Practical quantum realization of the ampere from the elementary charge

J. Brun-Picard (PhD), S. Djordjevic, D. Leprat, F. Schopfer, W. Poirier

GDR Mesoscopic Quantum Physics, Aussois, December 5-8

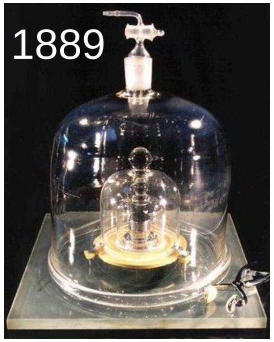
$\frac{h}{e^2}$ uantΩ Group

LNE

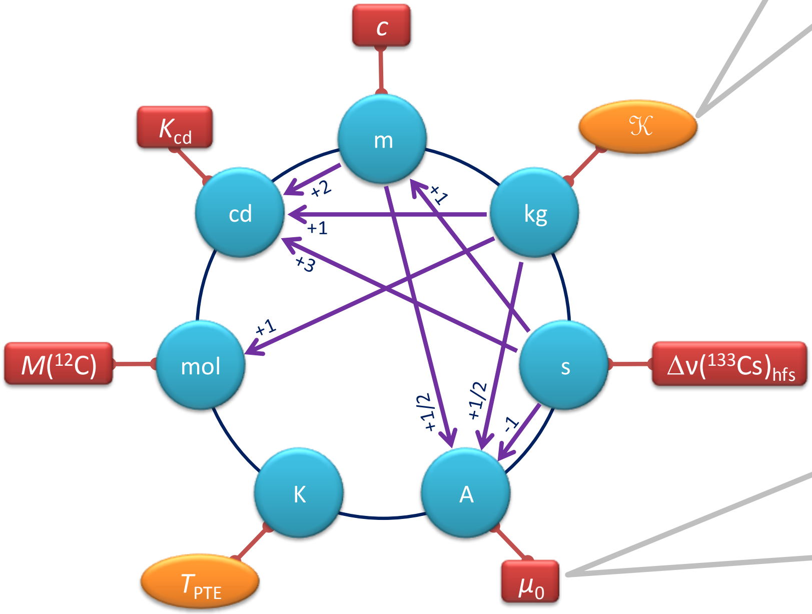
Le progrès, une passion à partager

International system of units (SI)

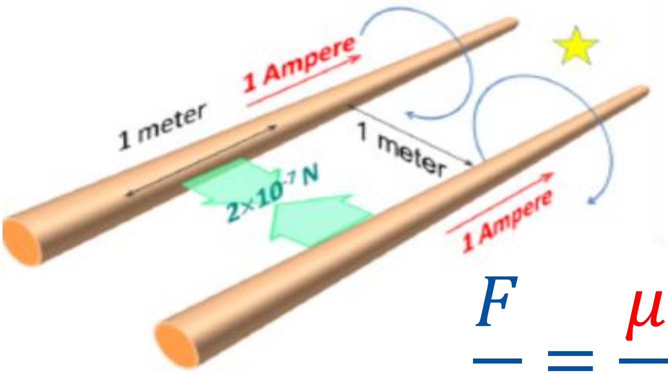
International prototype



Artefact with potential drift



$\mu_0 = 4\pi \times 10^{-7}$ H/m.

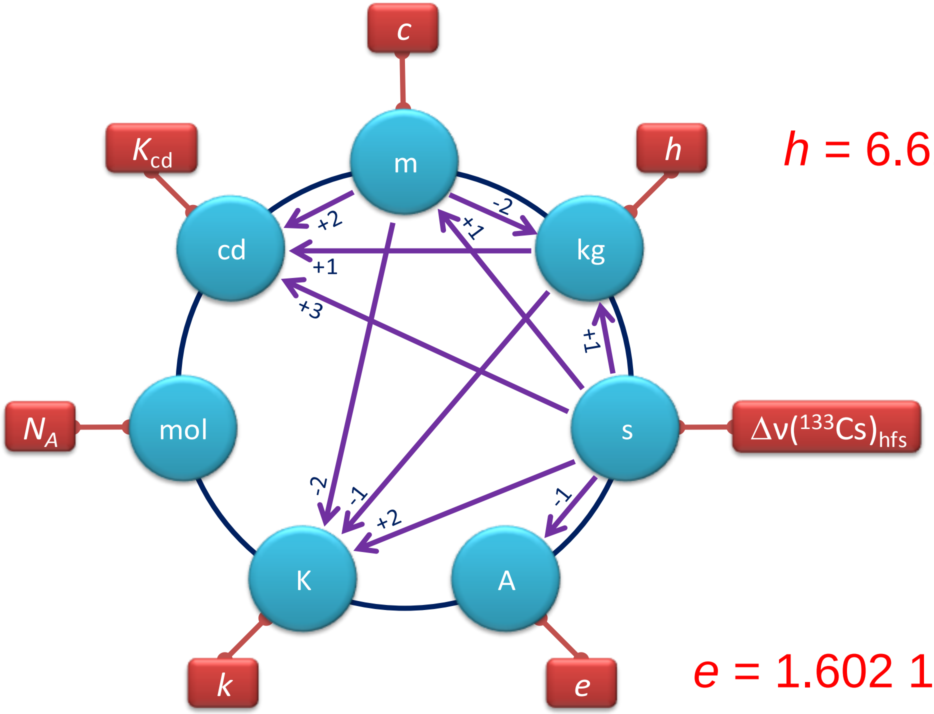


1948

$$\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$$

Realization of the ampere, linked to mechanical experiments (u ~ few 10^{-7})

Revision of the SI planned in 2018



$h = 6.626\ 070\ 040 \times 10^{-34}$ kg.m^2.s^{-1}

$e = 1.602\ 176\ 620\ 8 \times 10^{-19}$ A.s

- Redefinition of 4 base units (kg, A, K, mol) in terms of constants of physics with fixed numerical values (h,e,k,N_A)

Quantum electrical standards related to *h* and *e*

Present SI

Future SI

V

K_J

$u_{K_J} = 4 \times 10^{-7}$

μ_0

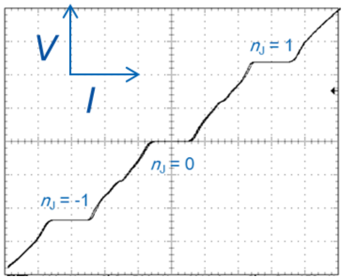
A

Ω

R_K

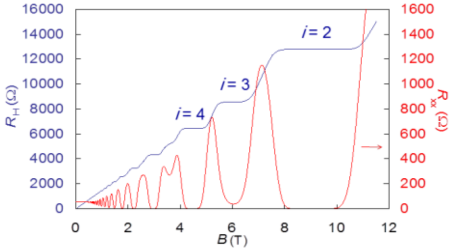
$u_{R_K} = 1 \times 10^{-7}$

AC Josephson effect



$$U_J = \frac{n_J}{K_J} f_J$$

Quantum Hall effect



$$R_H = \frac{R_K}{i_K}$$

(10^{-10})

$$K_J = \frac{2e}{h}$$

$u_{K_J} = 0$

V

A

e

Ω

$$R_K = \frac{h}{e^2}$$

$u_{R_K} = 0$

(10^{-9})

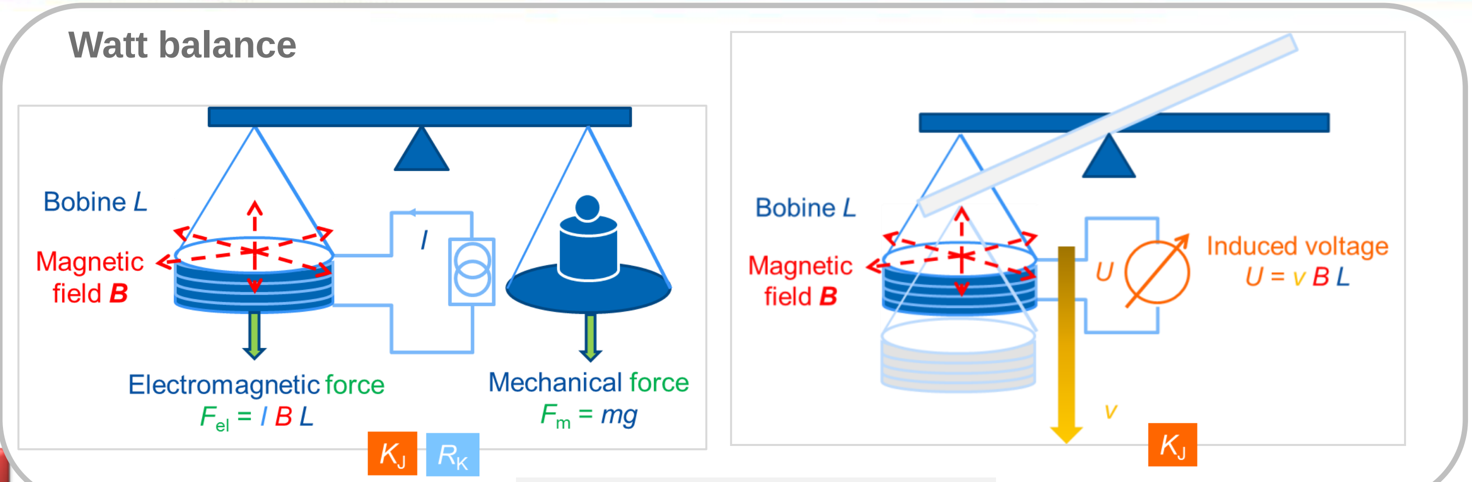
CODATA 2014

$K_J = 2e/h\,(1 + \varepsilon_J)$ $\varepsilon_J = (-0.9 \pm \mathbf{1.5}) \times 10^{-8}$

$R_K = h/e^2\,(1 + \varepsilon_K)$ $\varepsilon_K = (2.2 \pm \mathbf{1.8}) \times 10^{-8}$

