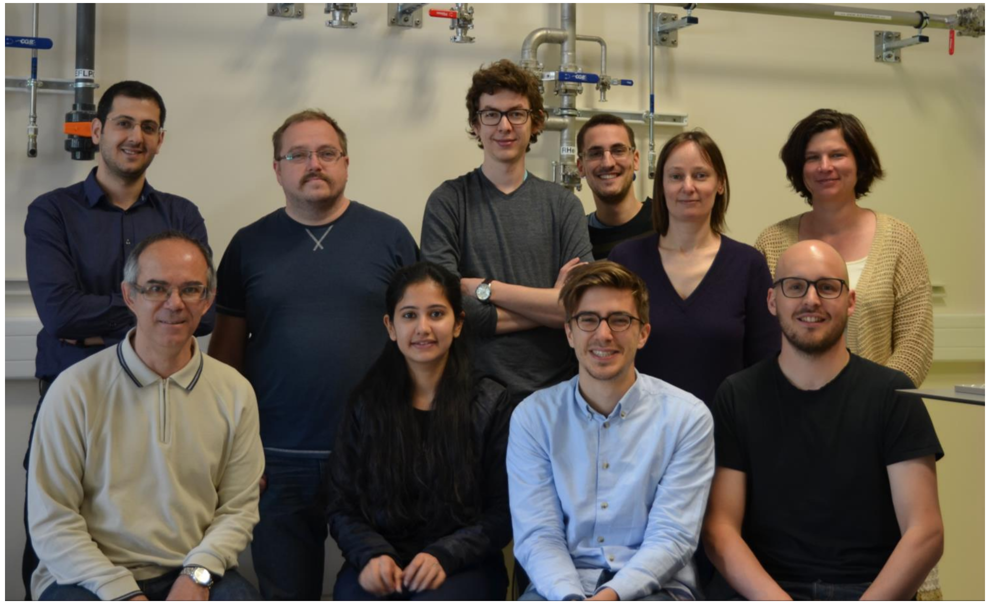
Weak and strong non-linear effects in Josephson junction chains

Wiebke Guichard
University Grenoble Alpes-Néel Institute
Grenoble

Superconducting quantum circuits team



Permanents: Olivier Buisson, Cécile Naud, Wiebke Guichard, Nicolas Roch
Non-permanents: Rémy Dasonneville, Javier Puertas-Martinez
Yuriy Krupko, Luca Planat, Farshad Foroughi
Former Students: Etienne Dumur, Thomas Weissl

Collaboration with theoreticians from LPMMC Grenoble

Denis Basko

Frank Hekking

Van Duy Nguyen

Gianluca Rastelli
(University of Konstanz)

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UNIVERSITÉ Grenoble Alpes
VILLE DE GRENOBLE
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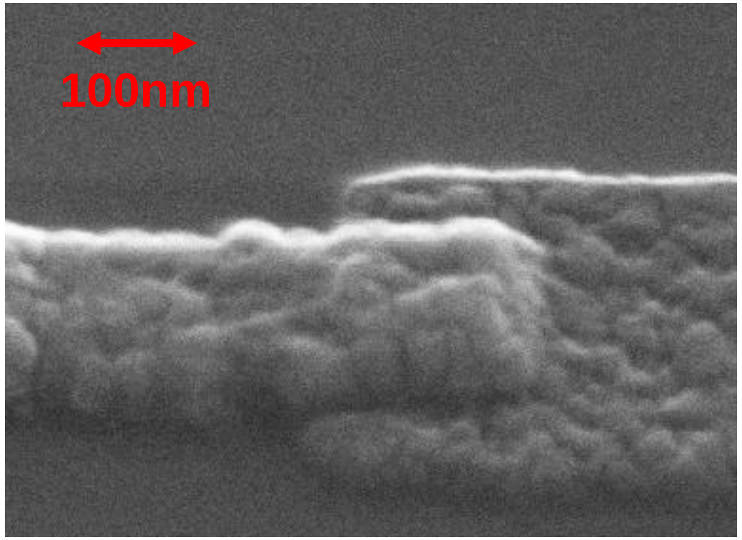
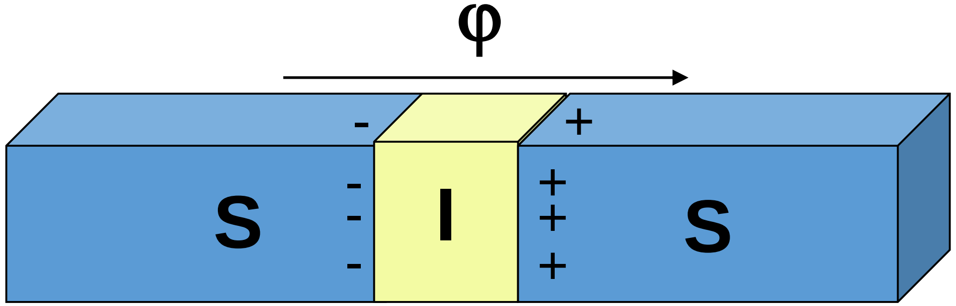
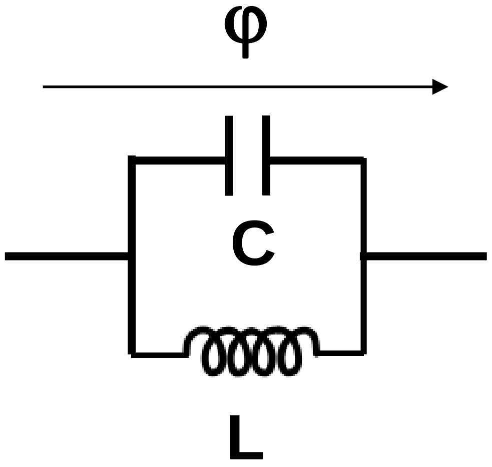
Artificial atoms with superconducting Josephson junction circuits



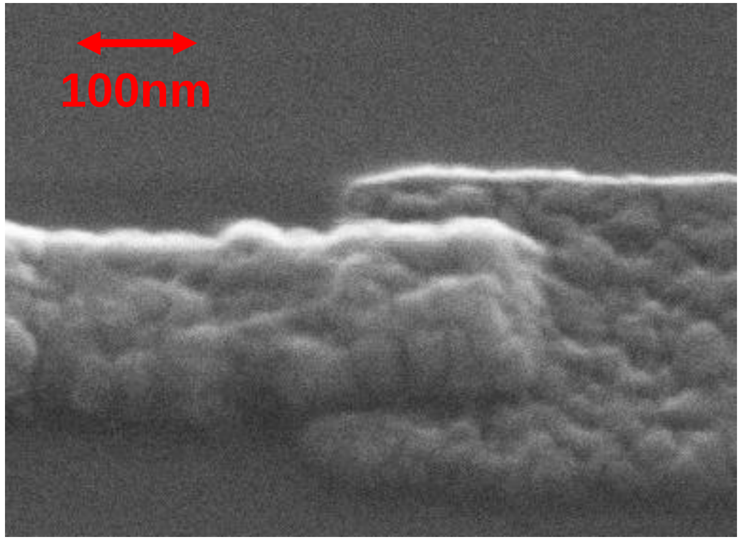


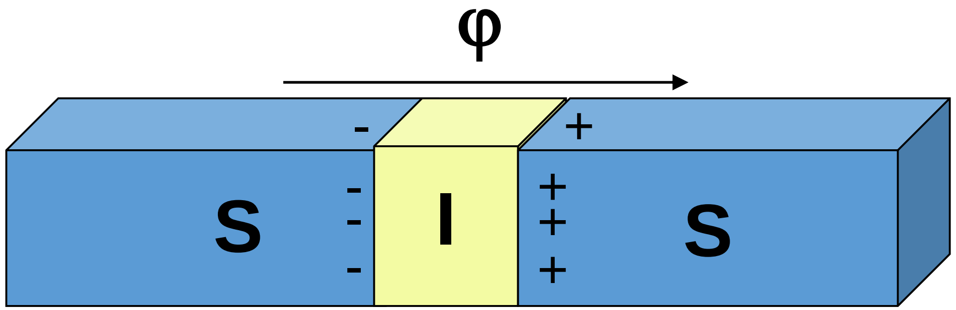


$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos\hat{\varphi}$$

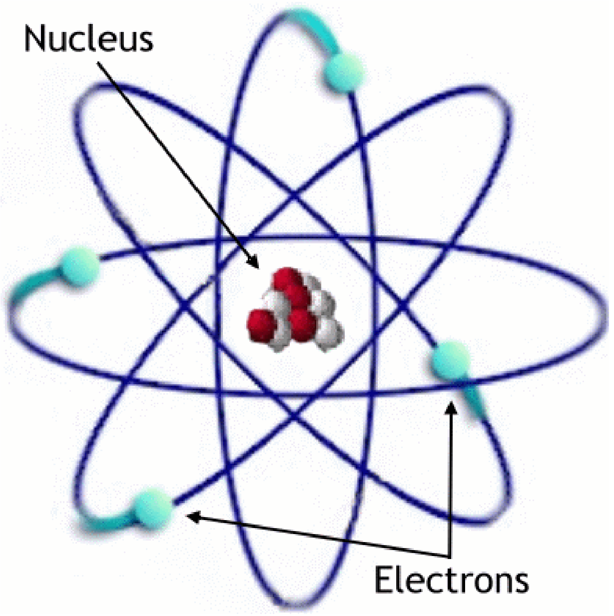
$$L(\varphi) = \frac{\hbar}{2eI_c}\frac{1}{\cos(\varphi)} = \frac{\hbar}{2eI_c}\frac{1}{1 - \frac{(I/I_c)^2}{2} + \ldots}$$

