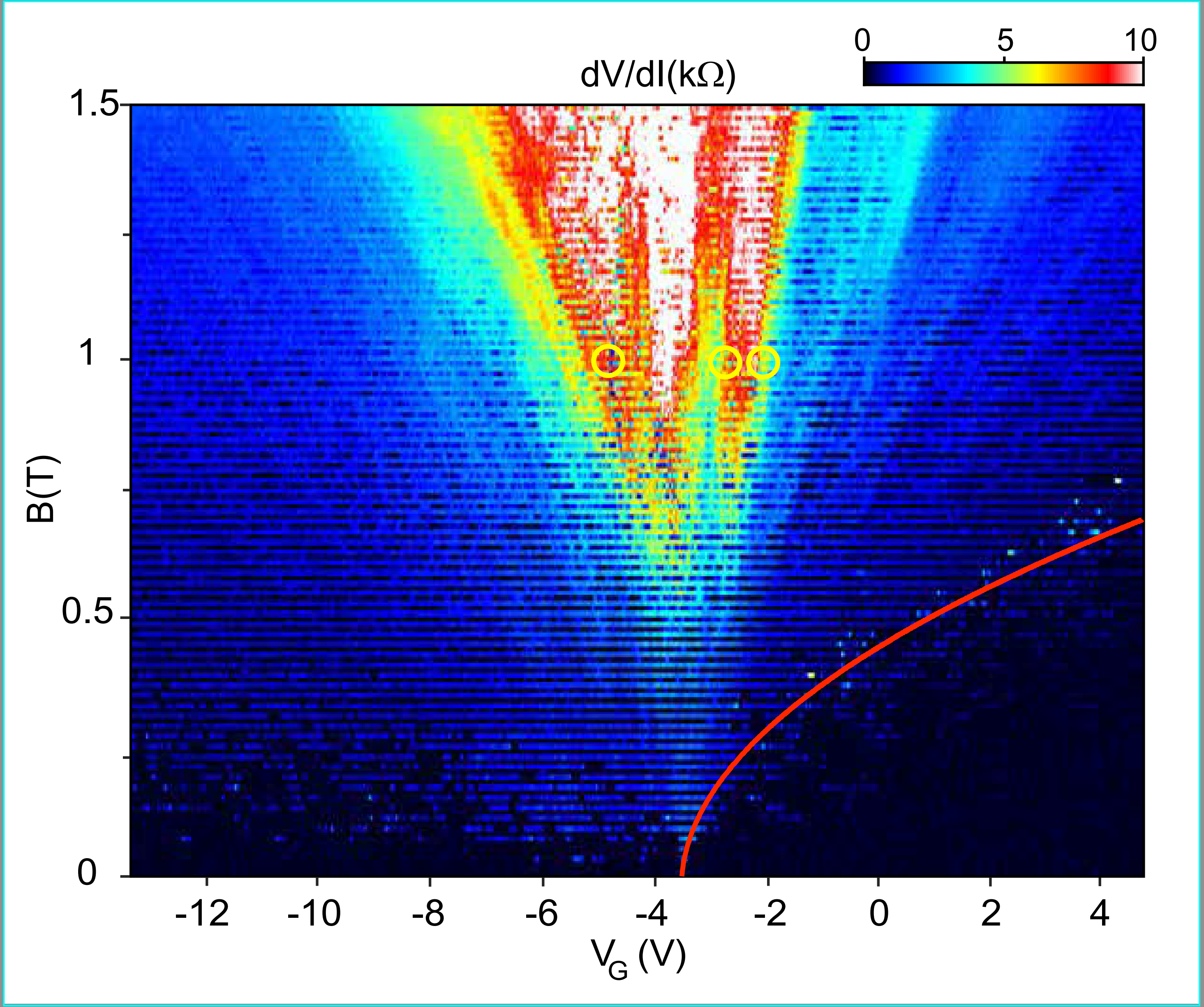


Measurement map for Junction 1



- Temperature dependence is conducted in three different locations
- Magnetic field interference is measured in three pockets

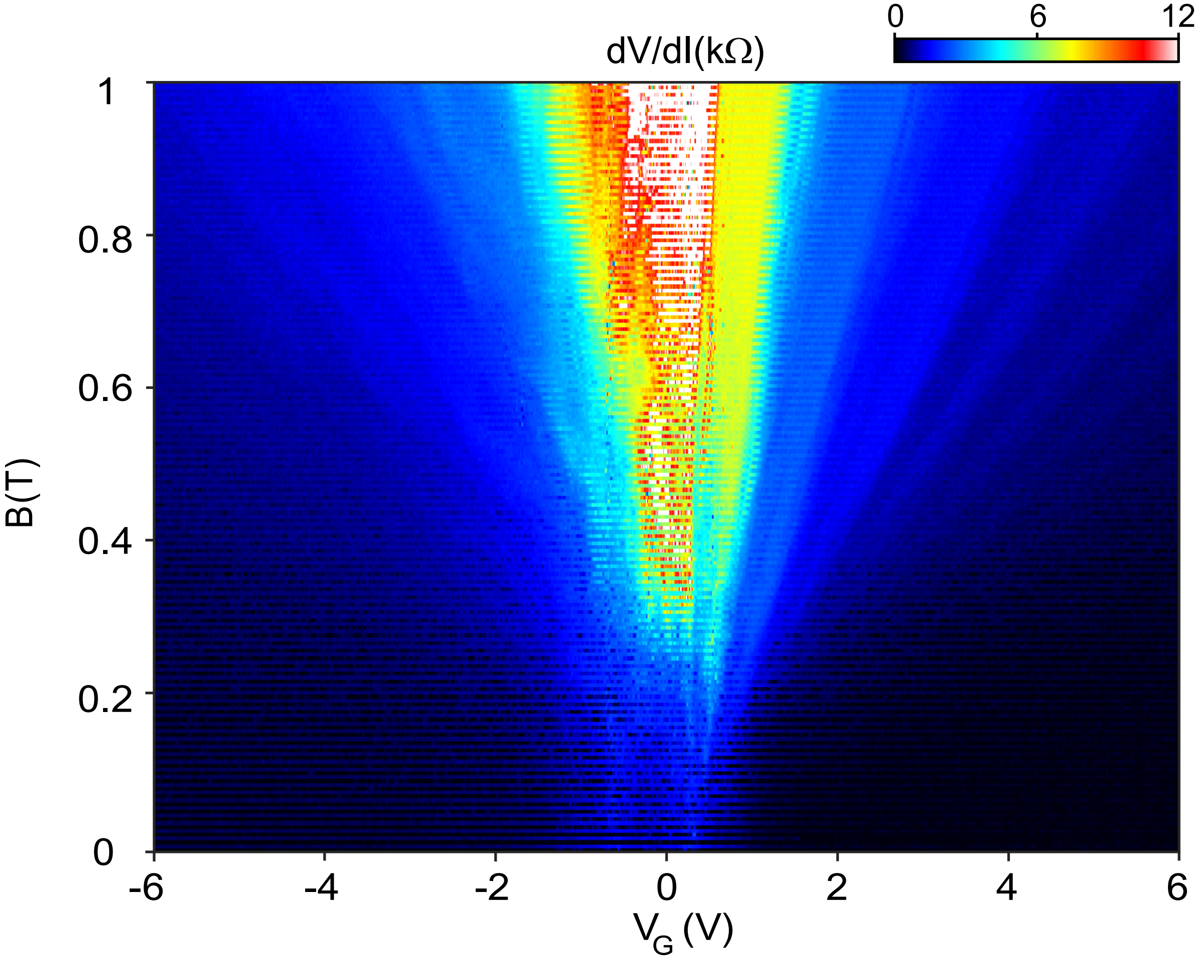
Alternating lines at high and low bias current