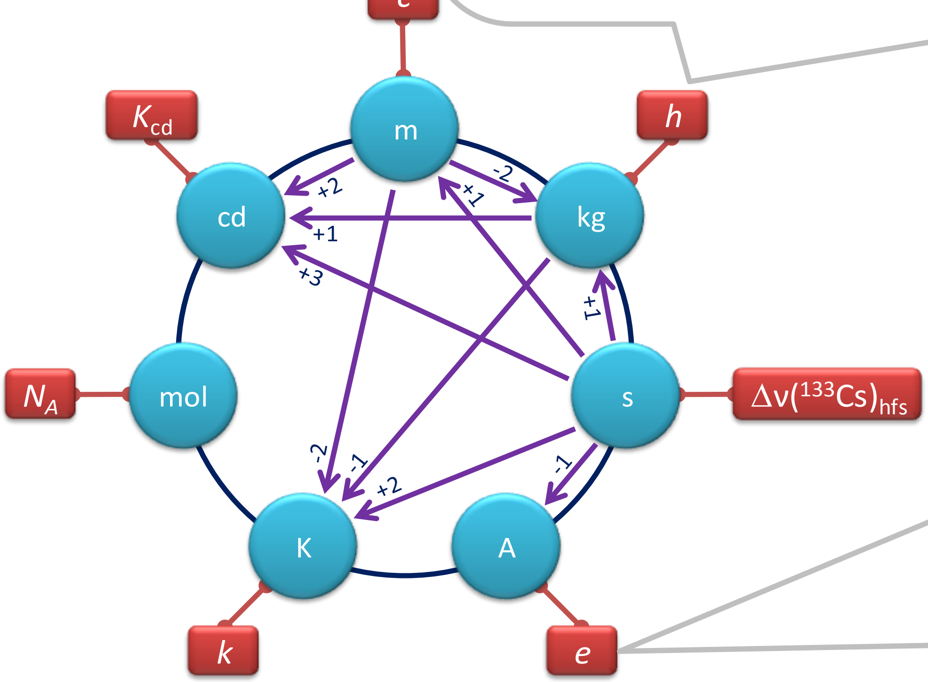
Consistent with absence of corrections

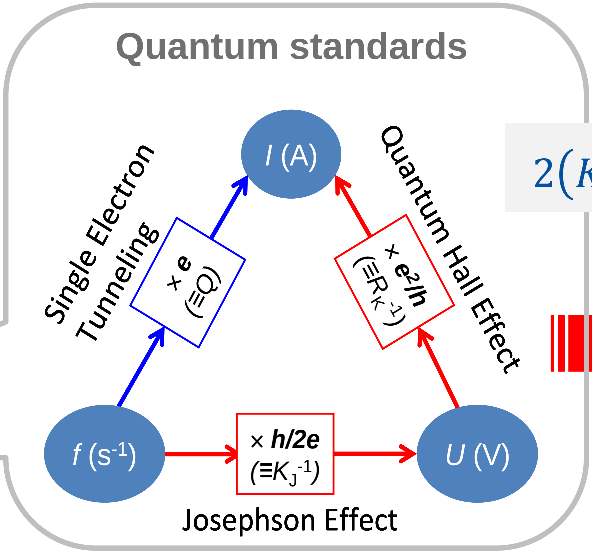
Some "Mise en pratique" in the future SI



$$mgv = UI \propto \frac{1}{K_J^2 R_K} = \frac{h}{4}$$

Uncertainty level few 10^{-8}

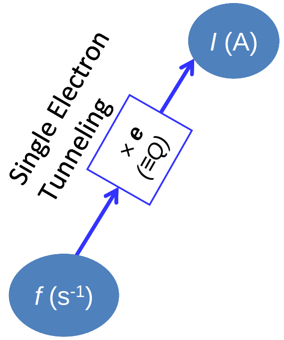




$$2(K_J R_K)^{-1} = e$$

Need 10^{-8} uncertainty level

Single-electron current sources



- Coulomb blockade at low temperatures
- Charge quantization in mesoscopic structures

- Transfer of n_Q electrons through the device at each cycle of a control parameter synchronized at frequency f_Q

- Small capacitances
- Barrier resistances > h/e^2
- Low temperature $kT < e^2/C$

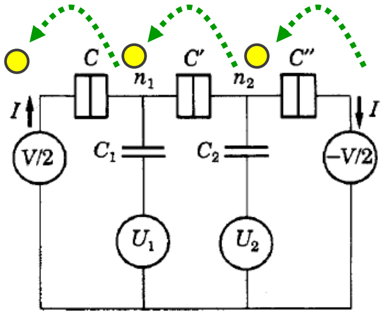
$$|I| = n_Q\, Q\, f_Q$$

$$\boxed{Q = e}$$

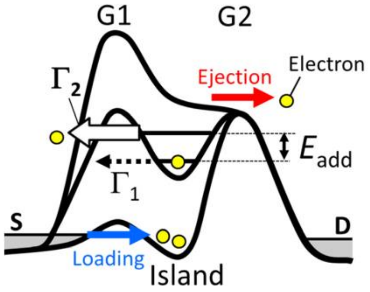
Metallic islands with fixed tunnel barriers (adiabatic pumping)

Trade-off between speed and precision

Tunable-barrier semiconductor devices (non-adiabatic pumping)

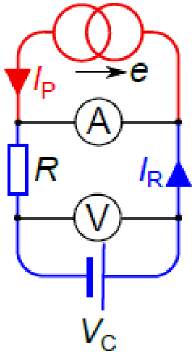


H. Pothier et al. EPL (1992)



Yamahata et al. APL (2016)

Tunable barrier electron pumps : state of the art



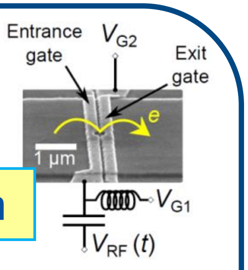
GaAs/AlGaAs

I = 151 pA

f = 0.945 GHz

B > 14 T

T ~ 300 mK



1,2 ppm

Giblin et al. Nature Com. 3, 930 (2012).

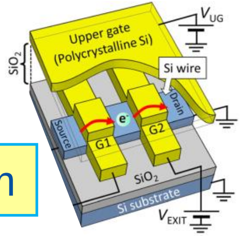
Si

I = 160 pA

f = 1 GHz

B = 0

T ~ 1.5 K



0,9 ppm

Yamahata et al. APL 109, 013101 (2016).

GaAs/AlGaAs

I = 87 pA

f = 545 MHz

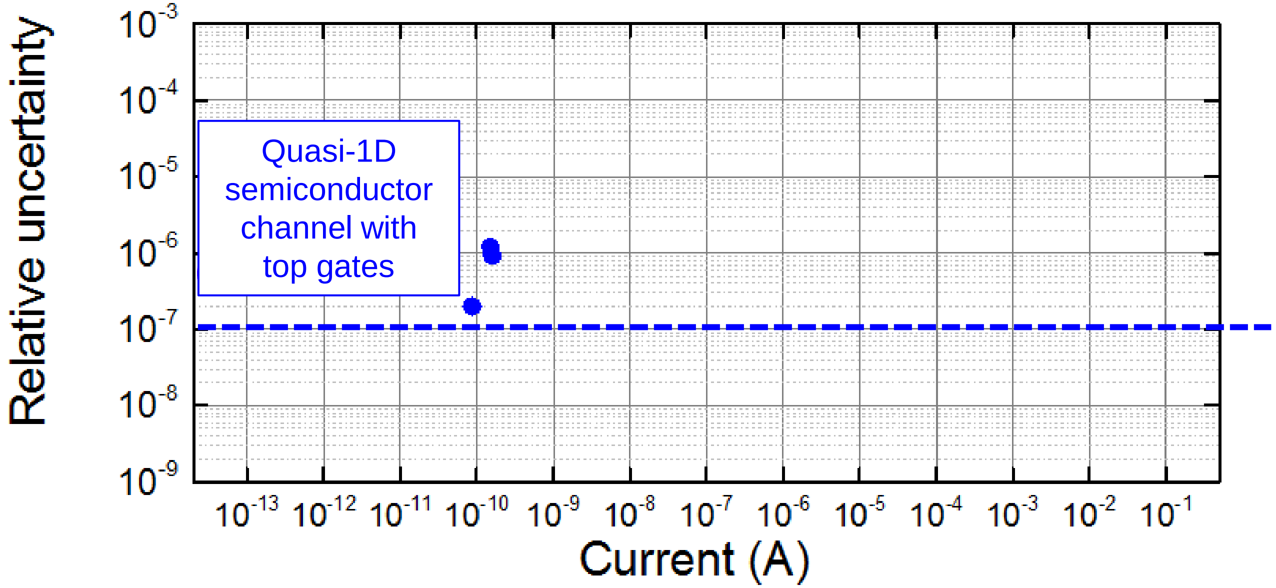
B > 16 T

T ~ 100 mK

T_m=5 h ⇒ 0,6 ppm

0,2 ppm

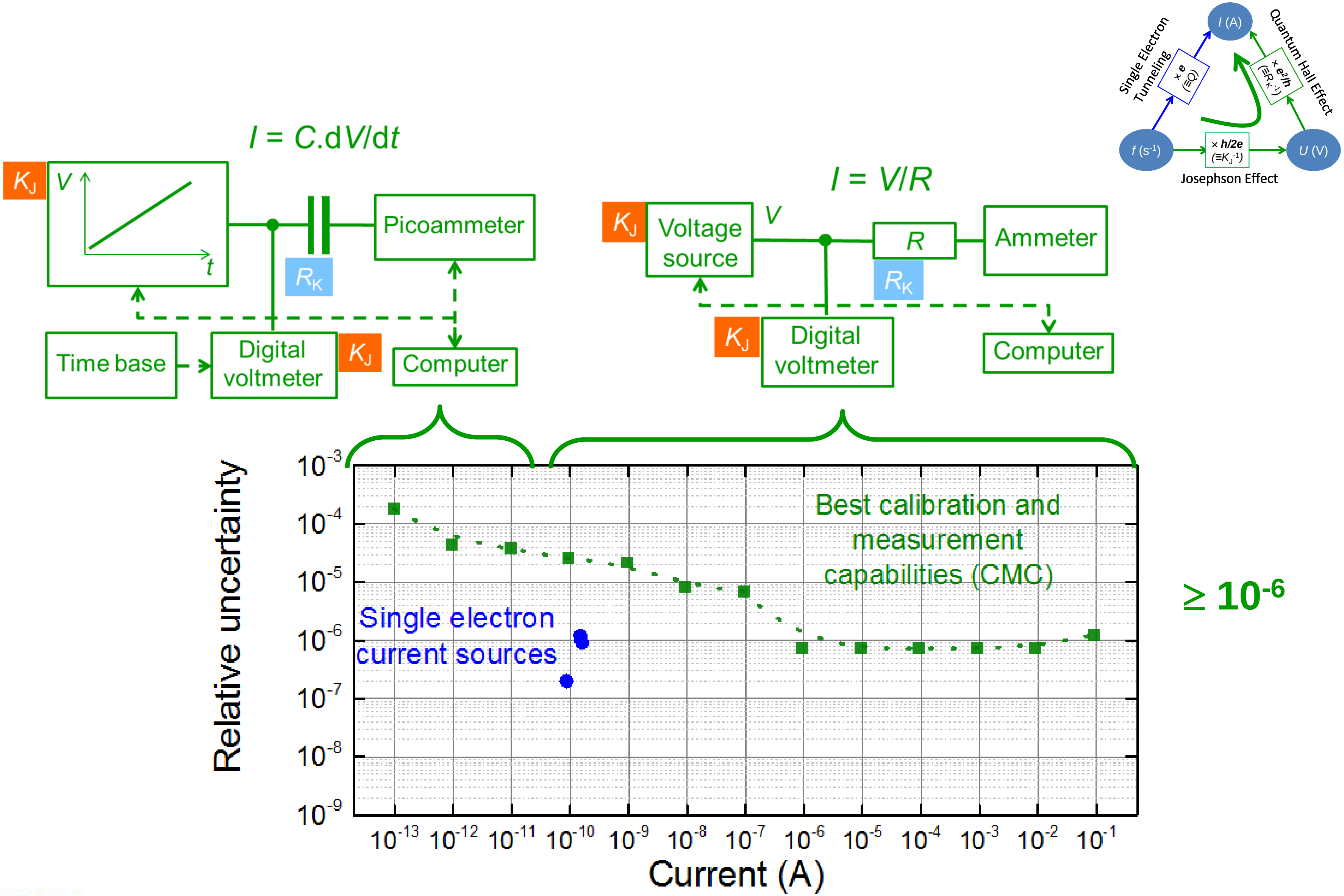
Stein et al., APL 107, 103501 (2015).