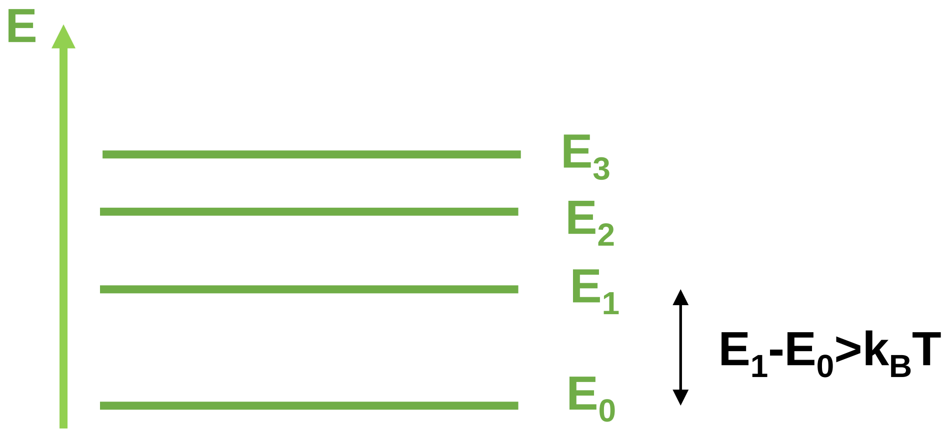
Artificial atoms with superconducting Josephson junction circuits





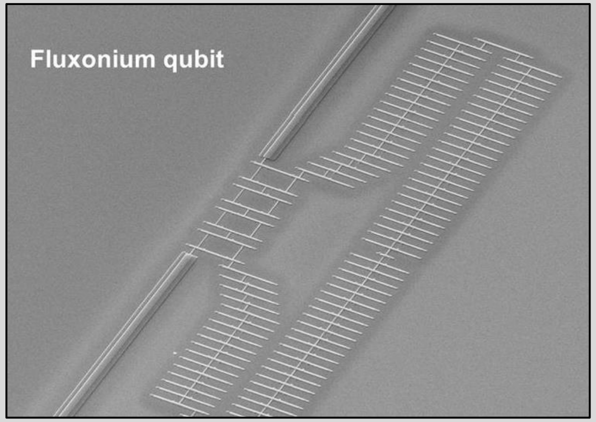
$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos\hat{\varphi}$$



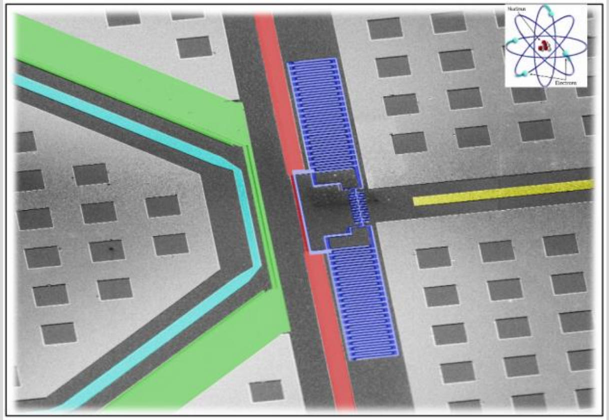


Recent experimental studies implying Josephson junction chains

Linear inductances in qubit-circuits

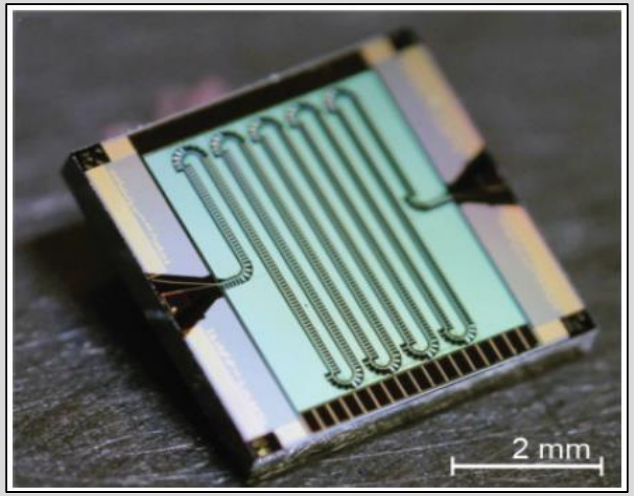


Fluxonium qubit
I. Pop et al, Nature, Vol 508,369 (2014)

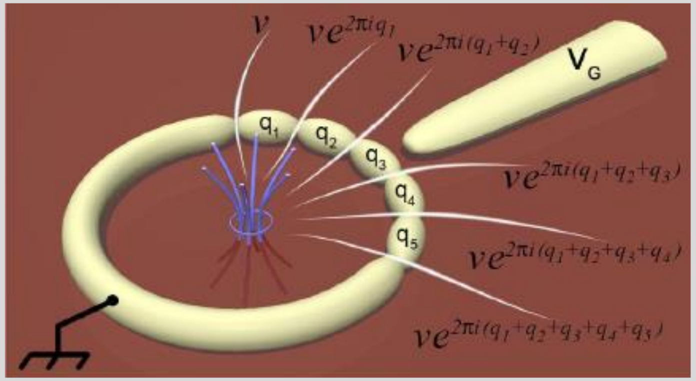


Artifical atom: two inductively coupled transmons
Étienne Dumur et al, Phys. Rev. B 92, 020515 (2015)

Non-linear effects



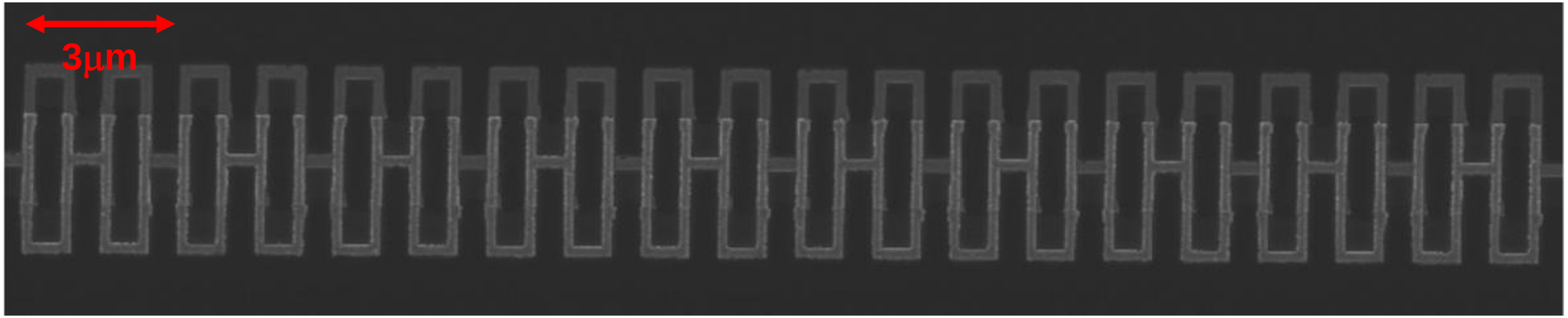
JJ-chain traveling-wave parametric amplifier
C. Macklin et al, Science, Vol 350, 307 (2015)



Quantum phase-slips in JJ chains
I. Pop et al, Nature Physics, Vol 6, 591, (2010)

Josephson junction chain: a versatile element for quantum circuits

Activities of the superconducting quantum circuit team at the Néel Institute



$$L = \frac{\hbar}{2eI_c}\frac{1}{\cos(\varphi)} = \frac{\hbar}{2eI_c}\frac{1}{1-\frac{(I/I_c)^2}{2}+\ldots}$$