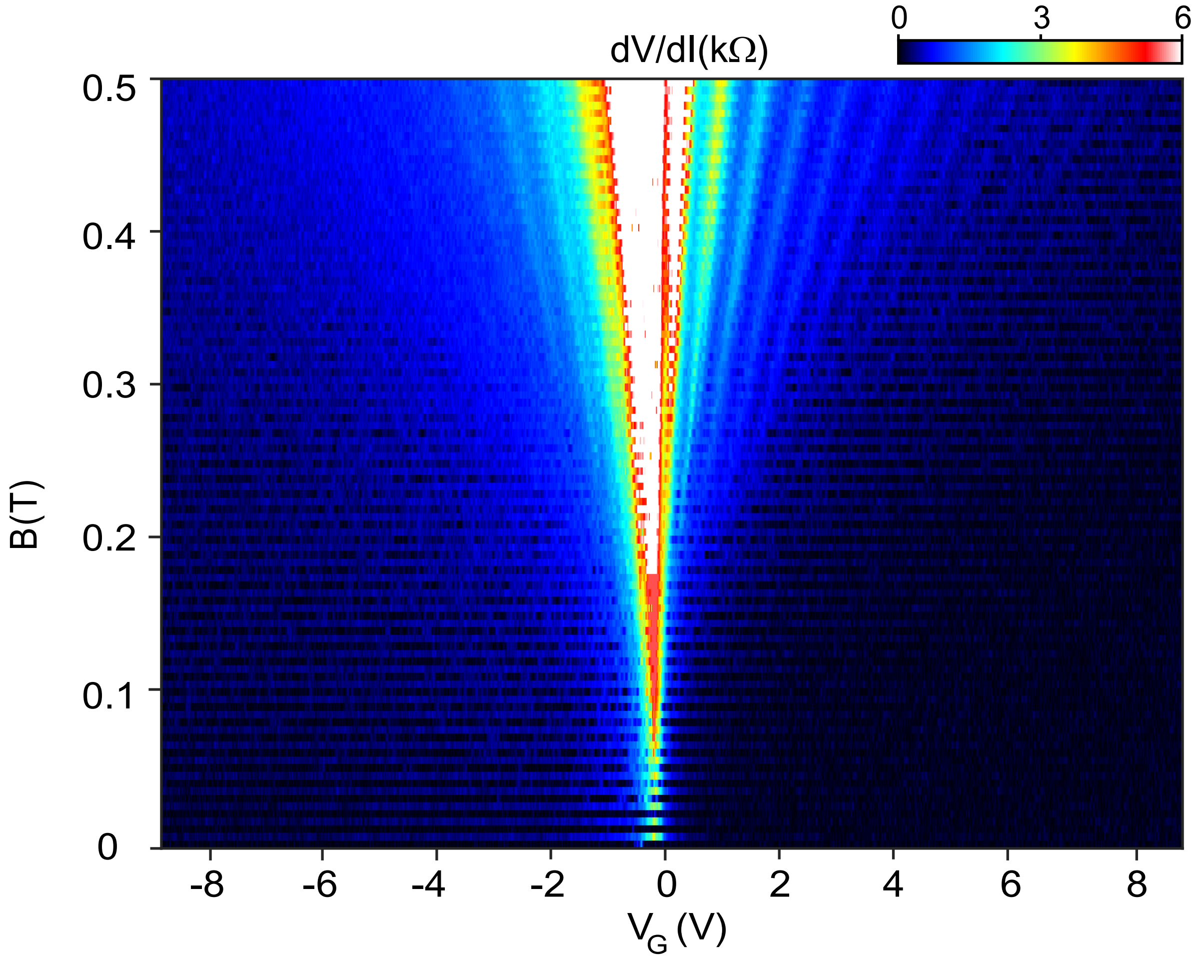


- At each B, the gate dependence is recorded at finite (few nA) and vanishing <100 pA current
- Alternating lines of high and low resistance