Quasi-1D semiconductor channel with top gates

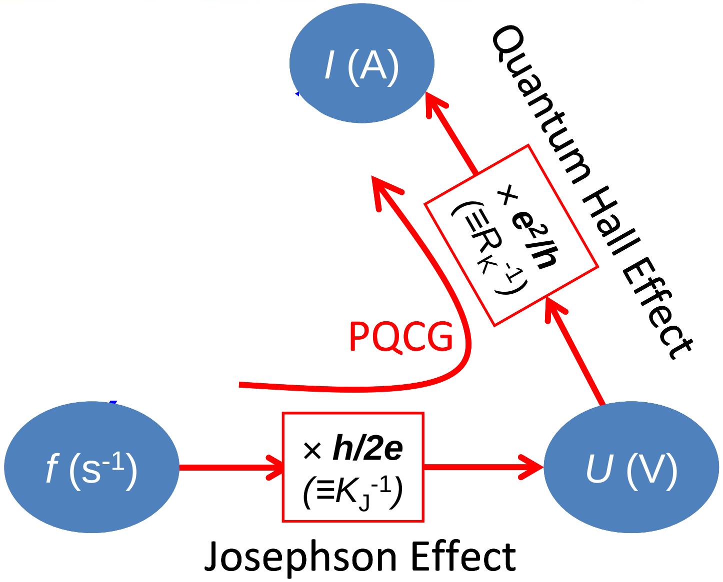
10^{-7}

Best calibration and measurement capabilities (CMC)



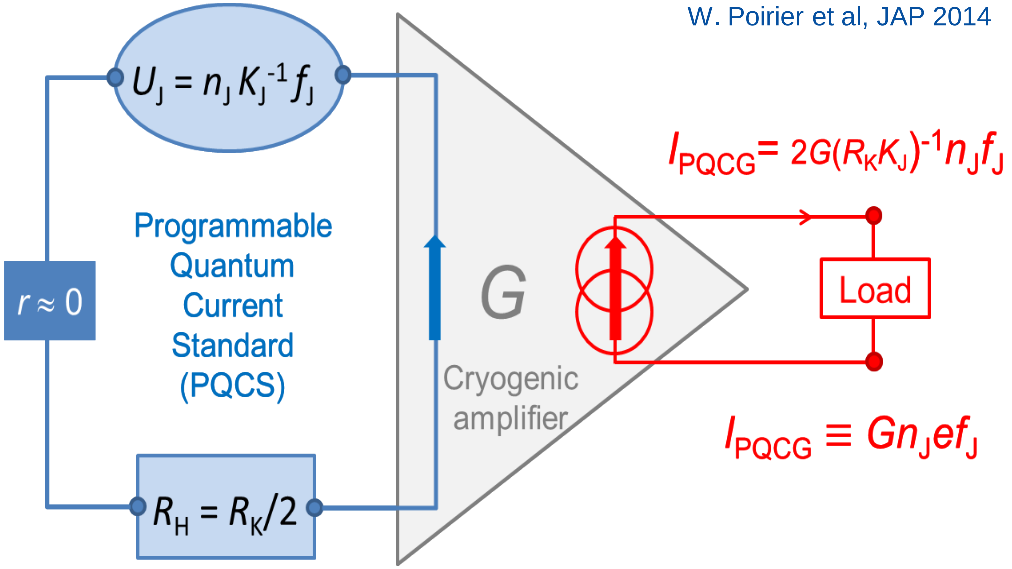
$\geq 10^{-6}$

Programmable quantum current generator principle

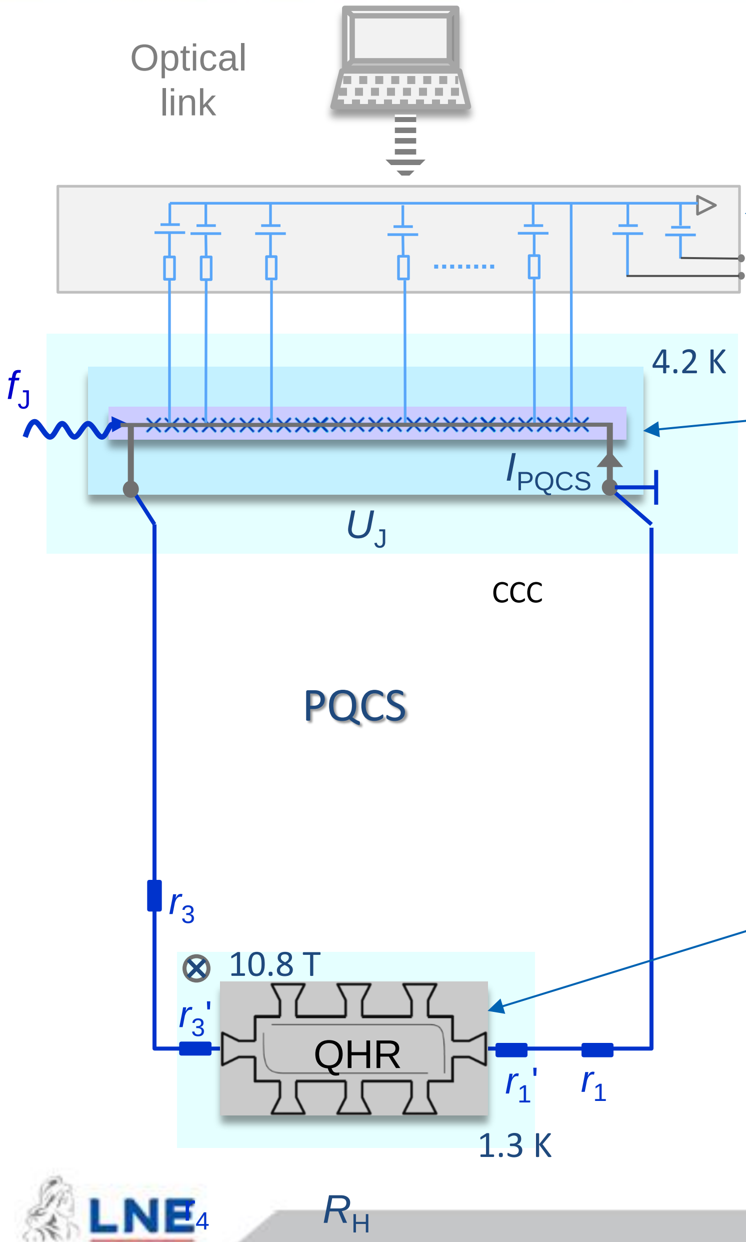


Programmable quantum current generator

W. Poirier et al, JAP 2014

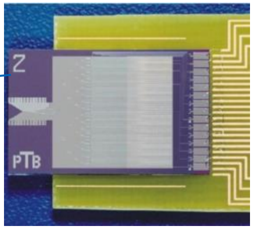


Implementation of the PQCG

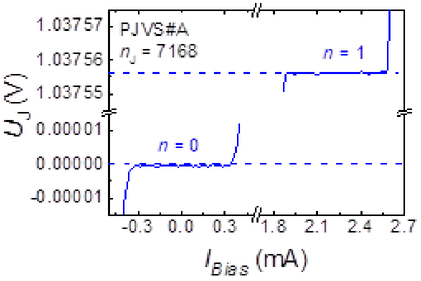


Programmable Josephson voltage standard (PJVS)

Home made
programmable
bias source
on batteries

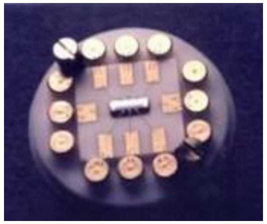


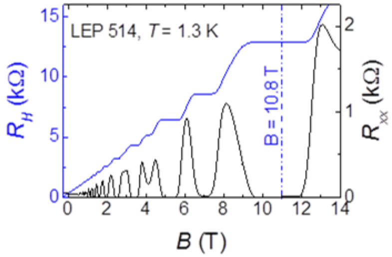
SINIS arrays
n_J = 7168 – 8192 jj,
f_J = 70 GHz,
14 segments,
I_c = 1.4 mA



$I_{PQCS} \leq$ 30 µA << Step amplitude

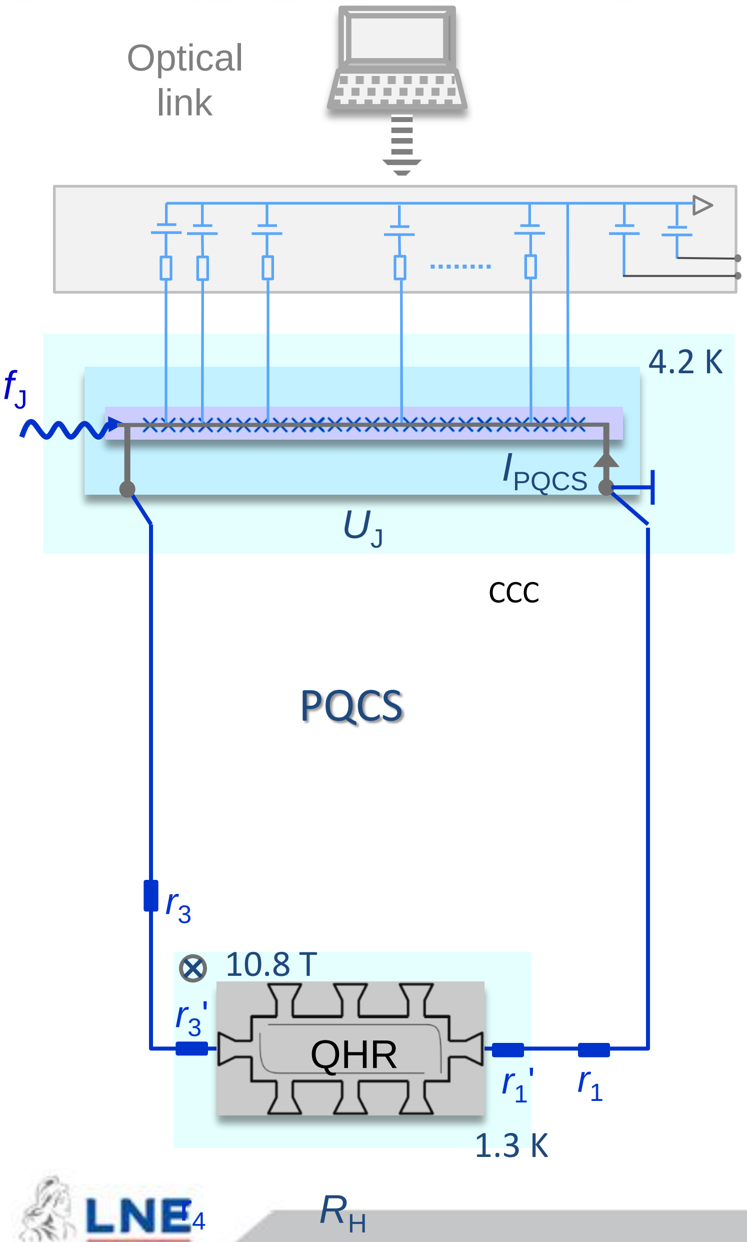
Quantum Hall resistance standard (QHRS)