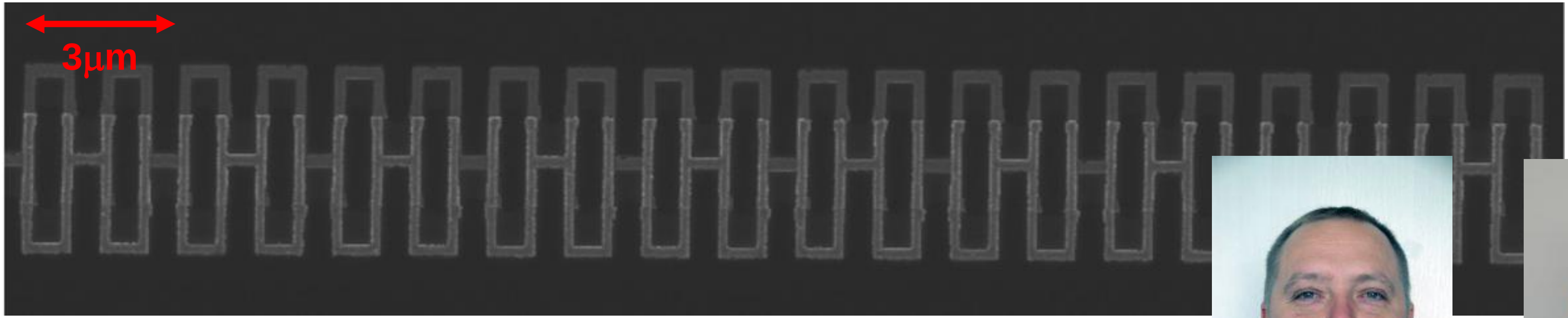
Large inductance
- -Fluxonium Qubit
- -V-shape artifical atom (Transmon) Novel Quantum measurements

Metamaterials
Bath of photonic modes
- -Dispersion relation
- -Spin-Boson model

Non-linear effects
- -Kerr effects between photonic modes
- -Amplification
- -Study of quantum phase-slips

Josephson junction chain: a versatile element for quantum circuits

Activities of the superconducting quantum circuit team at the Néel Institute



Yuriy Krupko
(Postdoc)

Farshad Foroughi
(Postdoc)

In this talk !

$$L = \frac{\hbar}{2eI_c}\frac{1}{\cos(\varphi)} = \frac{\hbar}{2eI_c}\frac{1}{1 - \frac{(I/I_c)^2}{2} + \ldots}$$

Large inductance
- **Fluxonium Qubit**

- **V-shape artifical atom (Transmon)**
  Novel Quantum measurements

Metamaterials
Bath of photonic modes
- **Dispersion relation**

- **Spin-Boson model**

Non-linear effects
- **Kerr effects between photonic modes**

- **Amplification**

- **Study of quantum phase-slips**

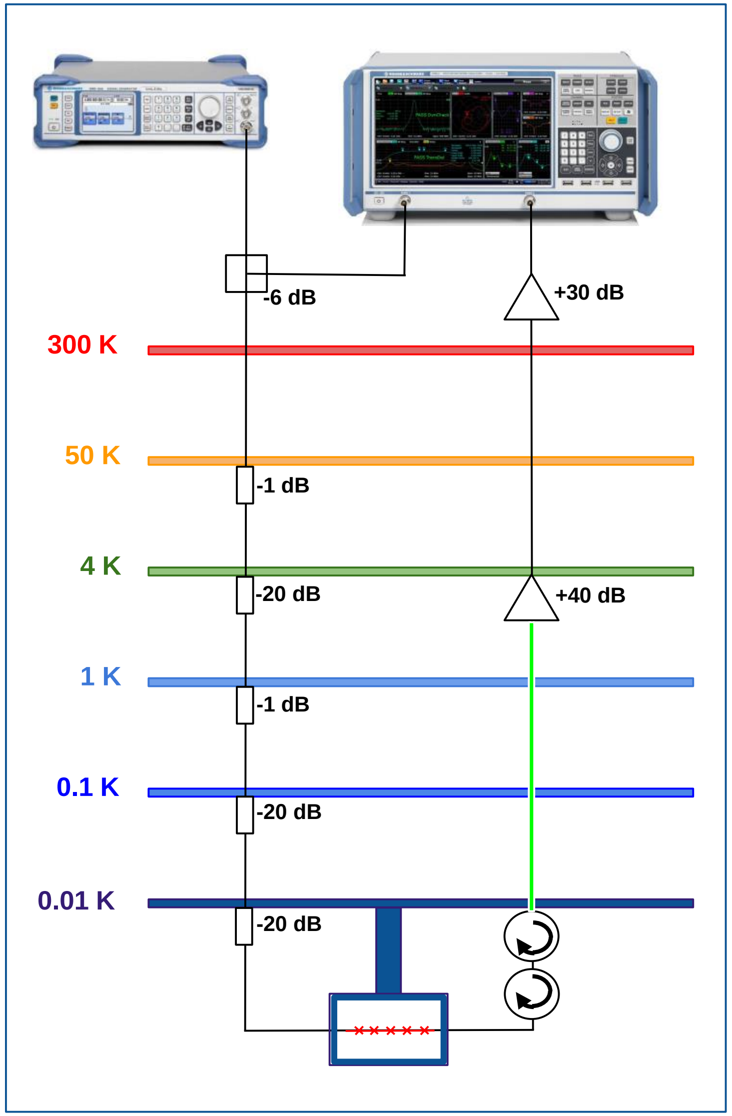
Outline

1) Linear effects: Dispersion of propagation modes in a Josephson junction chain

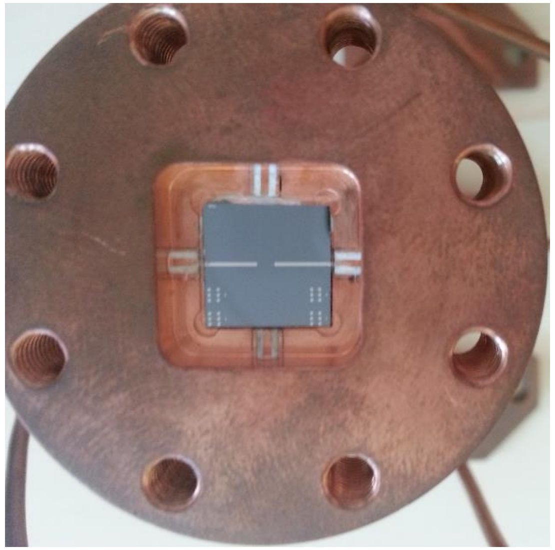
2) Weak non-linear effects: Self- and Cross Kerr effects in a Josephson junction chain

3) Strong non-linear effects: Quantum phase-slips

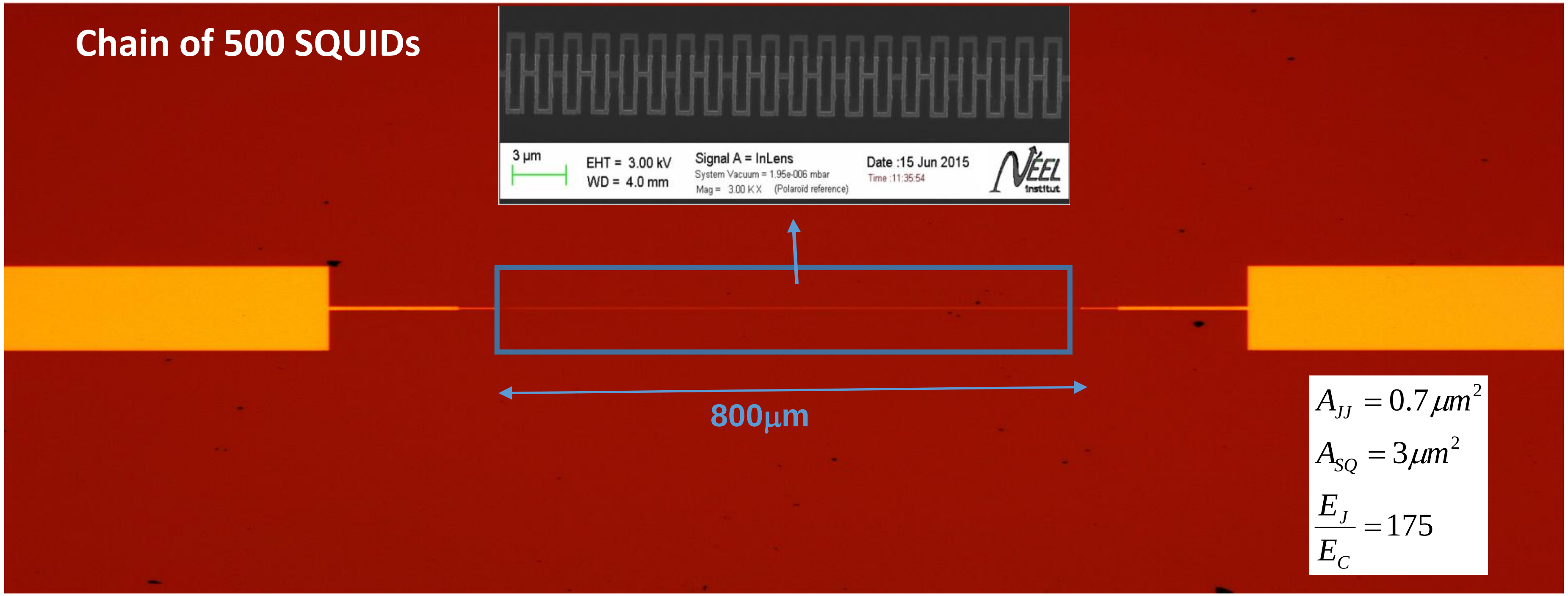
Experimental set-up: Transmission microwave measurements