	L	W
J1	0.3µm	2.4µm
J2	0.8µm	2.4µm
J3	0.65µm	4.5µm

Alternating lines at high and low bias current

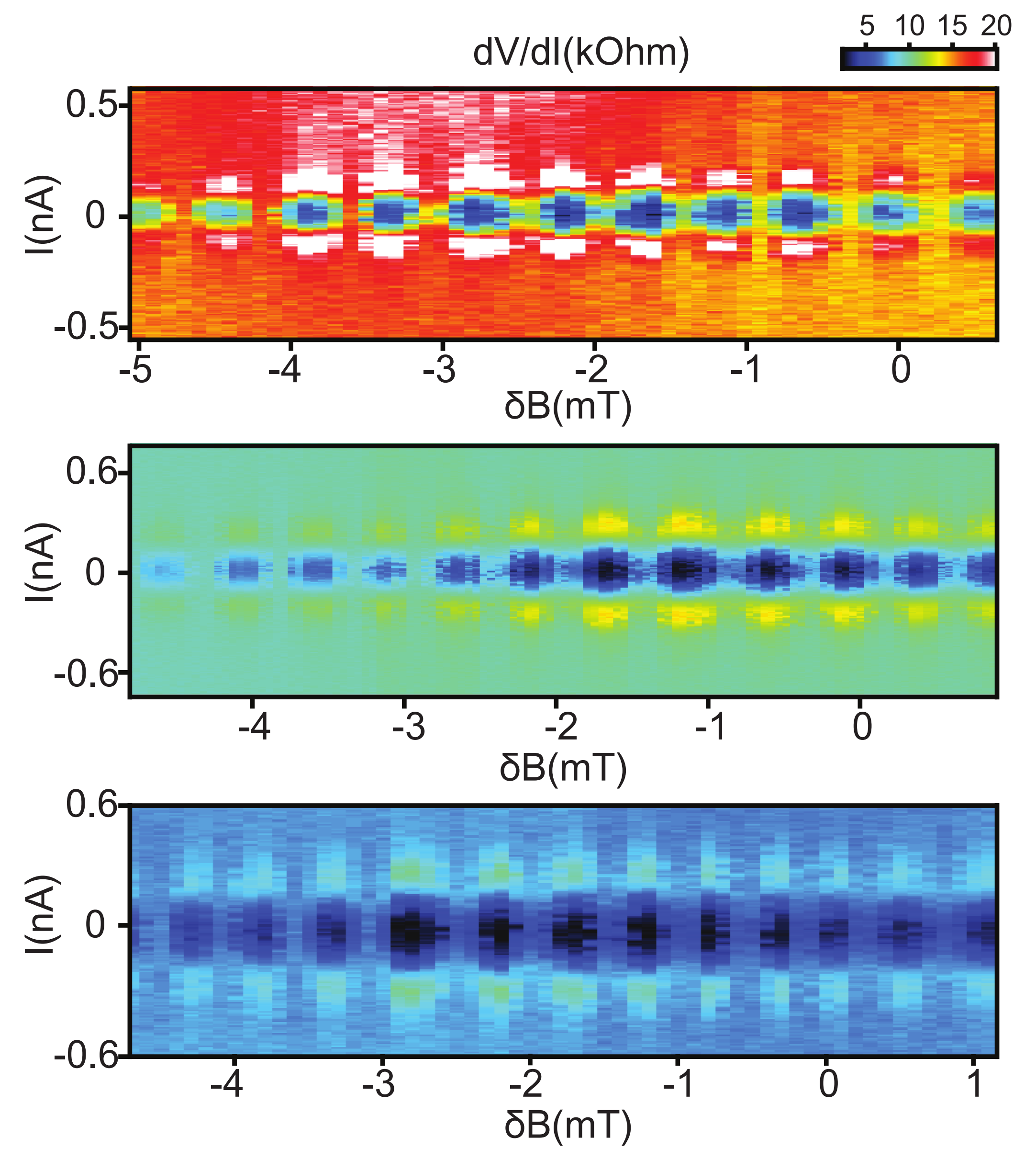




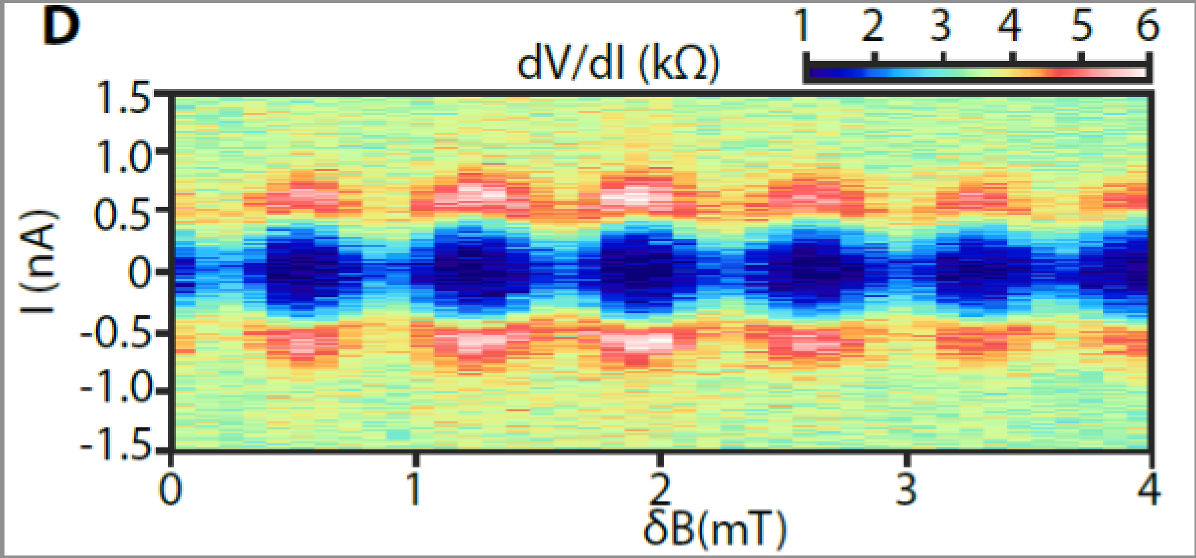
J2	0.8µm	2.4µm

J3	0.65µm	4.5µm

Interference at different filling factors

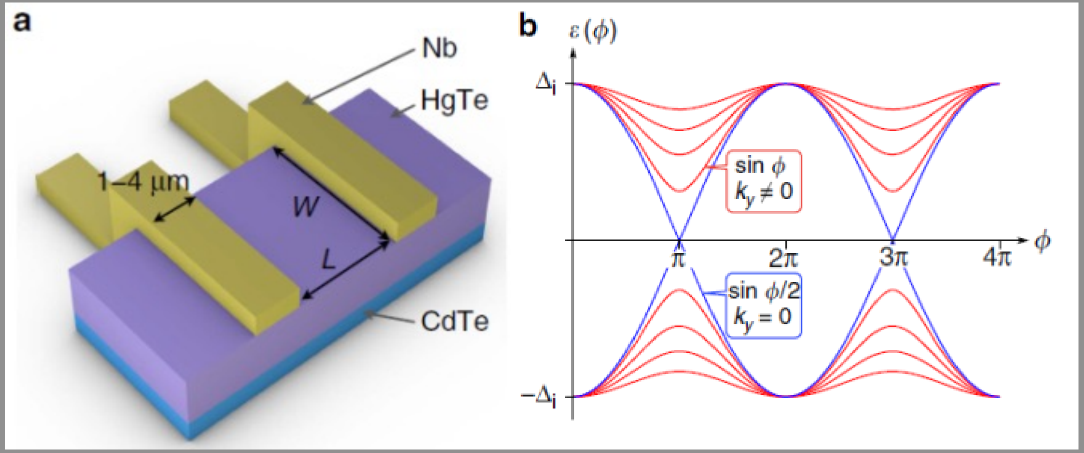


- ν = 2, 6 and 10 around 1T
- Periodicity is very close

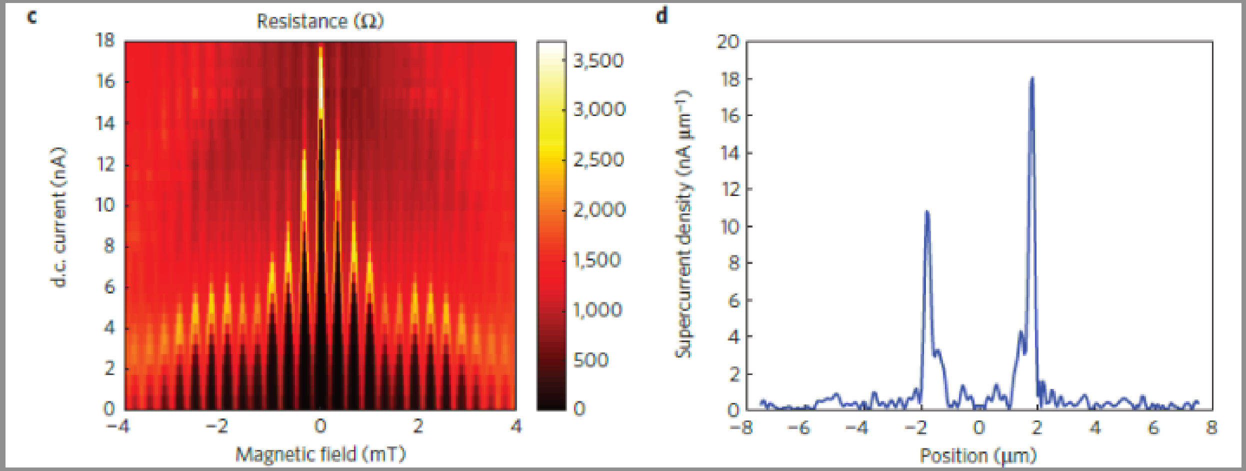


Superconductivity proximity to topological states

- Various proposals on coupling superconductivity to topological states
- Potential application in fault-tolerance quantum computation
- Majorana zero mode is predicted with broken symmetry
- In graphene, QH edge states + SC is one of the possible directions