GaAs/AlGaAs
$R_H = R_K/2$

Implementation of the PQCG



$$I_{PQCS} \approx n_J e f_J (1-\alpha)$$

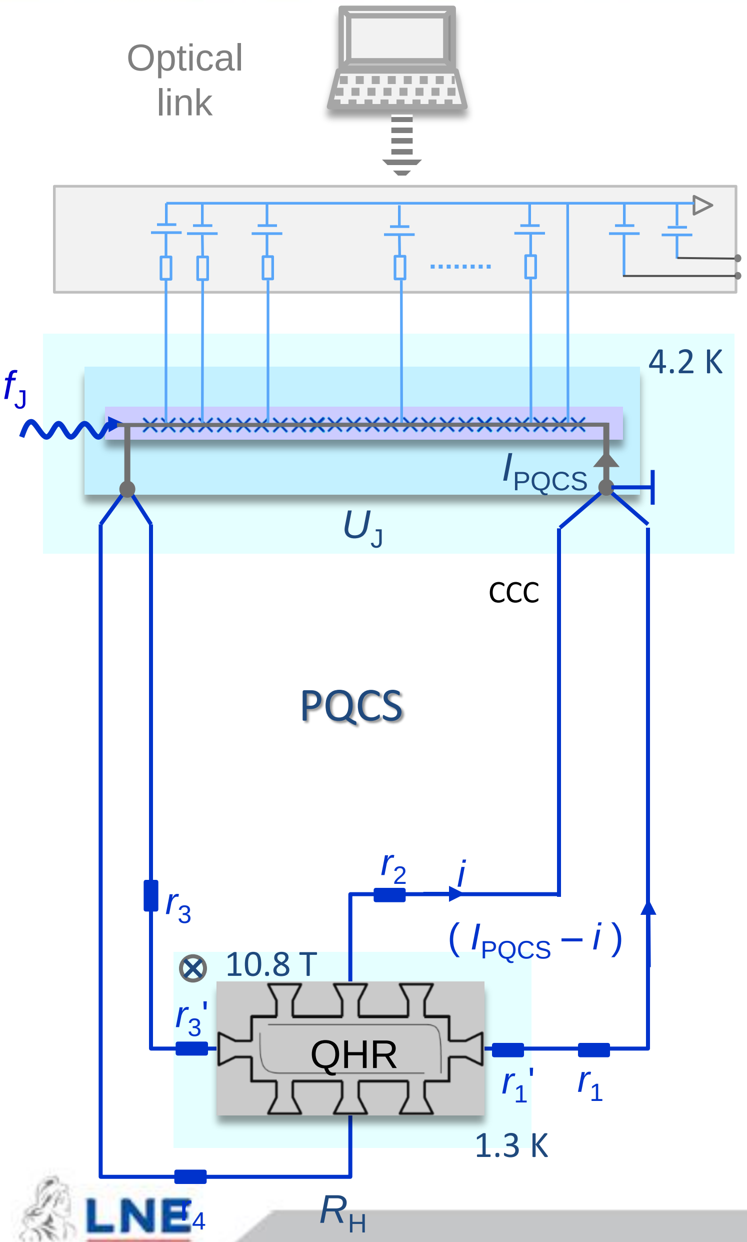
$$I_{PQCS} \approx \frac{U_J}{R_H}\left(1-\frac{r_1+r_1'+r_3+r_3'}{R_H}\right)$$

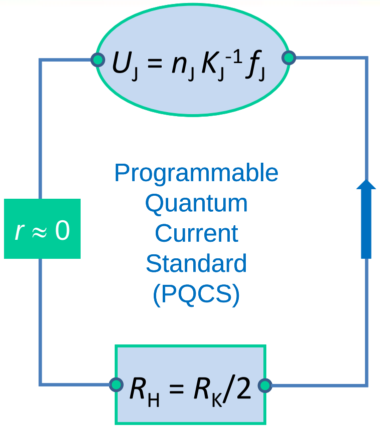
First order correction

$\alpha \sim 6.10^{-4}$

$u_\alpha = 4\times10^{-6}$

Multiple connection to the QHR

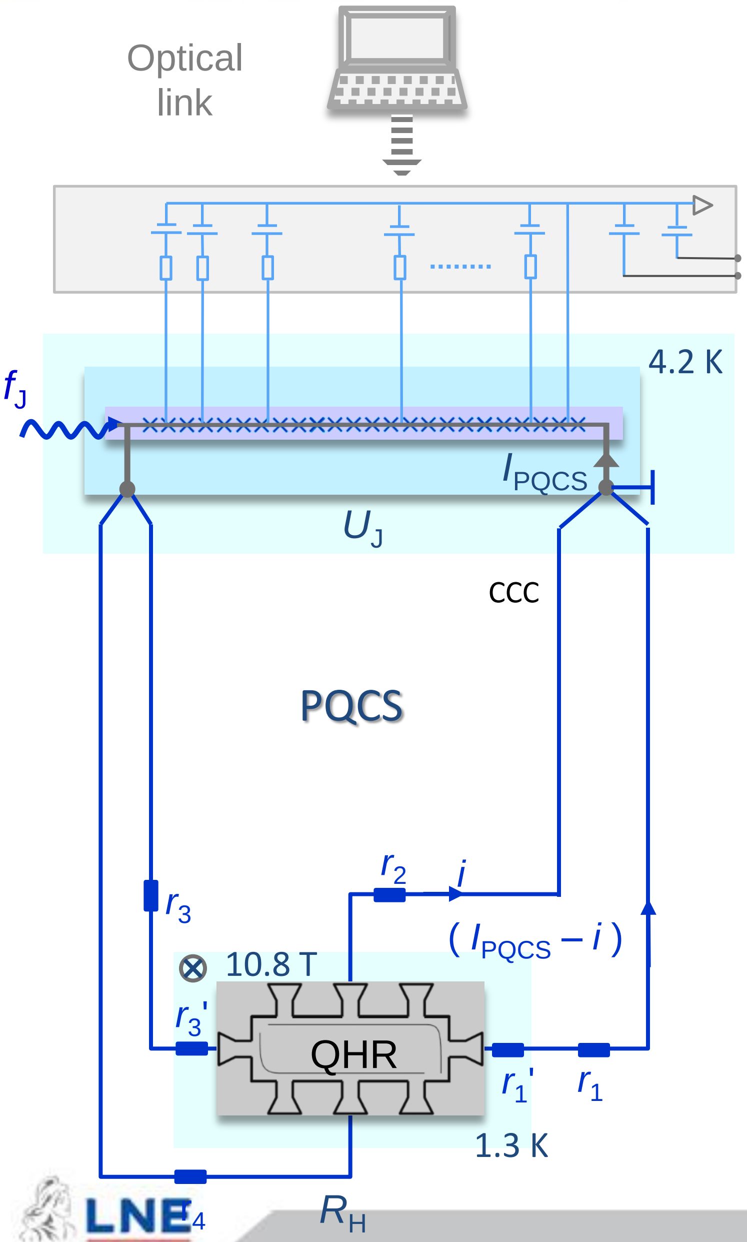




Double connection:

- 2 wires are connected to superconducting pads ensuring equipotentiality
- Connected to the same equipotential edge on the QHR

Multiple connection to the QHR



Chirality of edge-states for a given direction of magnetic field (Landauer-Buttiker)

⇒ I_{PQCS} mainly flows in the link $(r_1 + r_1')$

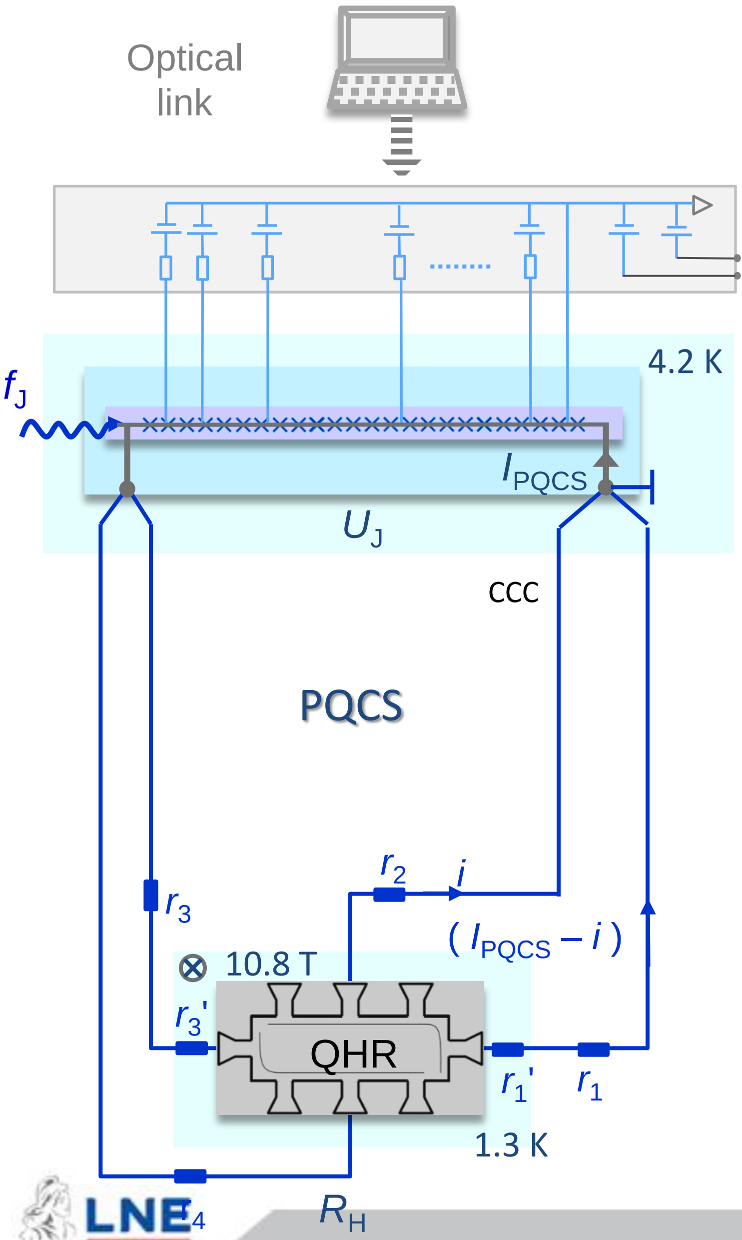
⇒ $V_{xx} = 0$ along an equipotential edge

⇒ Any two-terminal resistance between two terminals is R_H

$$i \approx \frac{(r_1 + r_1')}{R_H} I_{PQCS}$$

$$\Rightarrow \delta V_H \approx i \times r_2 = \frac{(r_1 + r_1')r_2}{R_H} I_{PQCS}$$