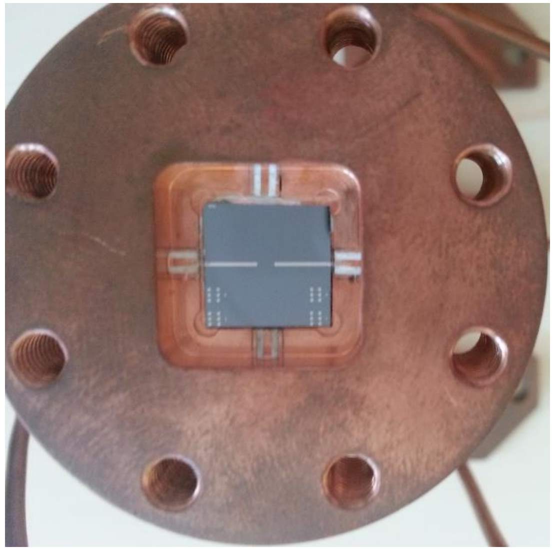
Dispersion of propagation modes in a Josephson junction chain



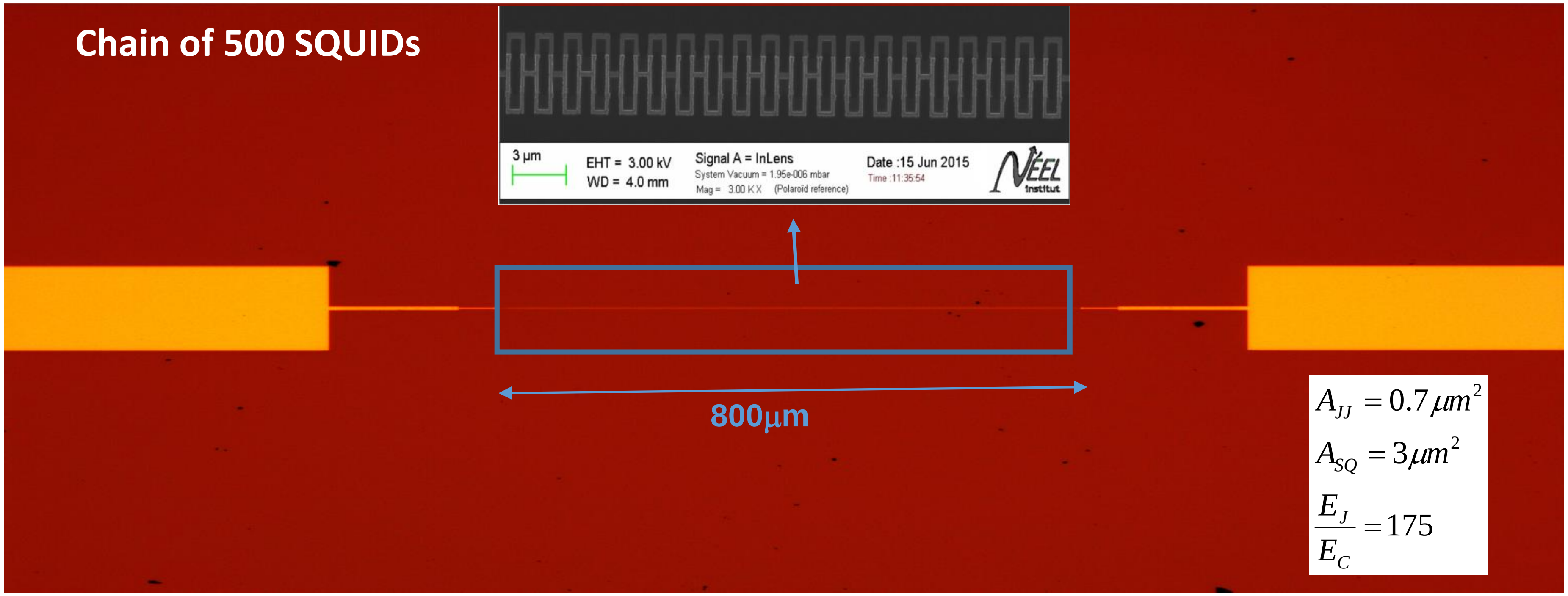
Sample holder

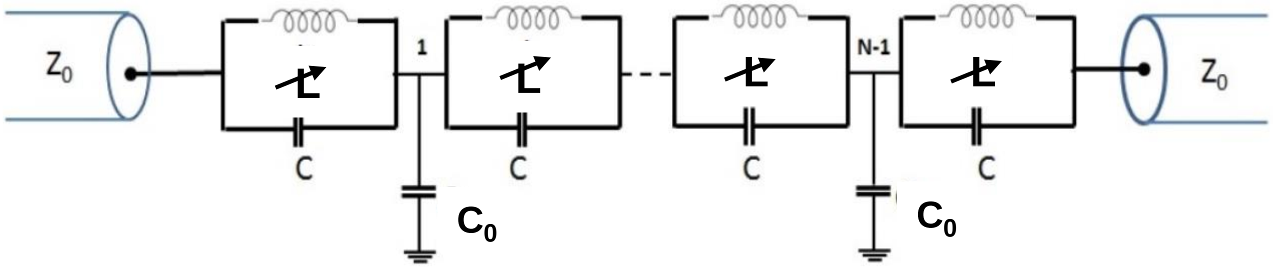


Dispersion of propagation modes in a Josephson junction chain

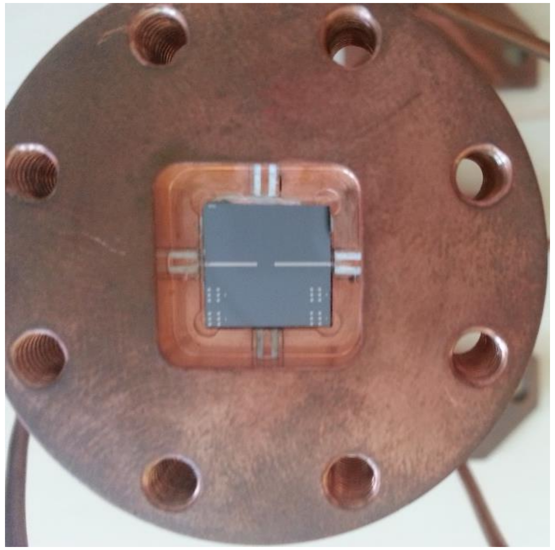


Sample holder

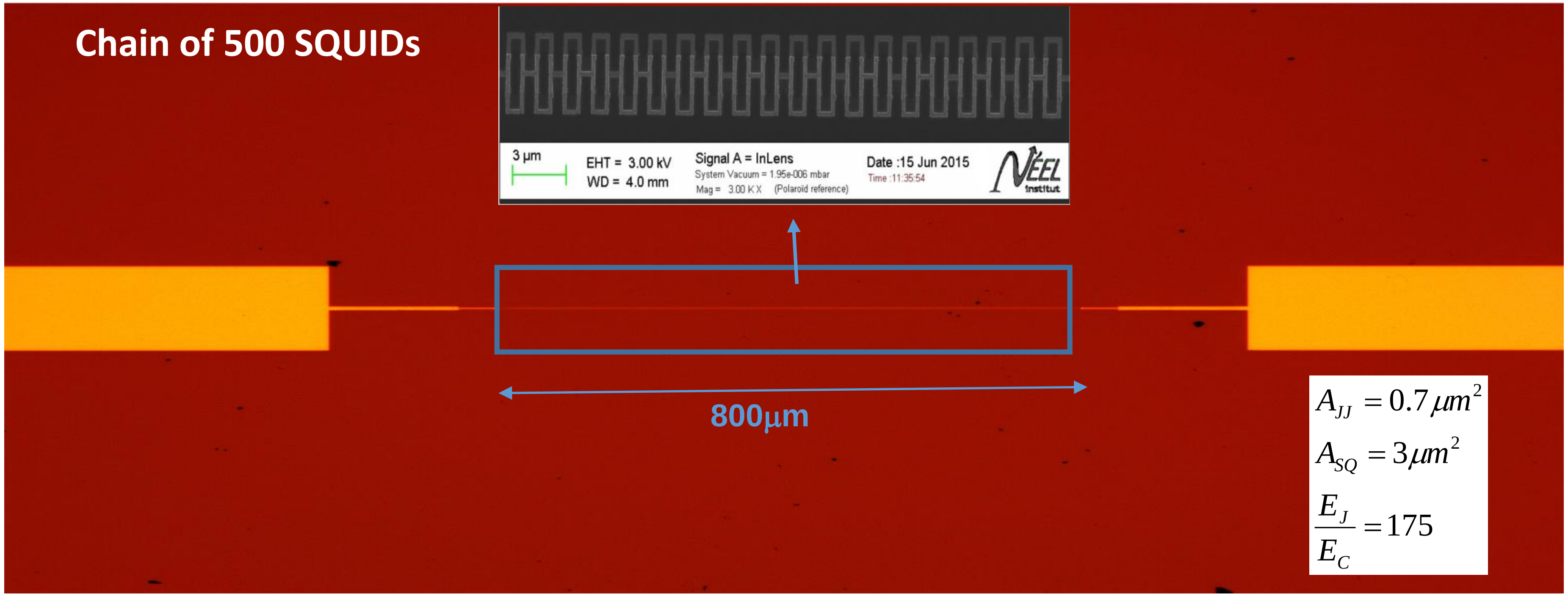


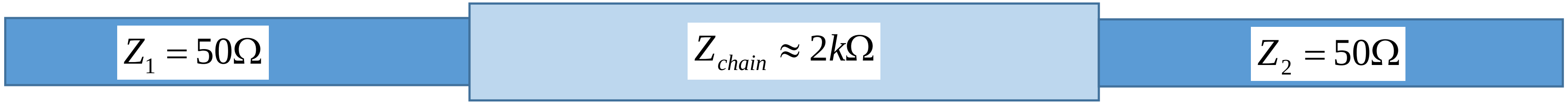


Dispersion of propagation modes in a Josephson junction chain



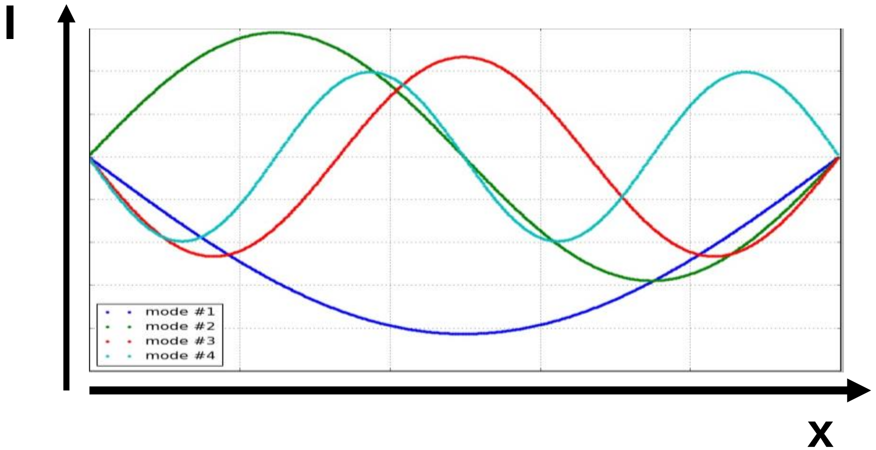
Sample holder



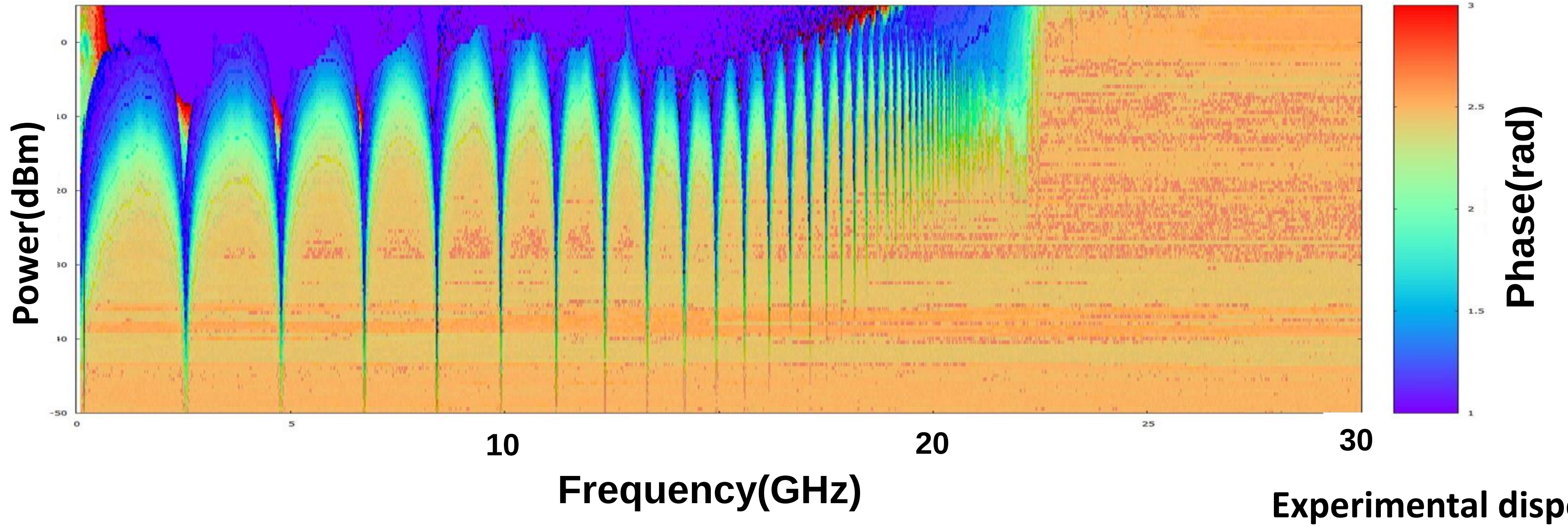


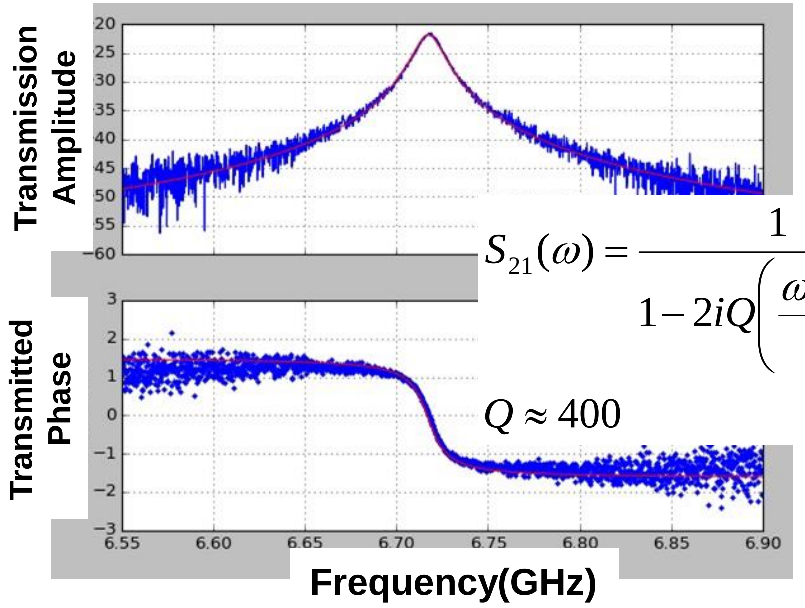
Fabry-Pérot Cavity