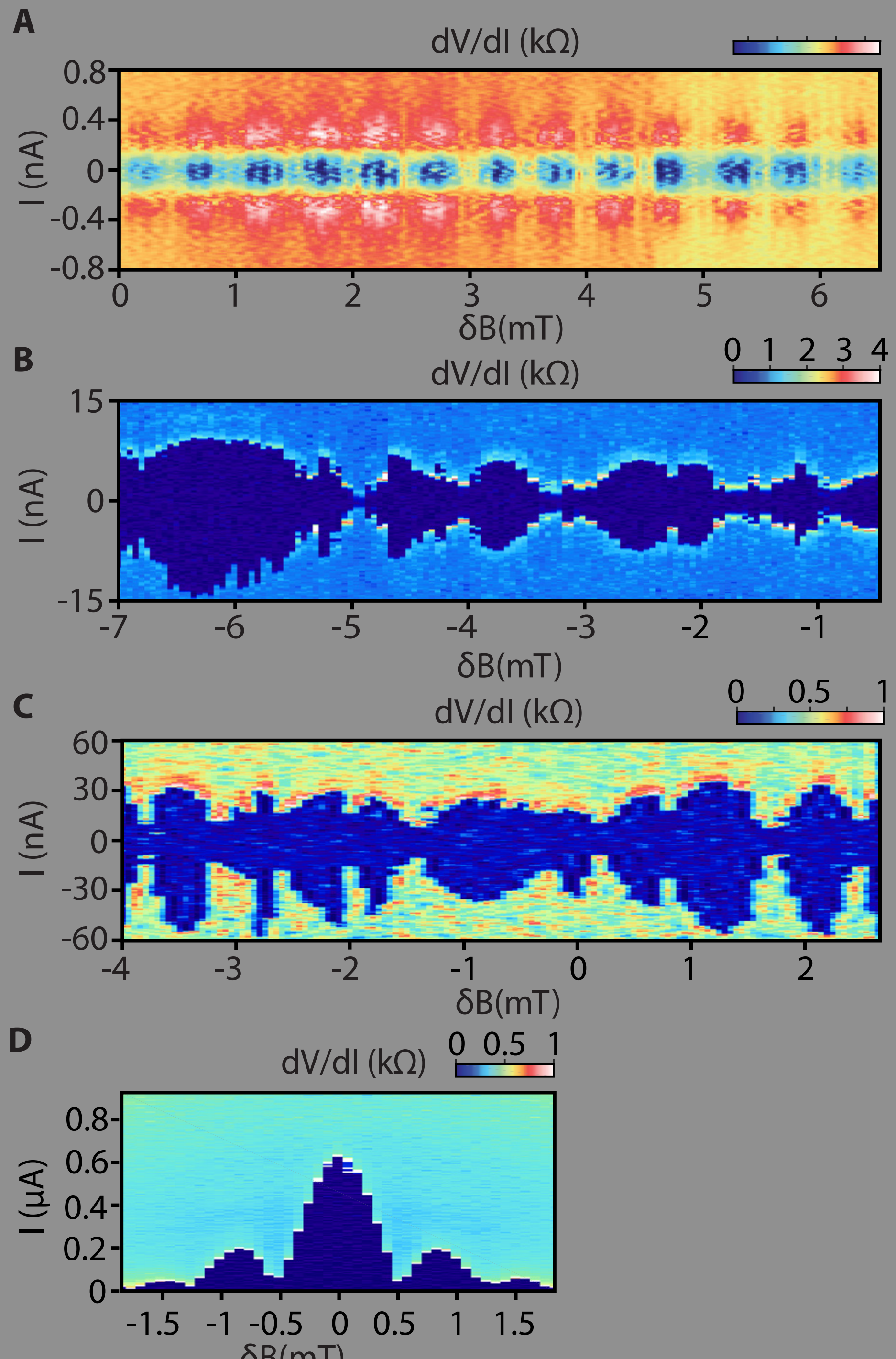


J. Wiedenmann et al,
Nat. Comm. 10303 (2016)



S. Hart et al Nat. Phys. **10** (2014)

Another set of interference



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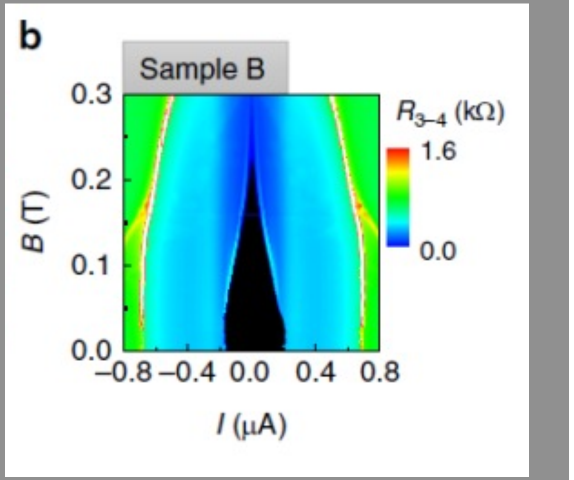
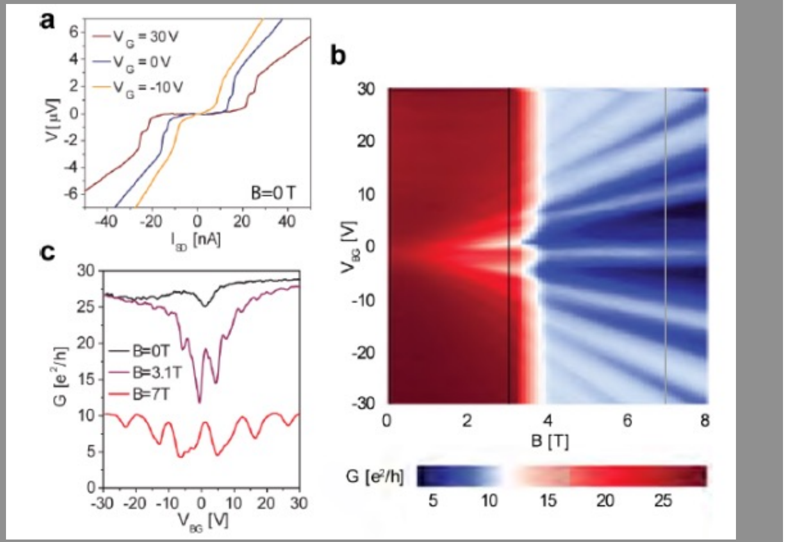
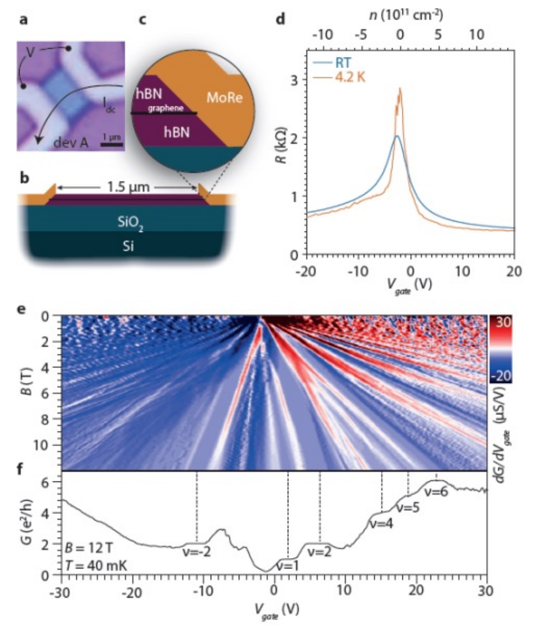


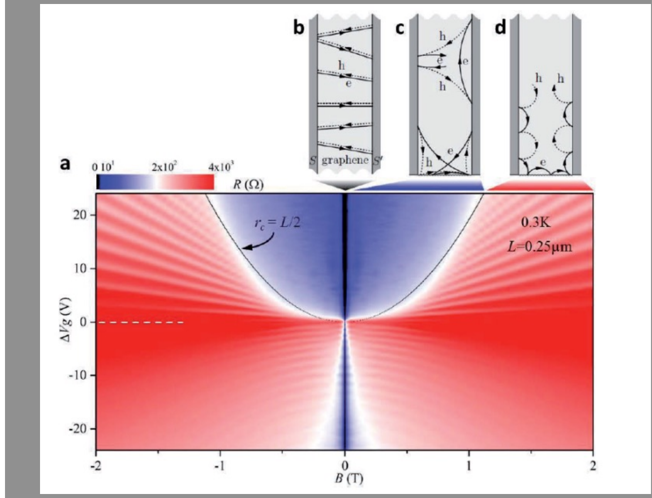
Z. Wan et al.
Nat. Comm. **6** 7426 (2015)



P. Rickhaus et al.
Nano Lett. (2012)



V. E. Calado et al.,
Nat. Nano. (2015)



Ben Shalom et al.
Nat. Phys. (2016)

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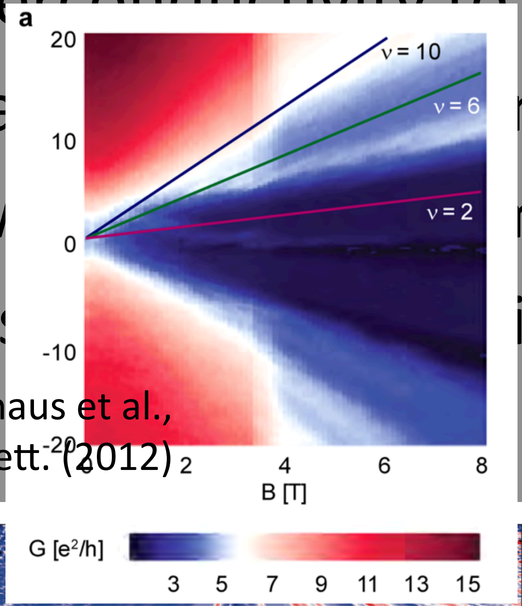

P. Rickhaus et al.,
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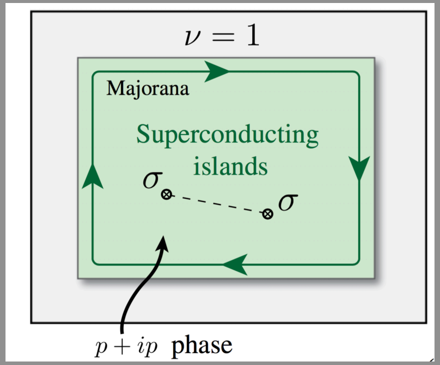
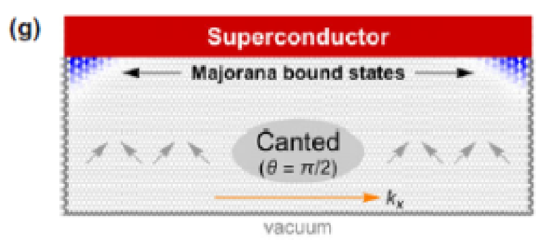
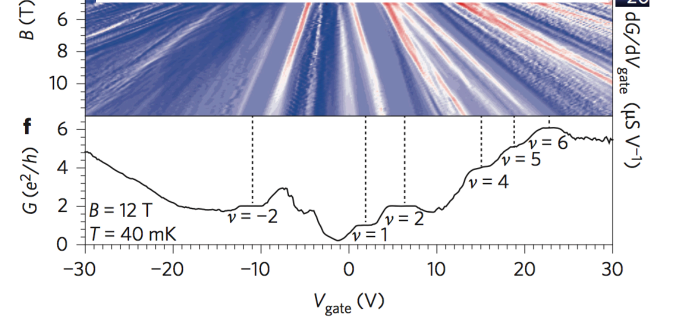


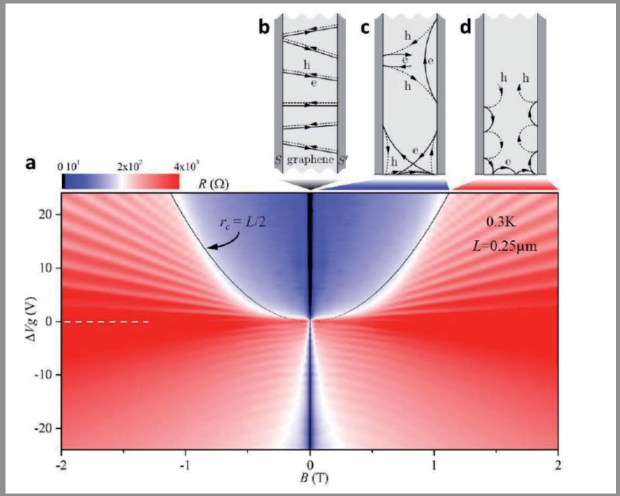
R.S.K. MONG et al.
PHYS. REV. X 4, 011036 (2014)