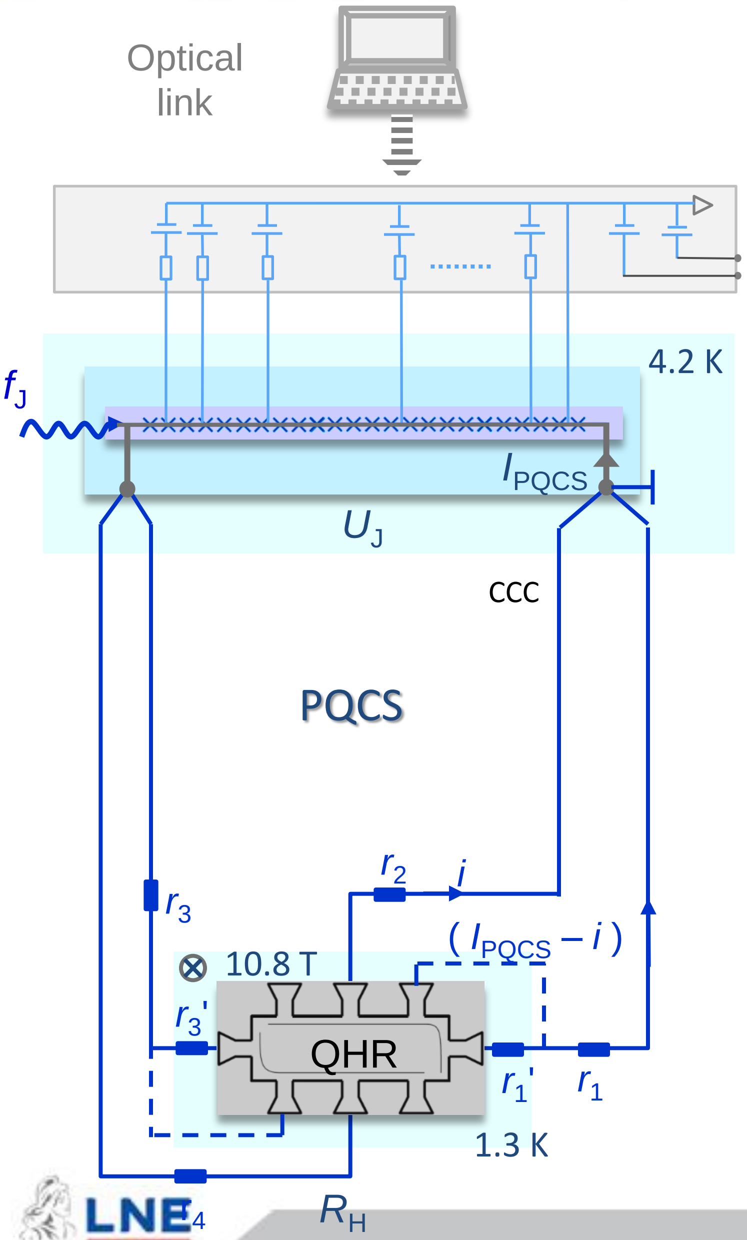
Multiple connection to the QHR



Double connection

$$I_{PQCS} \approx \frac{U_J}{R_H}\left(1 - \frac{(r_1 + r_1')r_2 + (r_3 + r_3')r_4}{R_H^2}\right)$$

Multiple connection to the QHR



Double connection

$$I_{PQCS} \approx \frac{U_J}{R_H}\left(1 - \frac{(r_1 + r_1')r_2 + (r_3 + r_3')r_4}{R_H^2}\right)$$

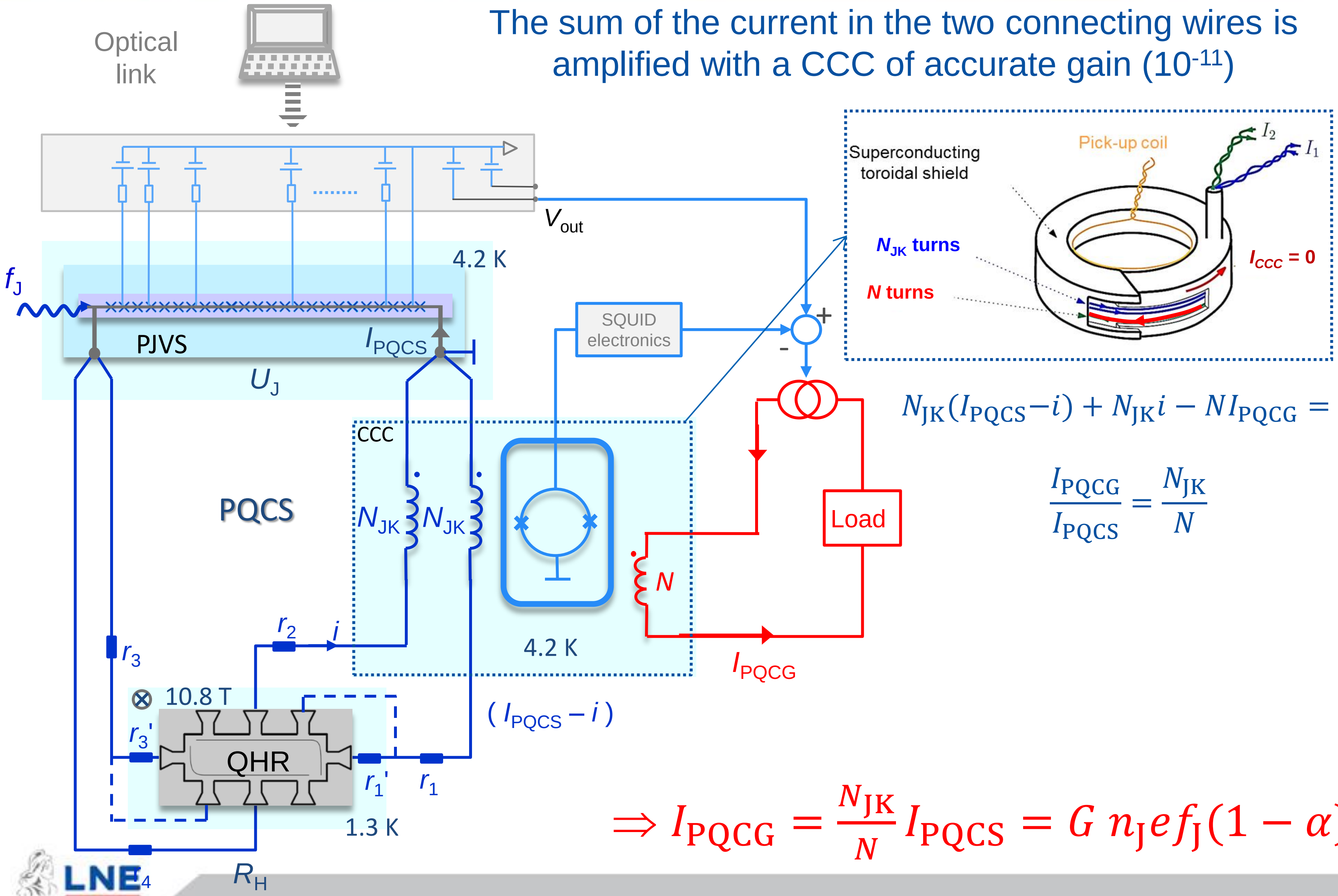
$$I_{PQCS} \approx n_J e f_J (1 - \alpha)$$

Second order correction $\alpha < 3 \times 10^{-7}$

$$u_\alpha = 2.5 \times 10^{-9}$$

Accurate amplification: Cryogenic current comparator

The sum of the current in the two connecting wires is amplified with a CCC of accurate gain (10^{-11})

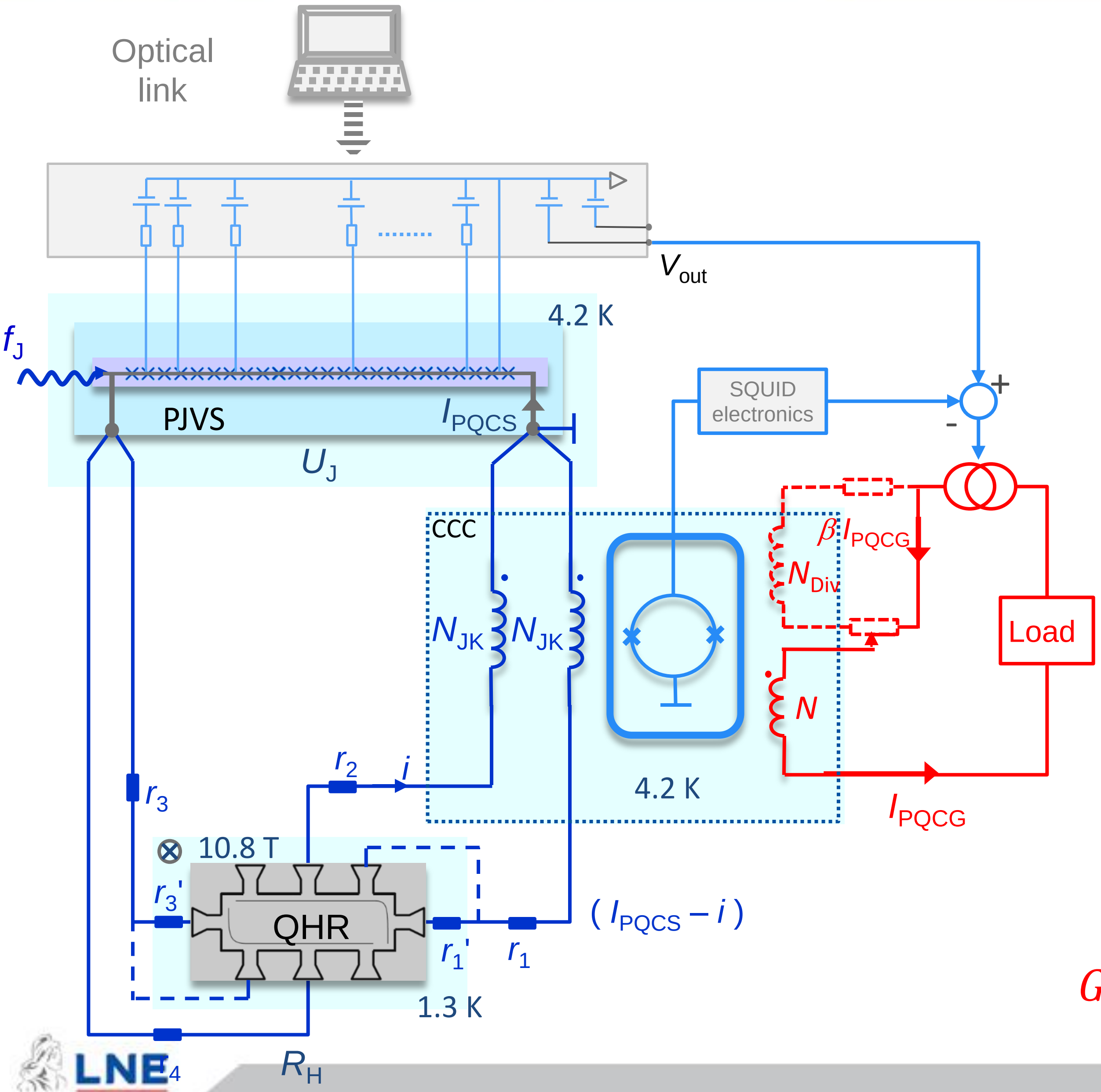


$$N_{JK}(I_{PQCS}-i)+N_{JK}i-NI_{PQCG}=0$$

$$\frac{I_{PQCG}}{I_{PQCS}}=\frac{N_{JK}}{N}$$

$$\Rightarrow I_{PQCG}=\frac{N_{JK}}{N}I_{PQCS}=G\,n_J e f_J(1-\alpha)$$

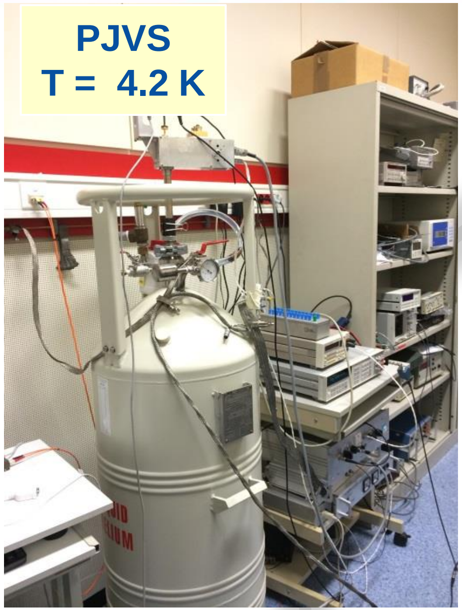
Accurate amplification: Cryogenic current comparator