Transmission through the cavity for frequencies of the stationary eigenmodes



Dispersion of propagation modes in a Josephson junction chain

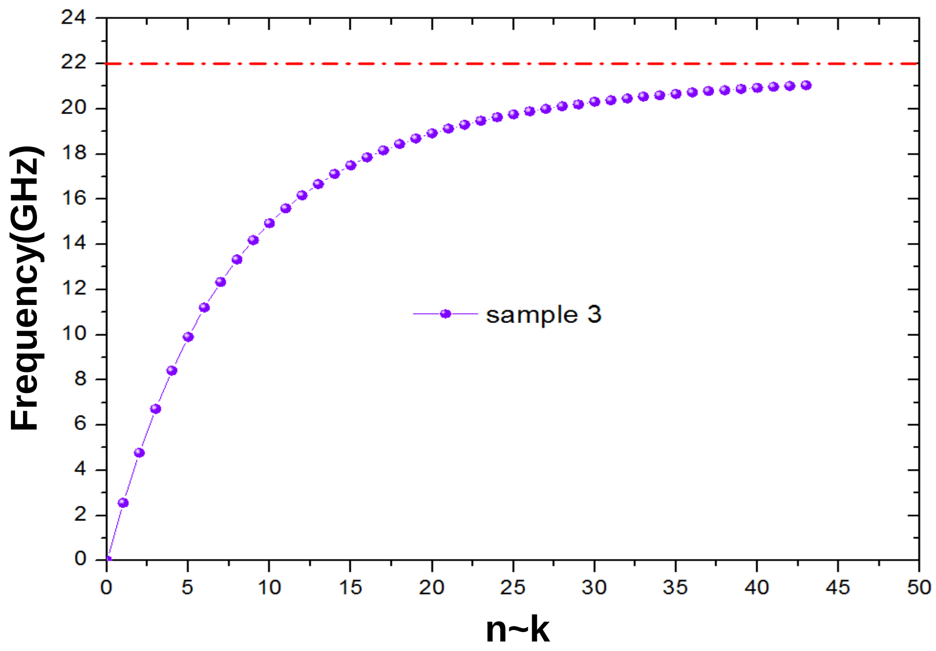




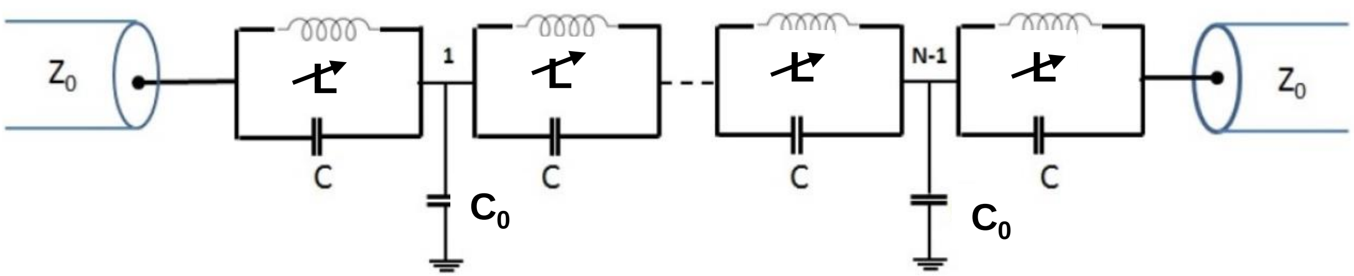
$$S_{21}(\omega) = \frac{1}{1 - 2iQ\left(\frac{\omega - \omega_r}{\omega_r}\right)}$$

$$Q \approx 400$$

Experimental dispersion curve $\omega(k)$



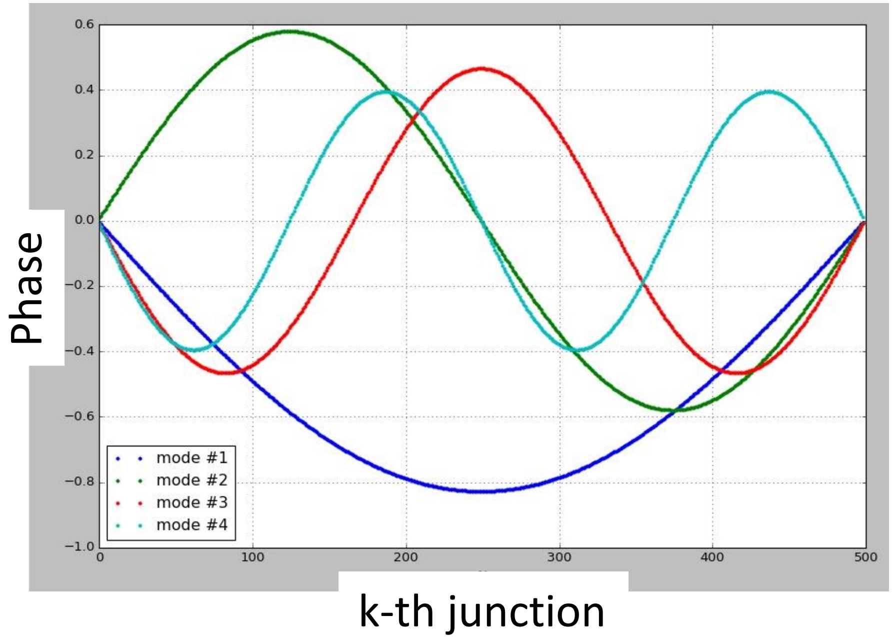
Standard model for propagation modes in a Josephson junction chain



$$H = \sum_{k=1}^{N} \hbar \omega_k (\hat{a}_k^+ \hat{a}_k + \frac{1}{2})$$

$$\hat{C}^{-1/2} \hat{L}^{-1} \hat{C}^{-1/2} \vec{\psi}_k = \omega_k^2 \vec{\psi}_k$$

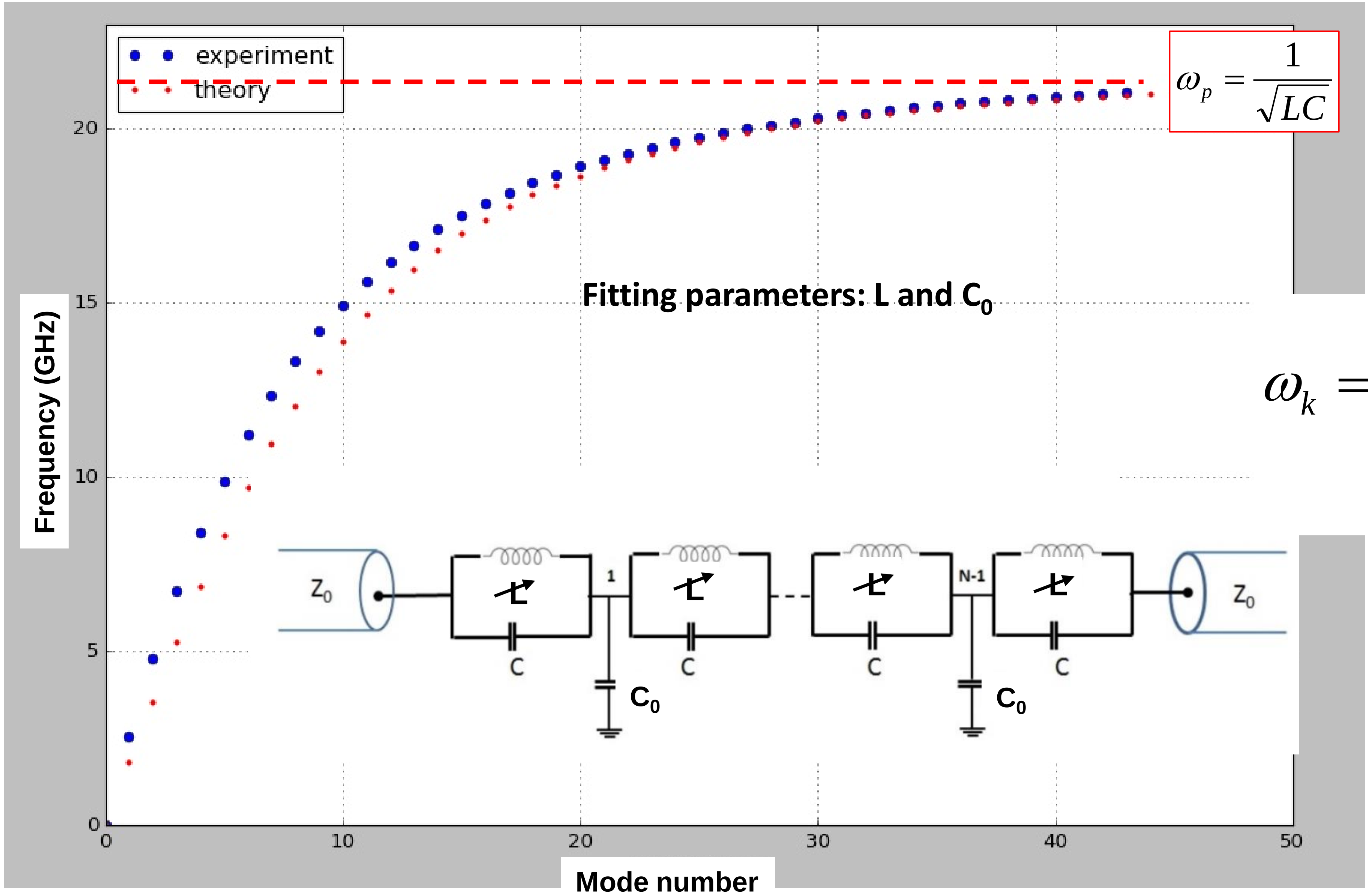
$$\omega_k = \omega_p \sqrt{\frac{1-\cos k}{1-\cos k + \frac{C_0}{2C}}}$$



$$\hat{L} = \begin{pmatrix} \frac{2}{L} & \frac{-1}{L} & 0 & \dots & & & 0 \\ \frac{-1}{L} & \frac{2}{L} & \frac{-1}{L} & 0 & \dots & & 0 \\ 0 & \frac{-1}{L} & \frac{2}{L} & \frac{-1}{L} & & & \\ \vdots & & \ddots & \ddots & \ddots & & \\ & & & & & \frac{2}{L} & \frac{-1}{L} \\ 0 & & & & & \frac{-1}{L} & \frac{2}{L} \end{pmatrix} \quad \hat{C} = \begin{pmatrix} C_0+C & -C & 0 & \dots & & & 0 \\ -C & C_0+2C & -C & 0 & \dots & & 0 \\ 0 & -C & C_0+2C & \ddots & & & \\ \vdots & & \ddots & \ddots & & & \\ & & & & & C_0+2C & -C \\ 0 & & & & & -C & C_0+C \end{pmatrix}$$