P. SAN-JOSE et al.
PHYS. REV. X 5, 041042 (2015)



V. E. Calado et al.,
Nat. Nano. (2015)

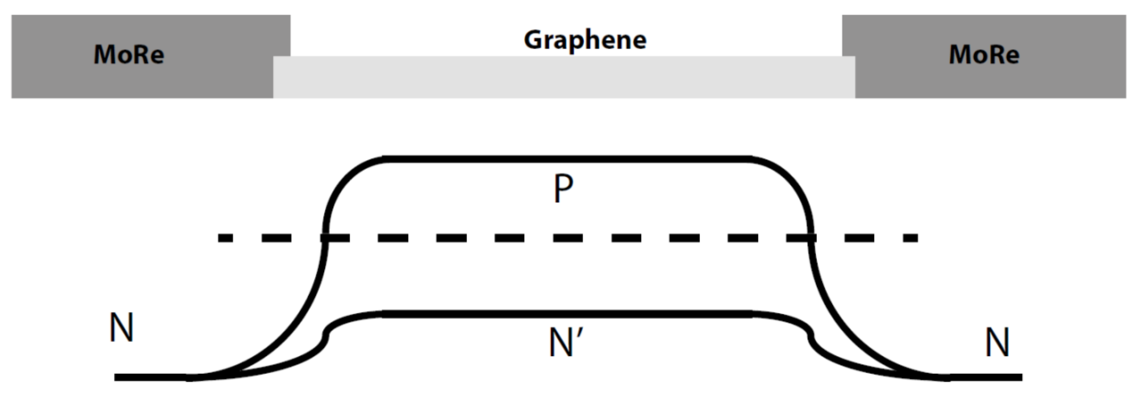


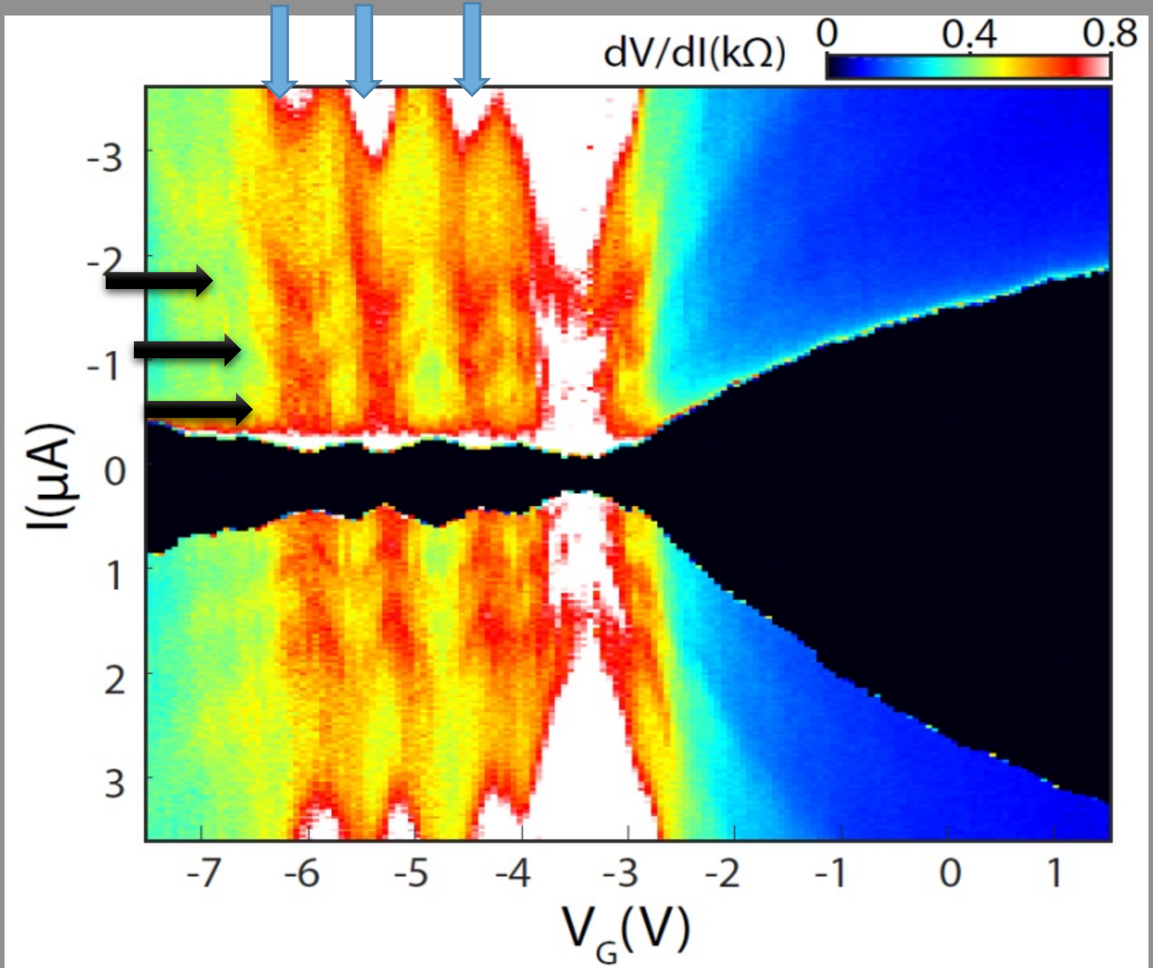
Ben Shalom et al.
Nat. Phys. (2016)

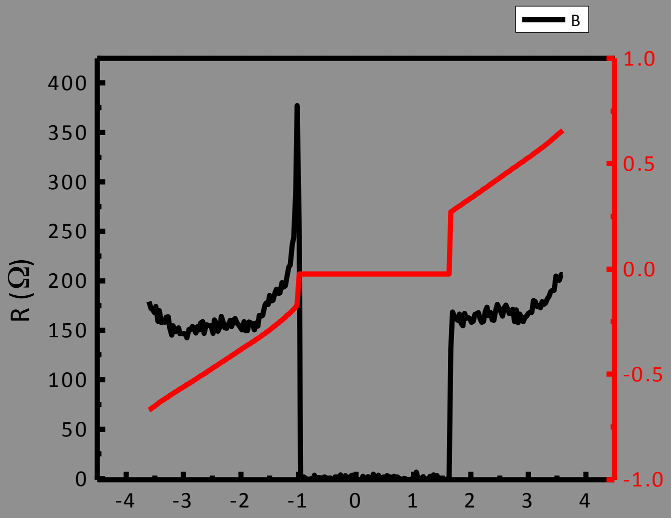
Graphene Josephson junction

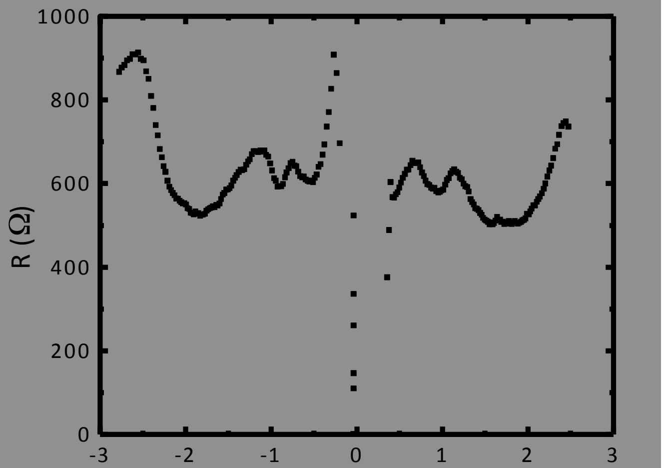
V. E. Calado et al., Nature Nano. (2015).
M. Ben Shalom et al., Nat. Phys. (2016).
M. T. Allen et al., Nat. Phys. (2016).

- Critical current is suppressed at DP
- P/N interface reduces supercurrent
- Multiple Andreev reflection
- Ballistic features: Fabry-Perot