Adding a current divider

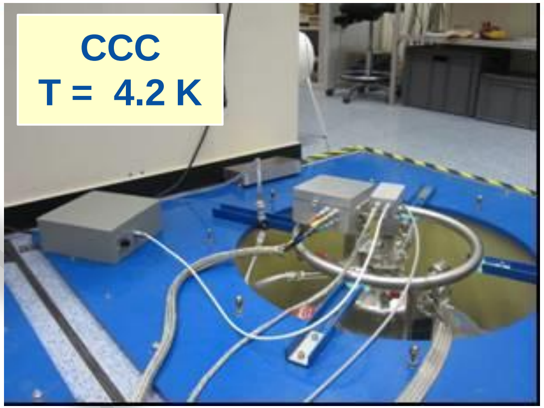
$$G \longrightarrow G_\beta = \frac{N_{JK}}{N + \beta N_{Div}}$$

Set-up



PJVS
T = 4.2 K

PQCS loop



CCC
T = 4.2 K

$\gamma_{\mathrm{CCC}} \sim 8\ \mu\mathrm{A.turn}/\phi_0$
$S_{\phi}^{\mathrm{SQUID}}: 10\ \mu\phi_0/\sqrt{\mathrm{Hz}}$

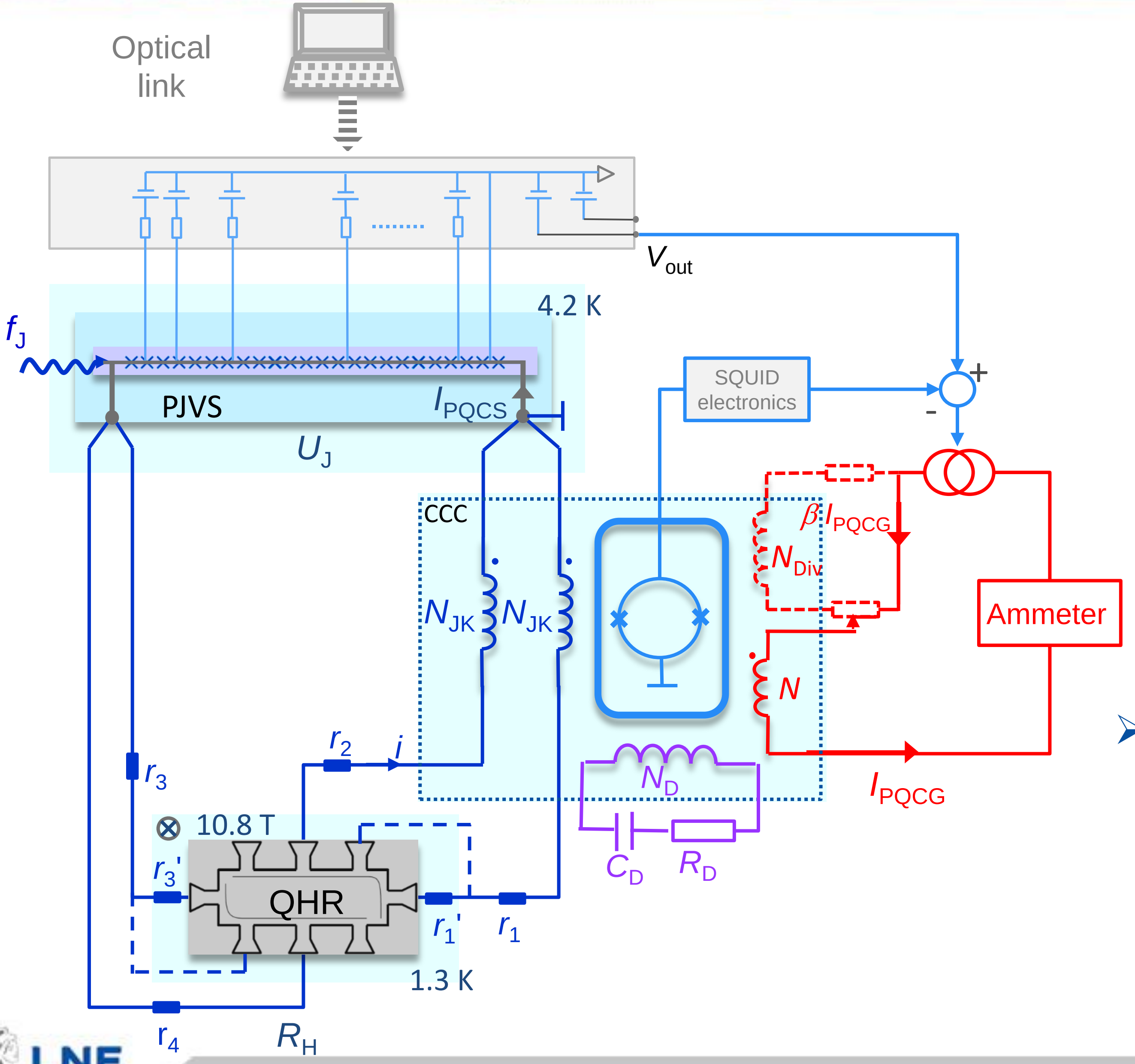
CCC (1,1,2,2,16,16,32,64,128, 160,160,1600,1600,2065,2065)

QHRS
T = 1.4 K
B = 10.8 T

⇒ Cable length ~ 25 m

⇒ Additionnal damping circuit

Damping circuit and stability



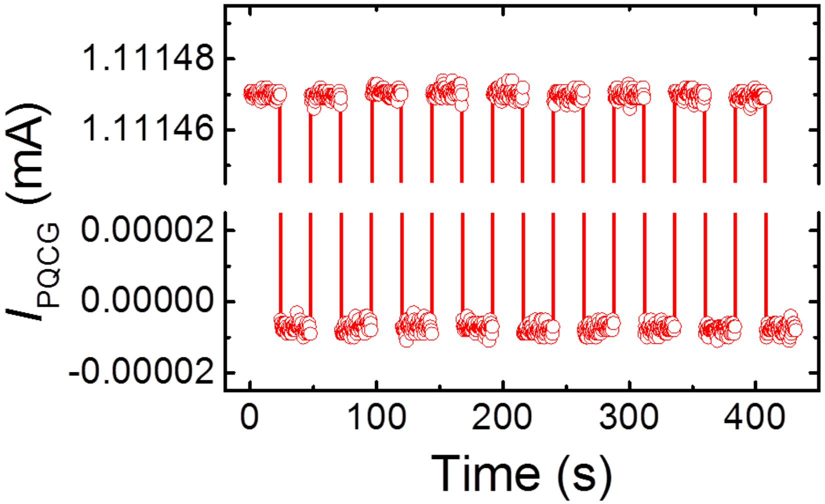
- Stability over a wide range of parameters

$N_{JK} = 129$

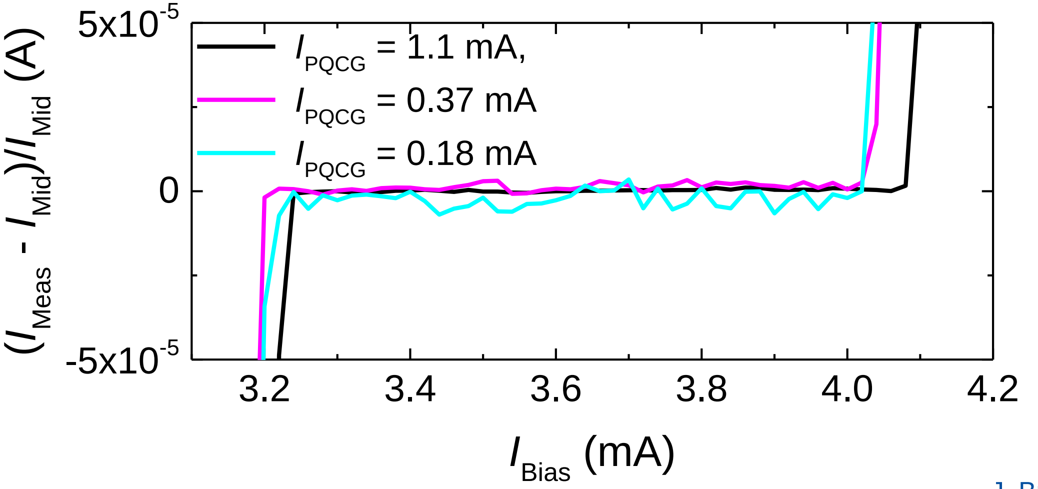
$N = 1$ to 4130

$0.03 < G < 129$

Output current of the PQCG

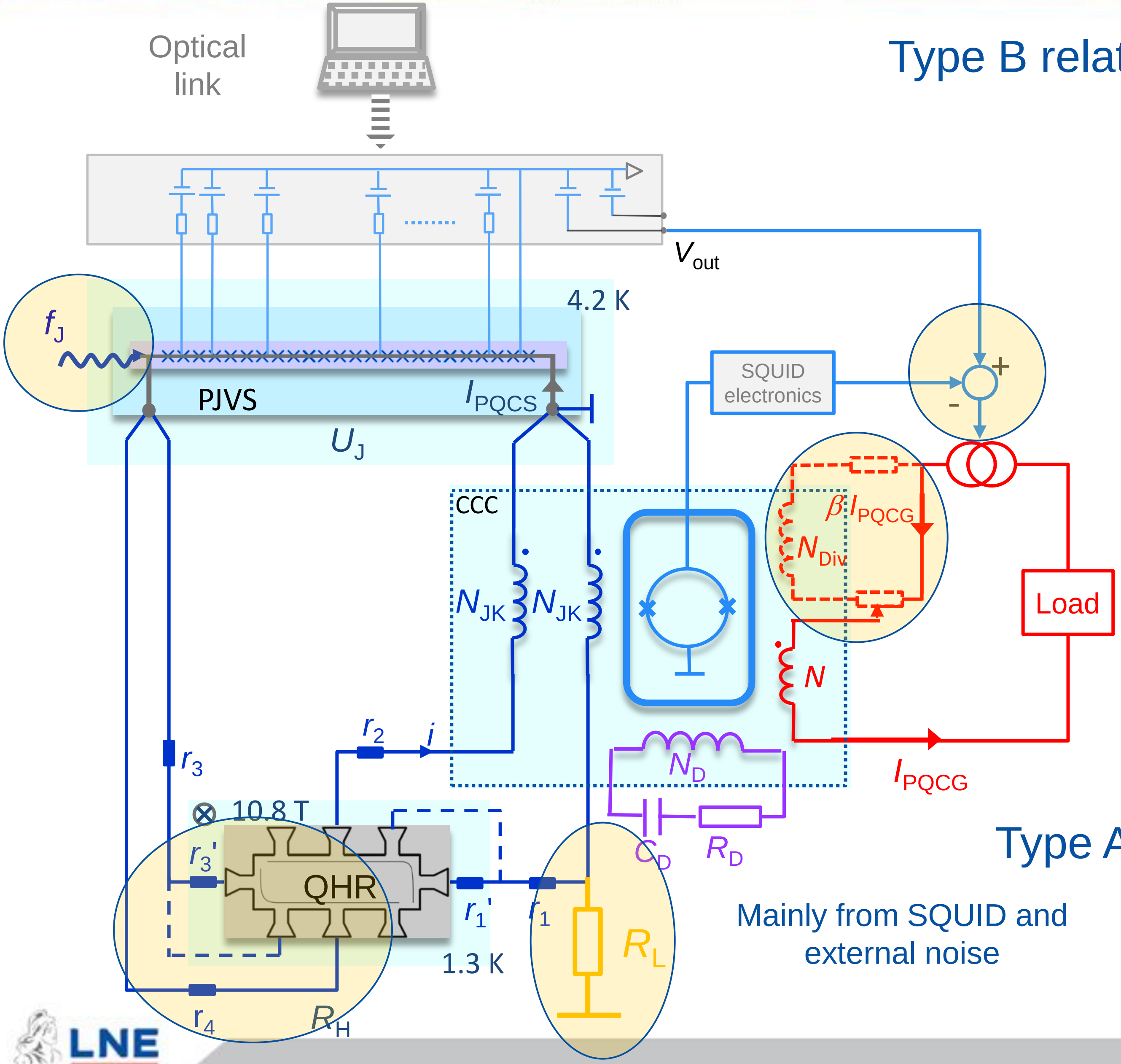


n_J=3073; N_{JK}=129;
N=4: I_{PQCS}=34 µA



J. Brun-Picard et al, accepted in PRX (2016)

Uncertainty budget



Mainly from SQUID and external noise

Type B relative Uncertainty < 10^{-8}

Double connection
$u_\alpha \sim 2.5\text{x}10^{-9}$

Current divider calibration
$u_\beta \sim 0.5\text{x}10^{-9}(N_{Div}/N)$
(from 0 to $8\text{x}10^{-9}$)