PhD-thesis of I. Pop (2011)
N.A. Masluk et al, Phys. Rev. Lett, 109,137002, (2012)
T. Weissl et al, Phys. Rev. B, 92,104508 (2015)

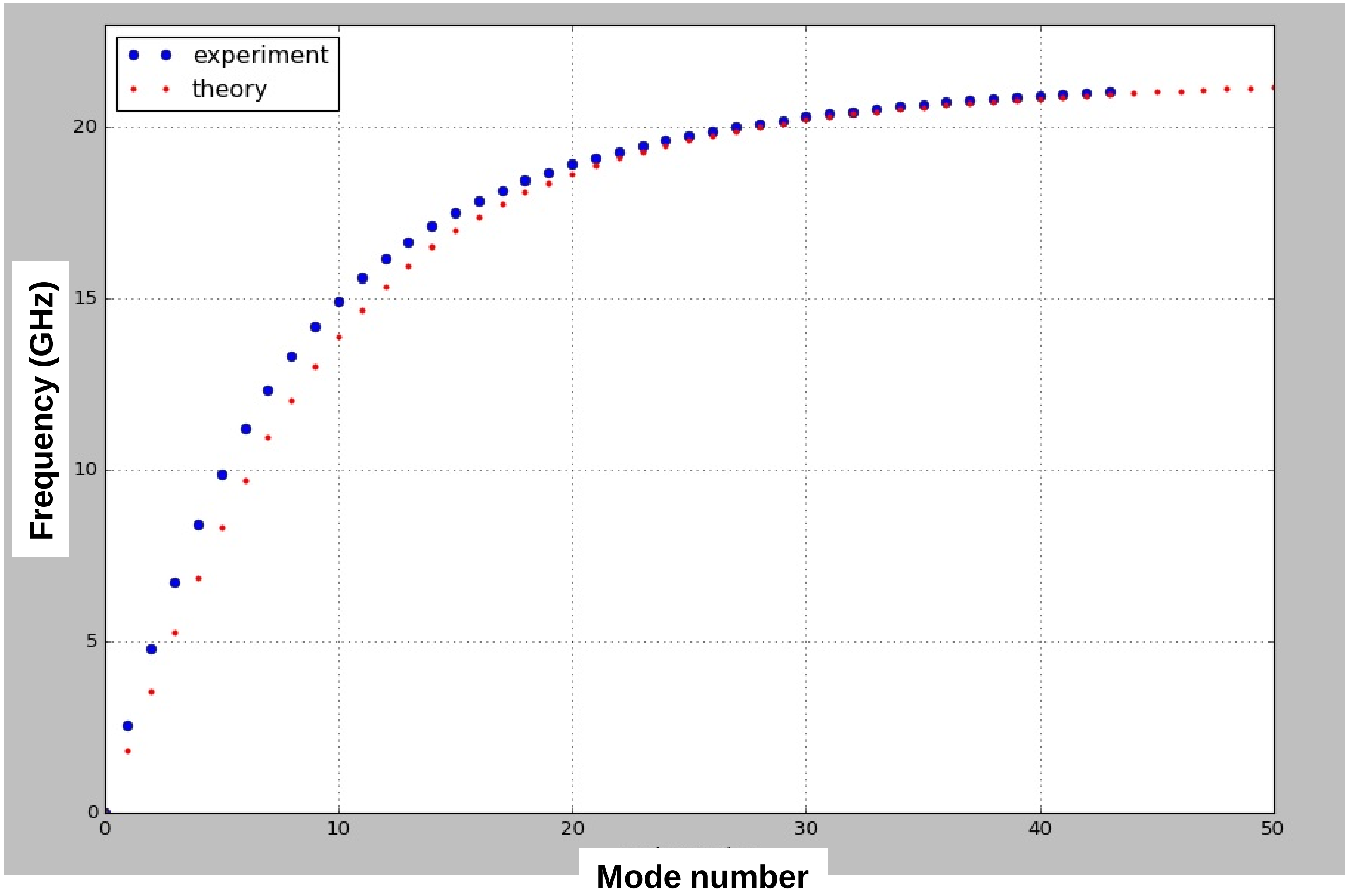
Dispersion: Comparison between theory and experiment

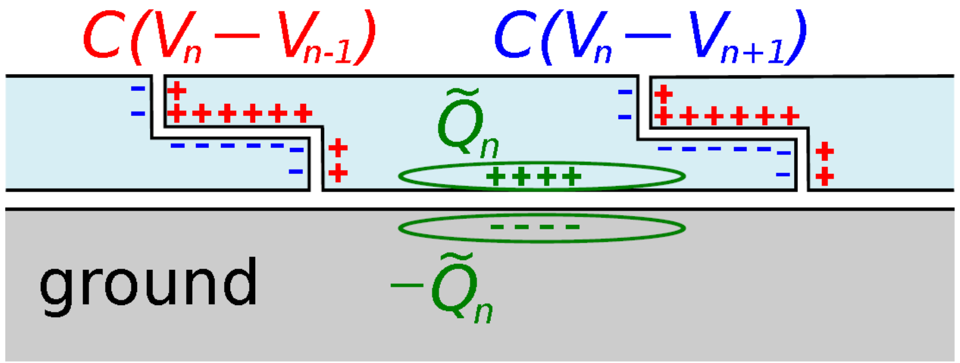


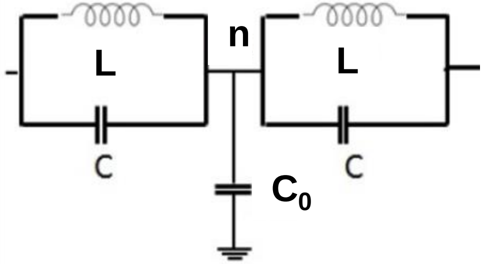
$$\omega_p = \frac{1}{\sqrt{LC}}$$

$$\omega_k = \omega_p \sqrt{\frac{1-\cos k}{1-\cos k + \frac{C_0}{2C}}}$$

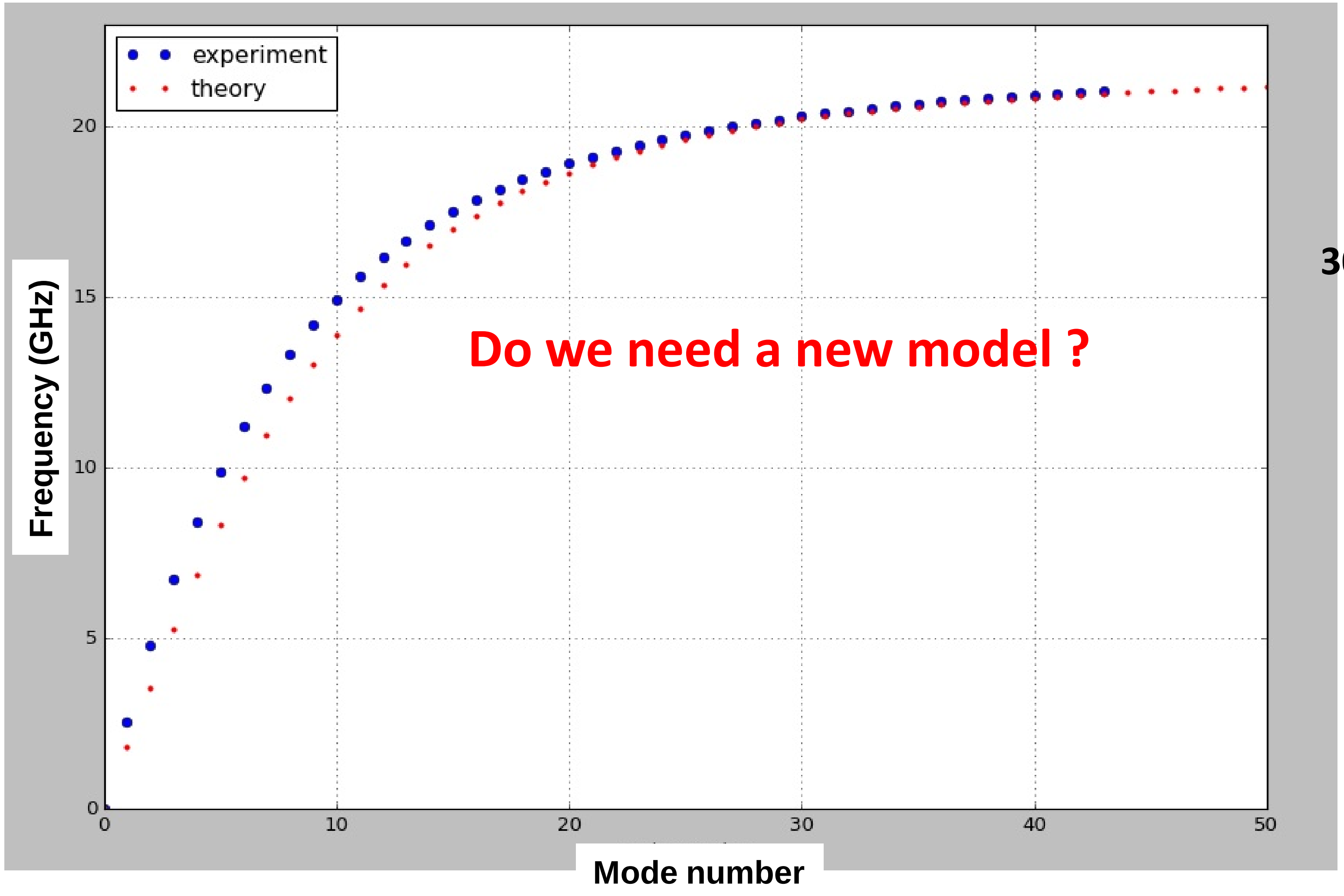
Dispersion: Comparison between theory and experiment