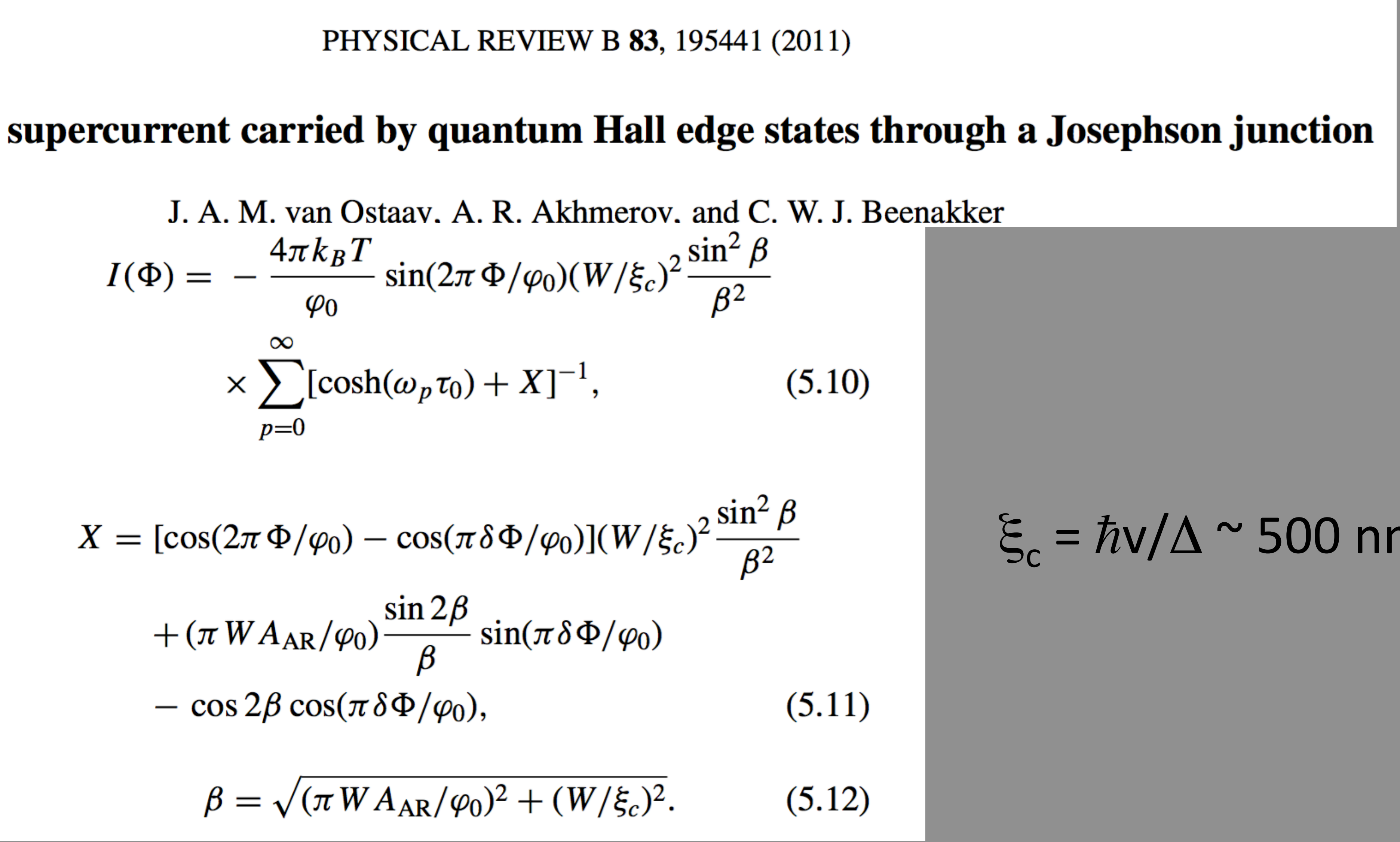








Supercurrent at the QH edge states



PHYSICAL REVIEW B **83**, 195441 (2011)

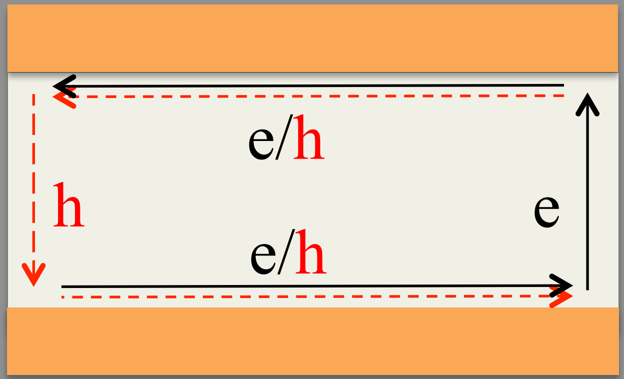
supercurrent carried by quantum Hall edge states through a Josephson junction

J. A. M. van Ostaav. A. R. Akhmerov. and C. W. J. Beenakker

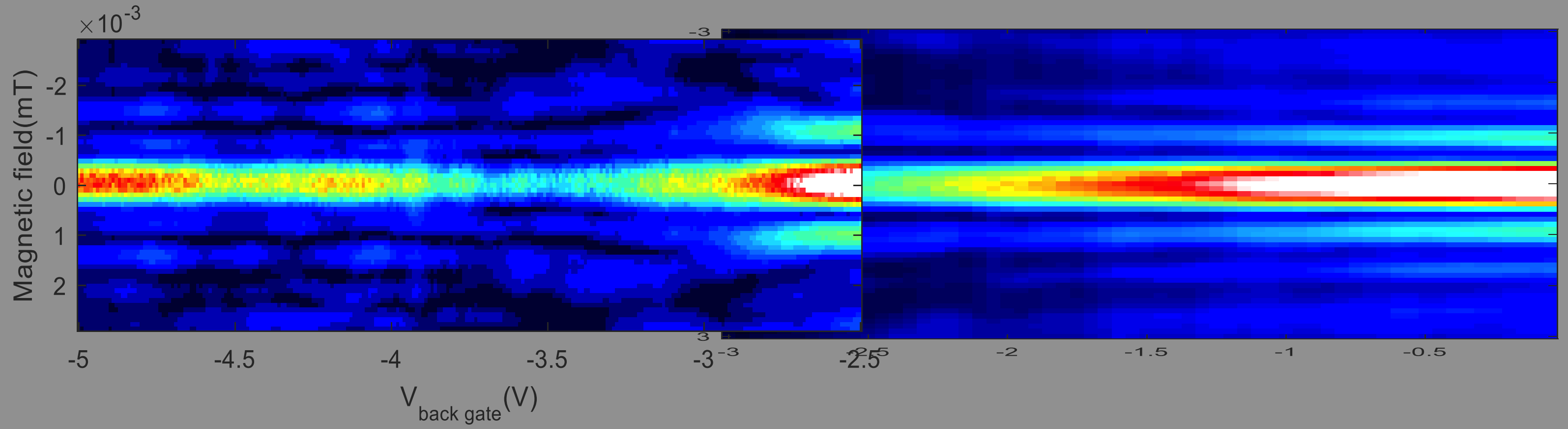
$$I(\Phi) = -\frac{4\pi k_B T}{\varphi_0}\sin(2\pi\Phi/\varphi_0)(W/\xi_c)^2\frac{\sin^2\beta}{\beta^2} \times \sum_{p=0}^{\infty}[\cosh(\omega_p\tau_0) + X]^{-1}, \tag{5.10}$$

$$X = [\cos(2\pi\Phi/\varphi_0) - \cos(\pi\delta\Phi/\varphi_0)](W/\xi_c)^2\frac{\sin^2\beta}{\beta^2} + (\pi W A_{\mathrm{AR}}/\varphi_0)\frac{\sin 2\beta}{\beta}\sin(\pi\delta\Phi/\varphi_0) - \cos 2\beta\cos(\pi\delta\Phi/\varphi_0), \tag{5.11}$$

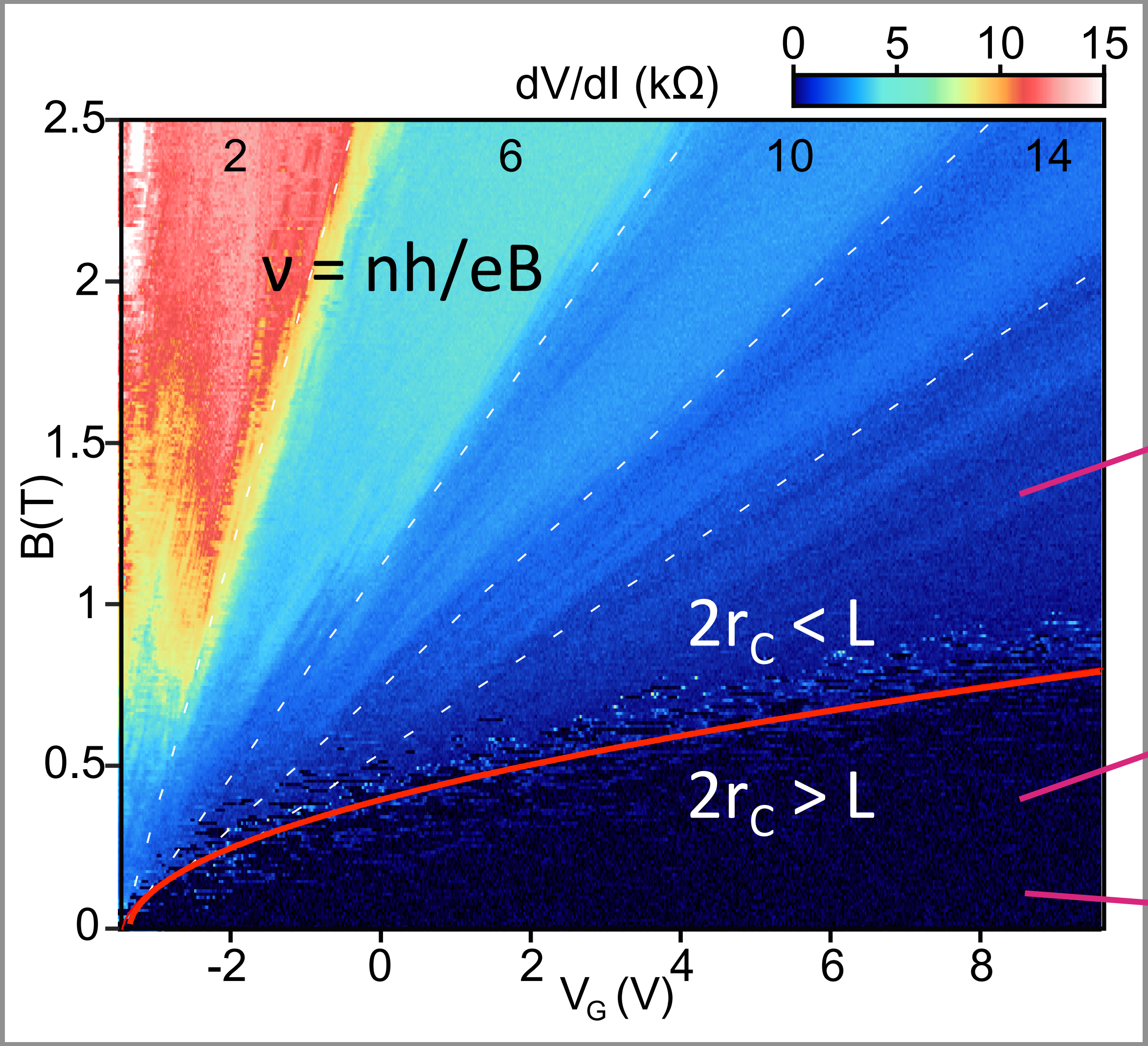
$$\beta = \sqrt{(\pi W A_{\mathrm{AR}}/\varphi_0)^2 + (W/\xi_c)^2}. \tag{5.12}$$