Feedback electronics
$u \sim 2.5\text{x}10^{-10}$

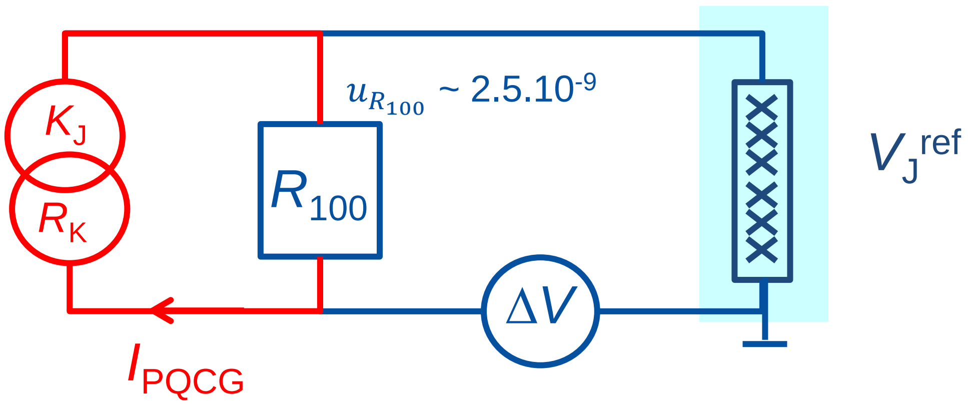
Frequency
$u_f \leq 10^{-10}$

Current Leakage
$u_L \sim (r_1+r_1')/R_L \sim 4.10^{-12}$

Type A relative Uncertainty

$S_I^{PQCG} < 4\text{x}10^{-8}/\sqrt{\text{Hz}}$

Accuracy measurements of I_{PQCG}



Adjusting the fraction β_0 of the current divider

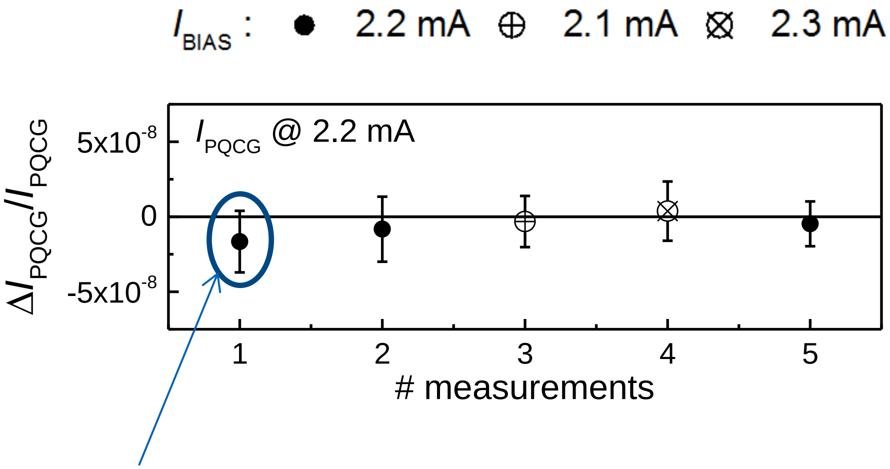
$$\Delta V = 0 \qquad \Rightarrow \qquad I_{\mathrm{PQCG}} = \frac{V_{\mathrm{J}}^{ref}}{R_{100}}$$

The measured value is compared to the calculated value $G_{\beta_0} n_{\mathrm{J}} e f_{\mathrm{J}} (1-\alpha)$

$$\boxed{\Delta I_{\mathrm{PQCG}} = I_{PQCG} - G_{\beta_0} n_{\mathrm{J}} e f_{\mathrm{J}} (1-\alpha)}$$

Accuracy results : Reproducibility

- 5 successive time series   (τ_{series} = 792 s)

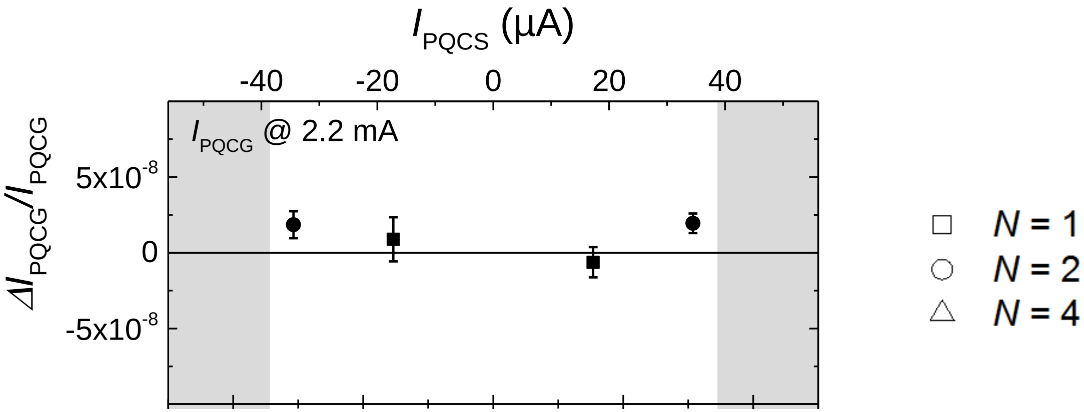


n_J = 3074
N_{JK} = 129
N = 2

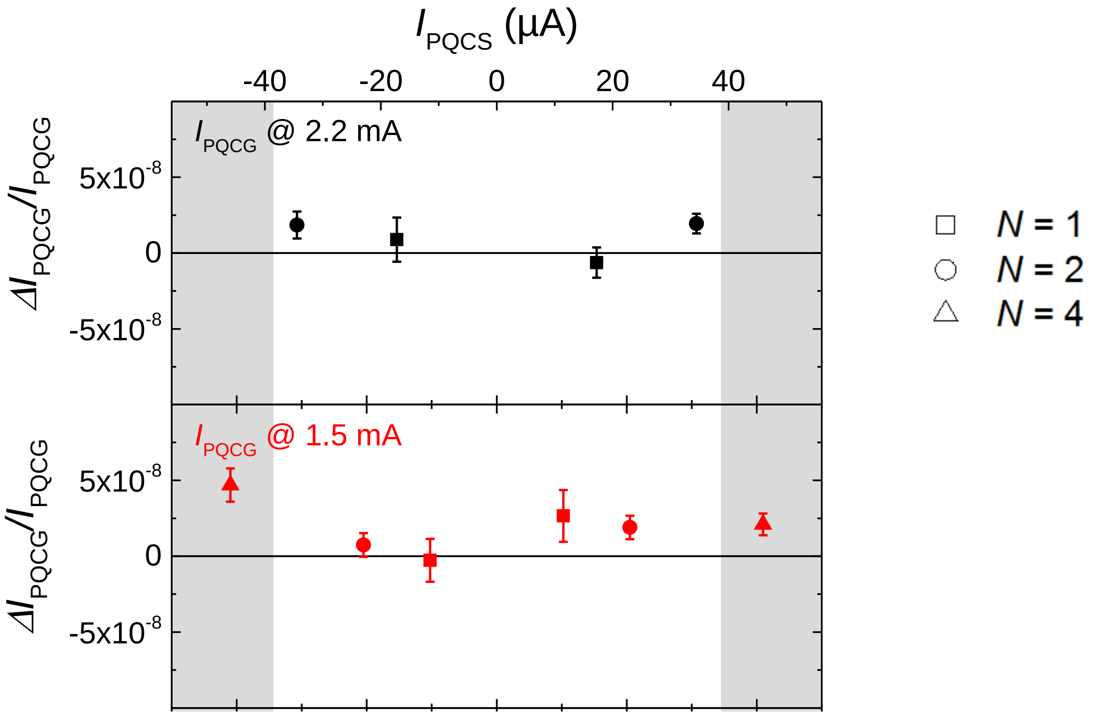
Error bars 1 s.d. dominated by noise ~ $2x10^{-8}$

- ➢ Accurate
- ➢ Reproducible
- ➢ Independent of the bias current of the Shapiro steps