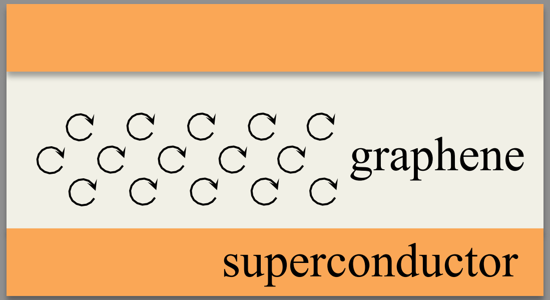
$\xi_c = \hbar v/\Delta \sim 500$ nm >> $l_0 \sim$ 20-30 nm


×10^-3
Magnetic field(mT)
-2
-1
0
1
2
-5
-4.5
-4
-3.5
-3
-2.5
V back gate(V)
-3
3
-3
-2.5
-2
-1.5
-1
-0.5

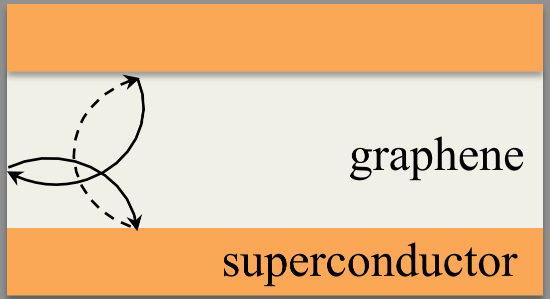
Resistance map in magnetic field and quantum Hall



QHE

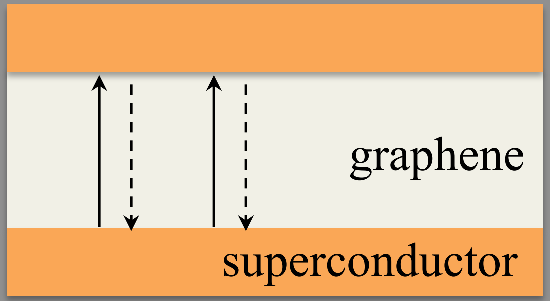


Semiclassical



M. Ben Shalom et al., Nat. Phys. (2016)

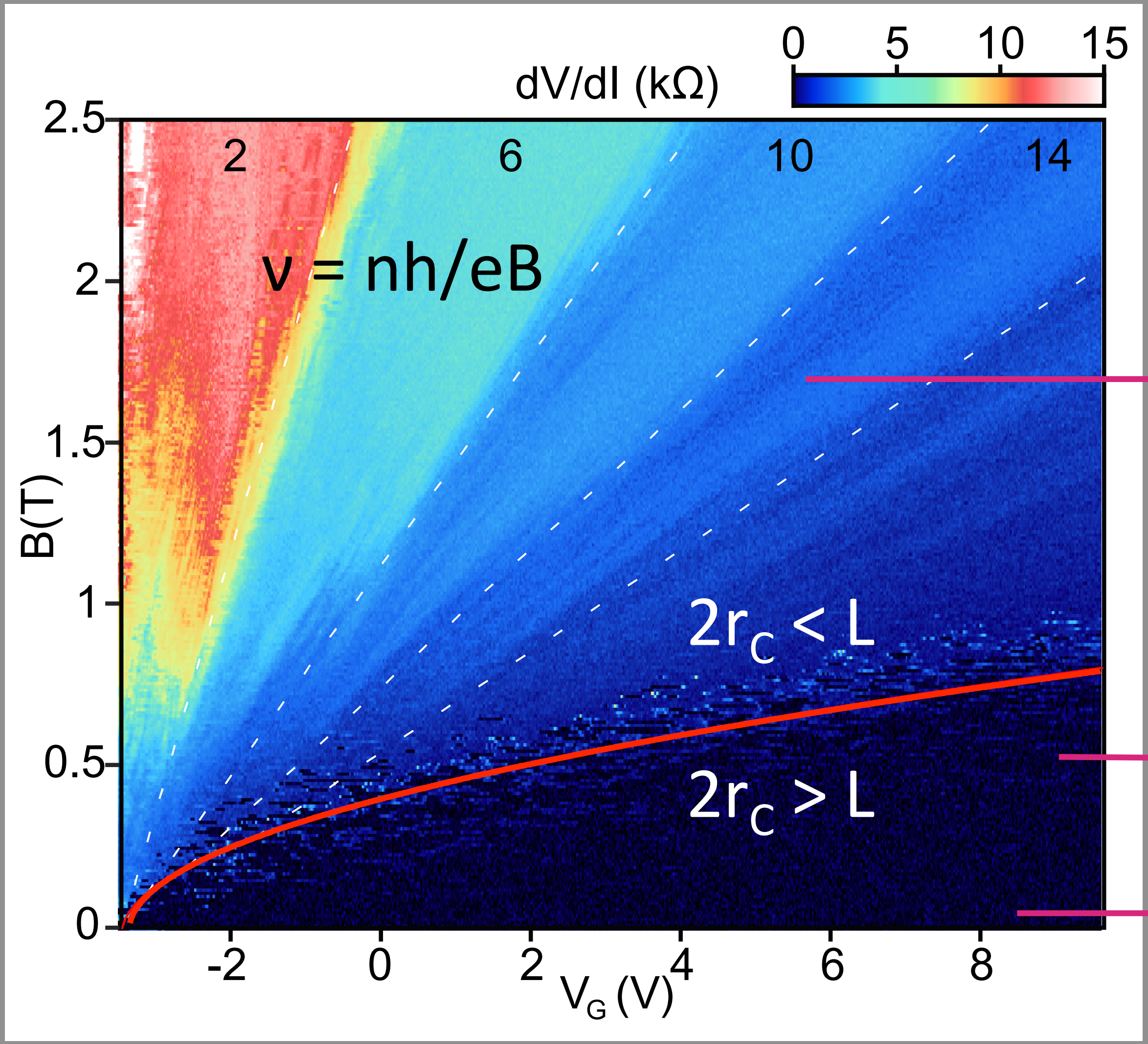
“Fraunhofer”



M.T. Allen et al., Nat. Phys. (2016)

- Semiclassical regime: cyclotron diameter $2r_C > L$
- QH regime: cyclotron diameter $2r_C < L$

Resistance map in magnetic field and quantum Hall



Semiclassical: cyclotron diameter $2r_C > L$

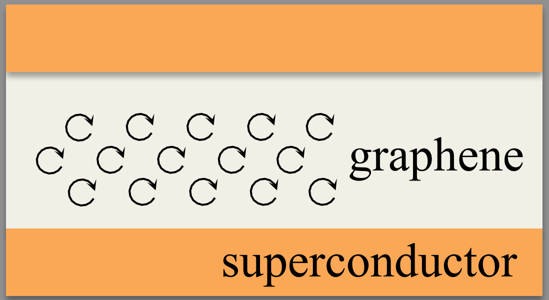
QH regime: cyclotron diameter $2r_C < L$

QHE

P. Rickhaus et al., Nano Lett. (2012)

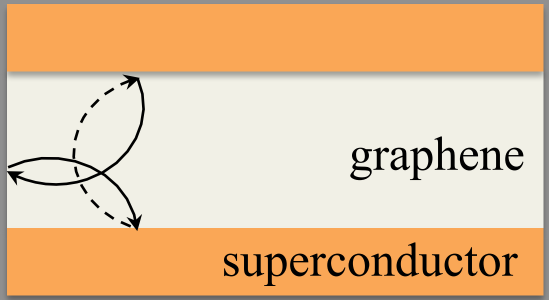
K. Komatsu et al., Phys. Rev. B (2012)