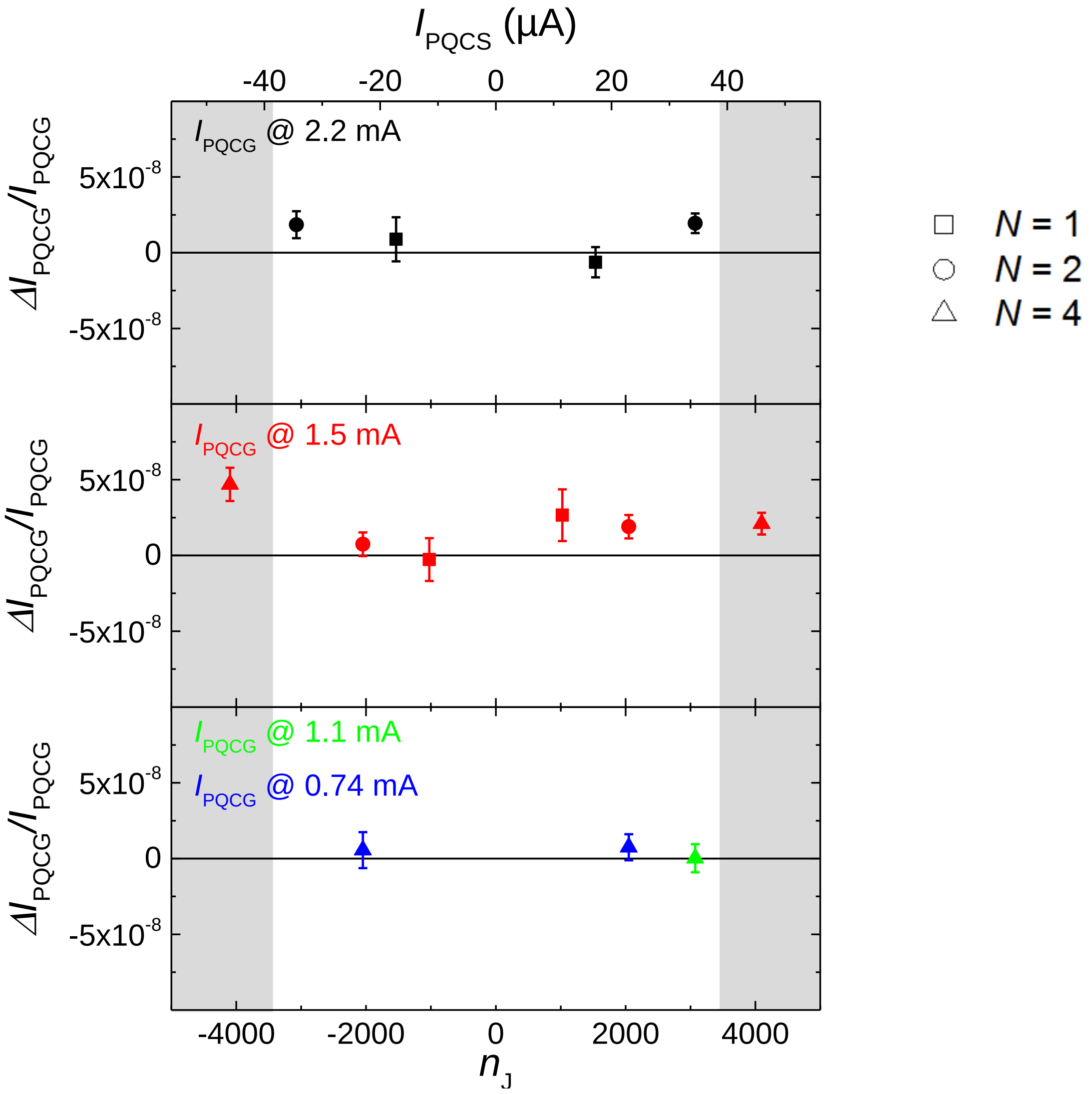
Accuracy of I_{PQCG} : Independence with I_{PQCS} and N



Accuracy of I_{PQCG} : Independence with I_{PQCS} and N

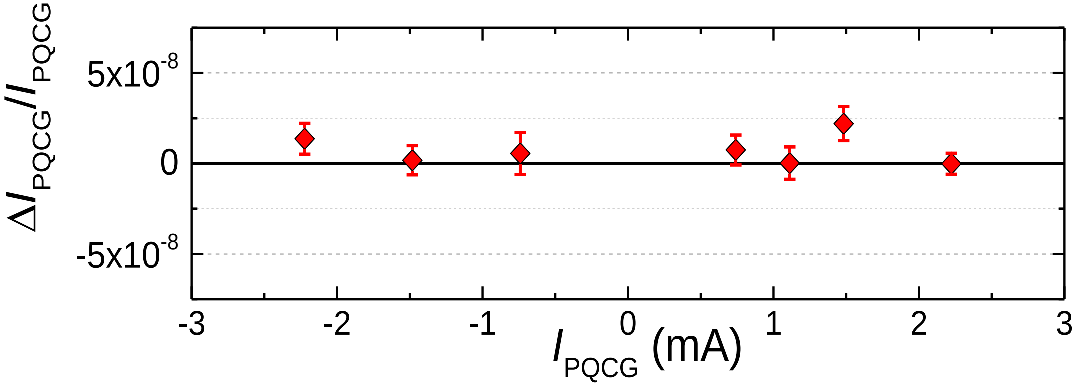


Accuracy of I_{PQCG} : Independence with I_{PQCS} and N

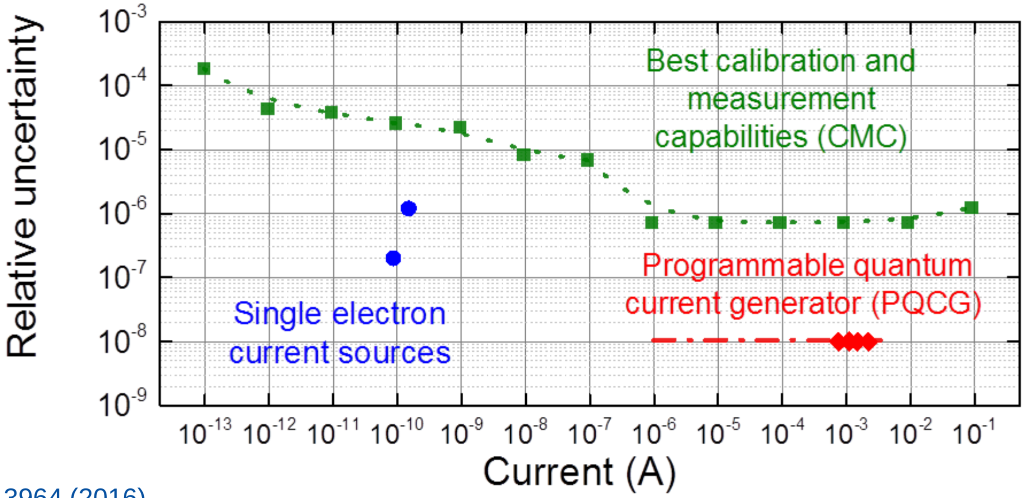


10^{-8}-accurate quantized current in mA range

=> Averaging of data for $n_J \leq 3074$



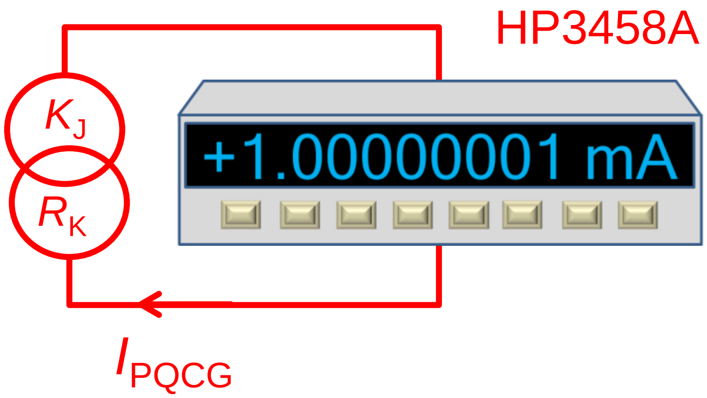
- No significant deviation within a measurement uncertainty of 10^{-8}
- Weithed mean : (6 ± 6) x 10^{-9}



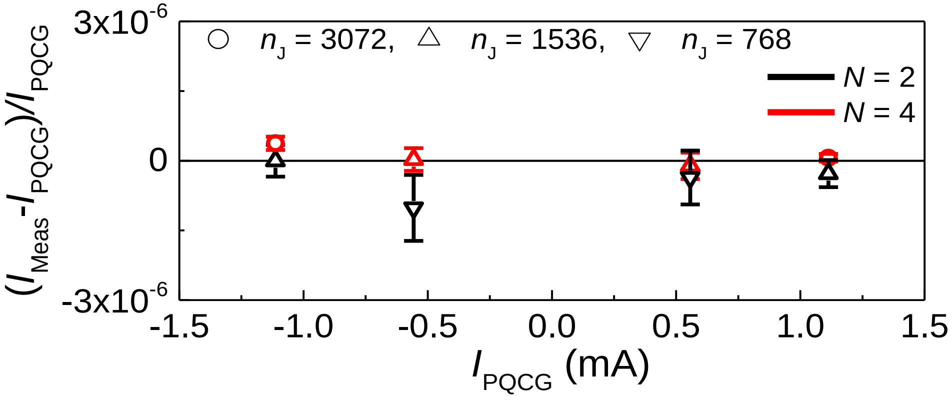
From the uncertainty budget, 10^{-8} -accurate down to µA range

J. Brun-Picard et al, arXiv:1606,3964 (2016)

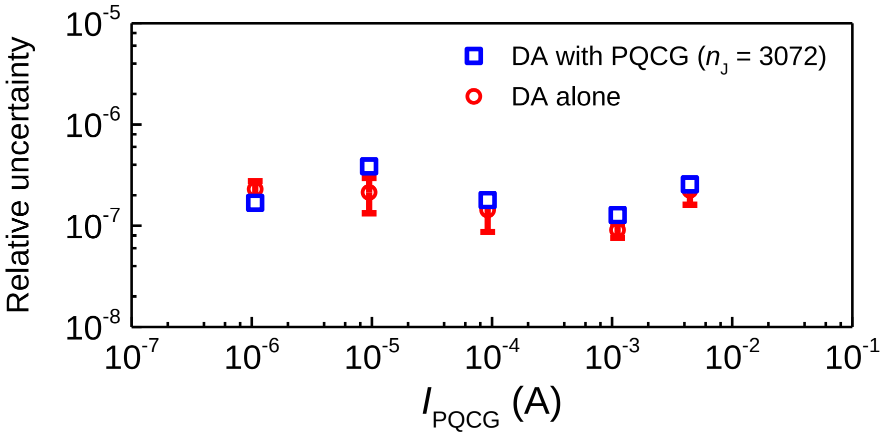
Digital ammeter calibration



Calibration in mA range



Calibration uncertainties over 4 decades



- 1µA - 10 mA: 10^{-7}uncertainty: limited by the DA

Conclusion and perspectives

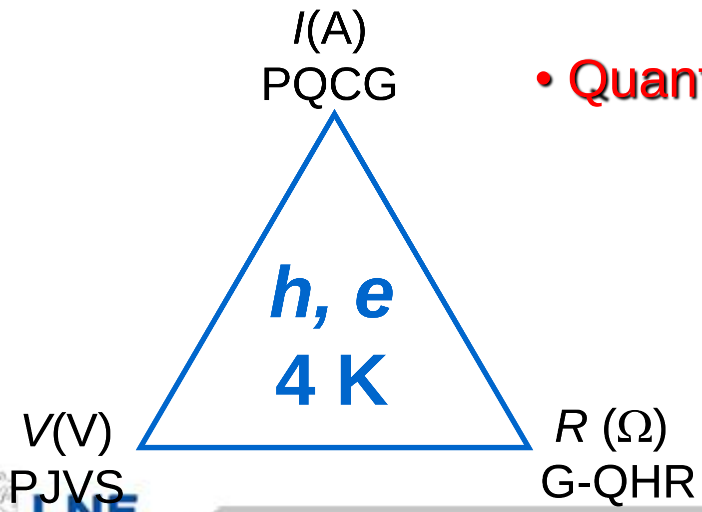
- The PQCG realizes a quantum « mise en pratique » of the ampere from *e* within a 10^{-8} uncertainty (with improvements (damping at low T, triple connection) => 10^{-9} uncertainty)

- The PQCG can calibrate Digital Ammeters

- A strong support to a new SI based on solid-state quantum phenomena and the requirement for adopting the relationships $R_K=h/e^2$ and $K_J=2e/h$

- Pulse-driven JVS + AC QHE and AC transformer = AC PQCG

- The PQCS as a reference for other applications: quantum ammeter (**metrological triangle**), quantum resistance comparisons



I(A)
PQCG

h, e
4 K

V(V)
PJVS

R (Ω)
G-QHR

- Quantum Generator / Multimeter

Support and funding

- Métrologie française

- JRP e-SI-Amp

The EMPIR initiative is co-funded by the European Union's Horizon 2020 research and innovation programme and the EMPIR Participating States