V. E. Calado et al., Nat. Nano. (2015)



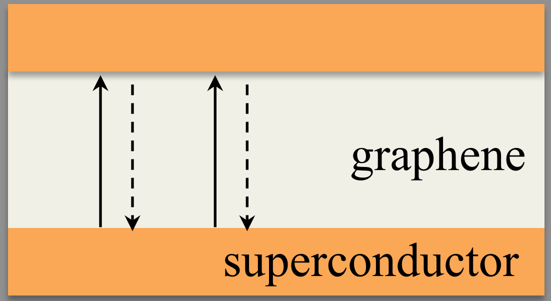
Semiclassical

M. Ben Shalom et al., Nat. Phys. (2016)

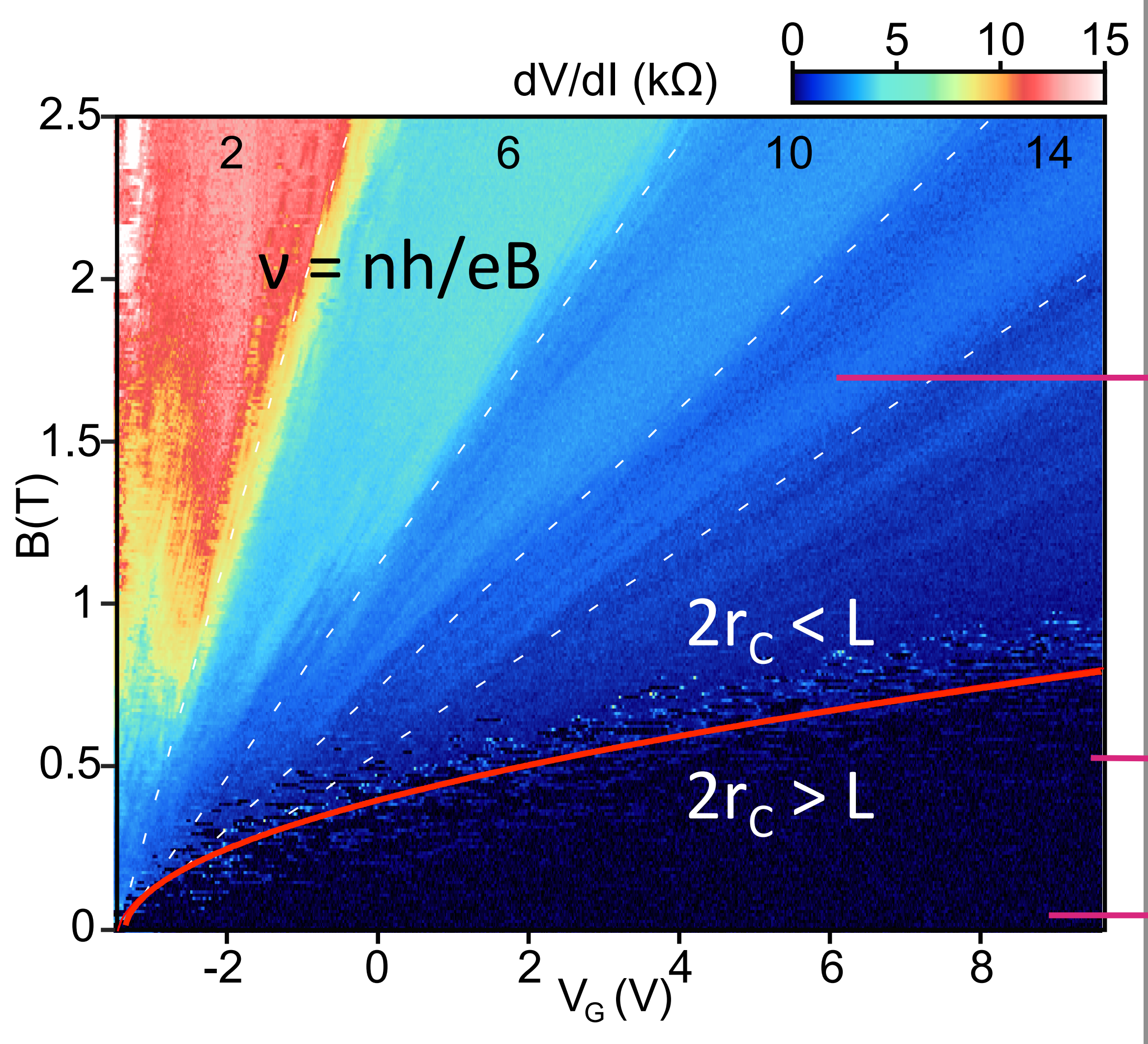


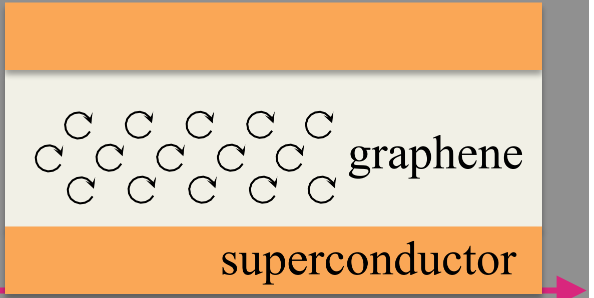
“Fraunhofer”

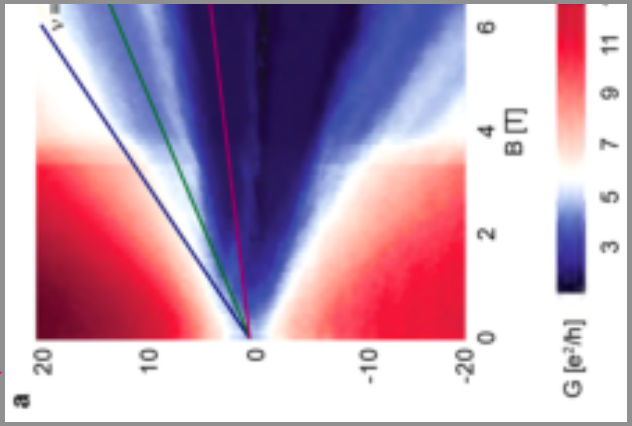
M.T. Allen et al., Nat. Phys. (2016)

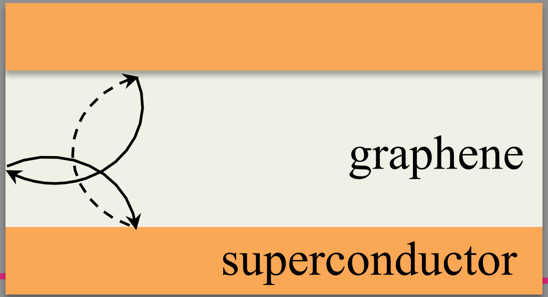


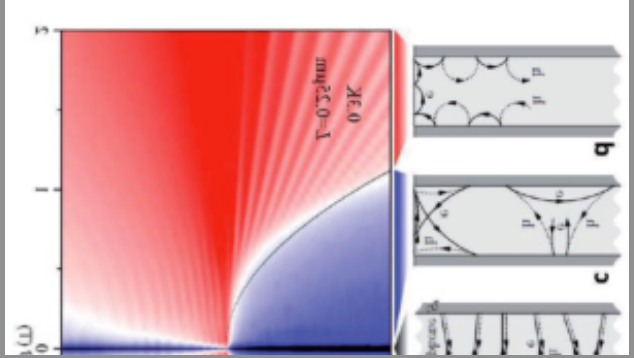
Resistance map in magnetic field and quantum Hall

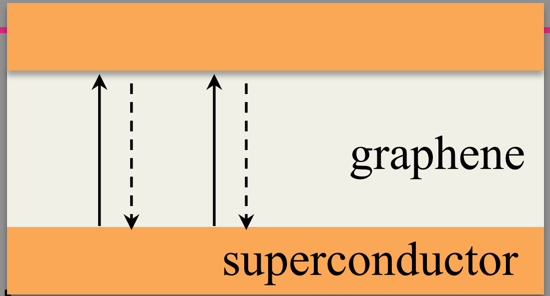


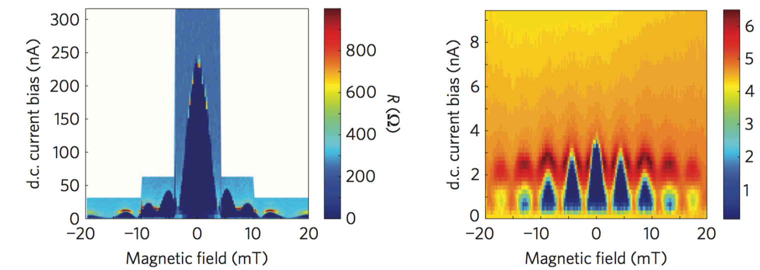








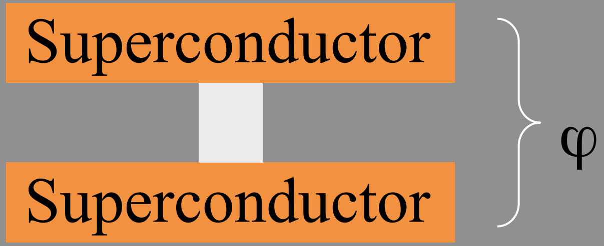




QH regime: cyclotron diameter $2r_C < L$

Semiclassical: cyclotron diameter $2r_C > L$

Glossary: phase, voltage, critical current



$d\varphi / dt = 2eV/\hbar$

$E = -E_J \cos(\varphi) - (\hbar/2e)\, I\, \varphi$

$E_J = \hbar\, I_C / 2e$

All microscopic properties

$I = 0$

$I > 0$

$I = I_S$
switching current

$I = I_C^{(0)}$
true critical current