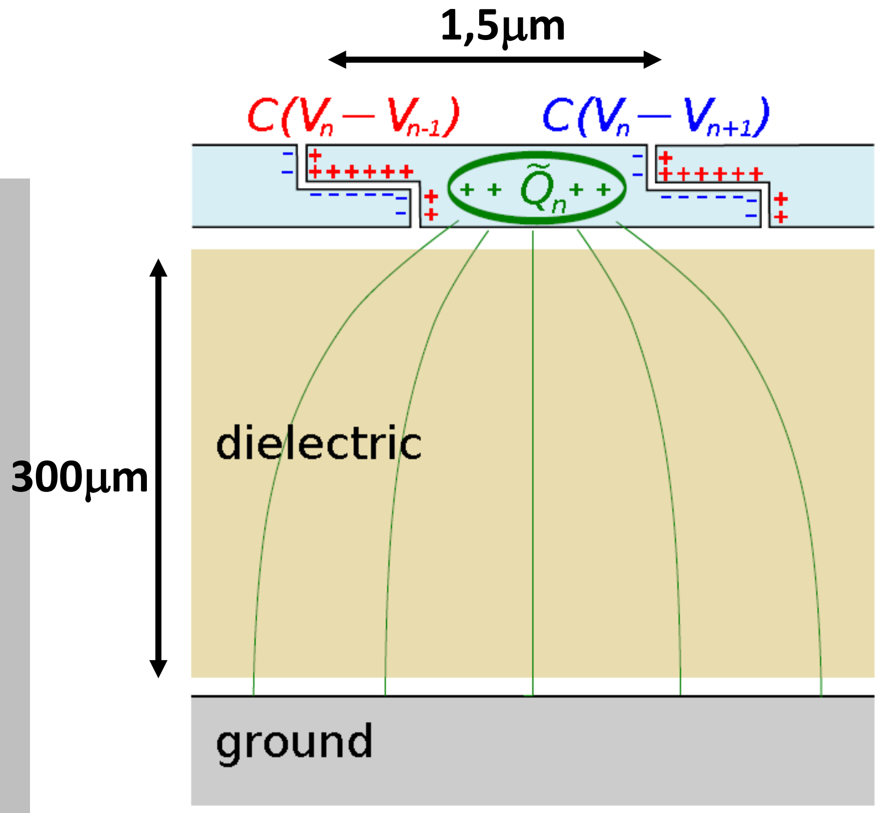


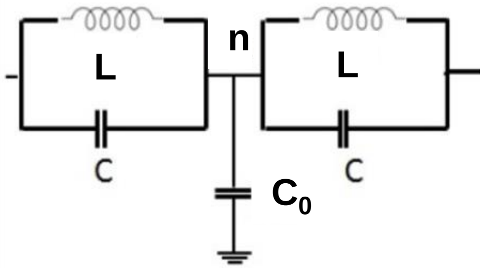




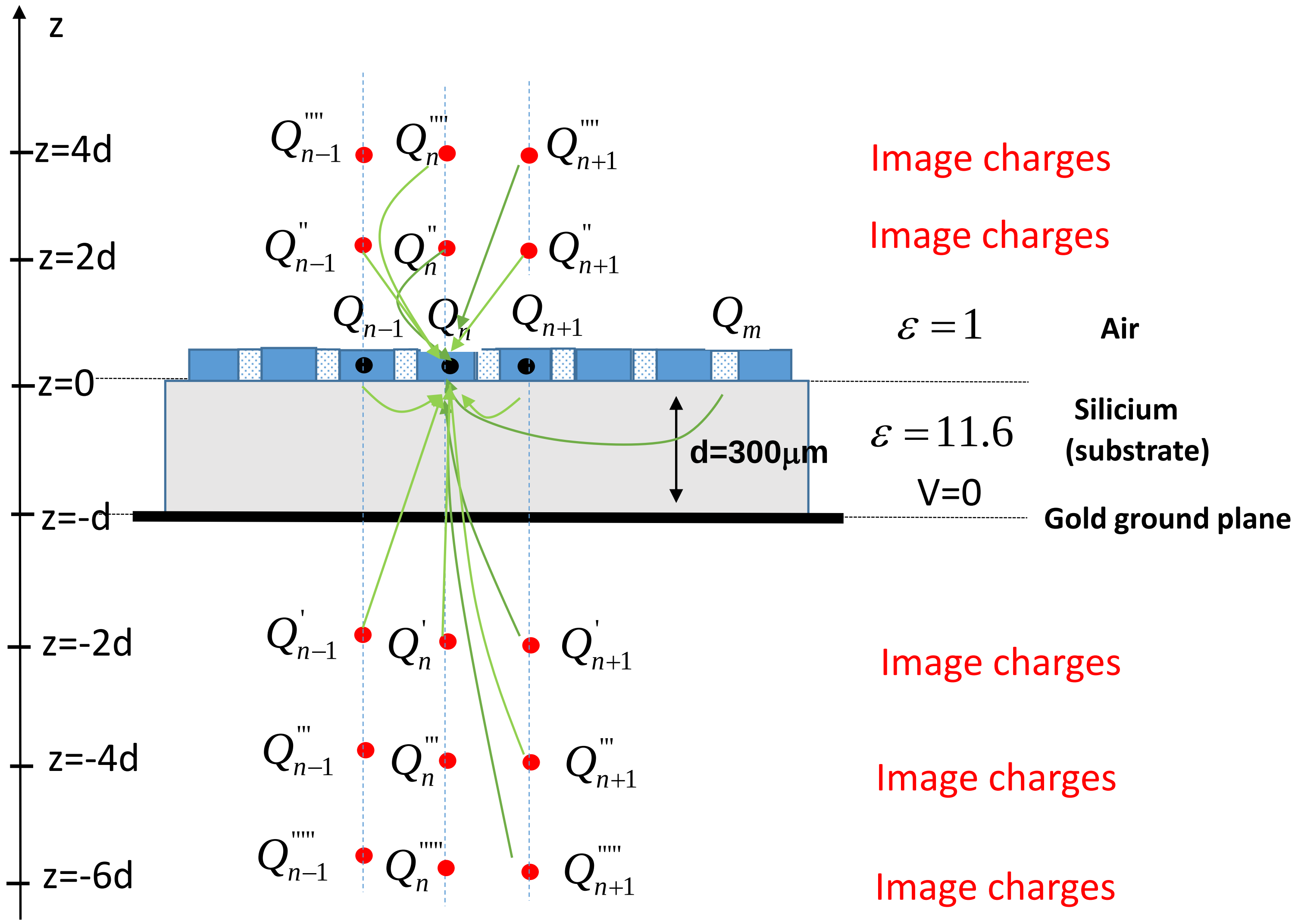
Dispersion: Comparison between theory and experiment







Remote ground model



Two interfaces → Multiple image charges

→ Longe range interaction

→ **New fitting parameters: L and a_0**

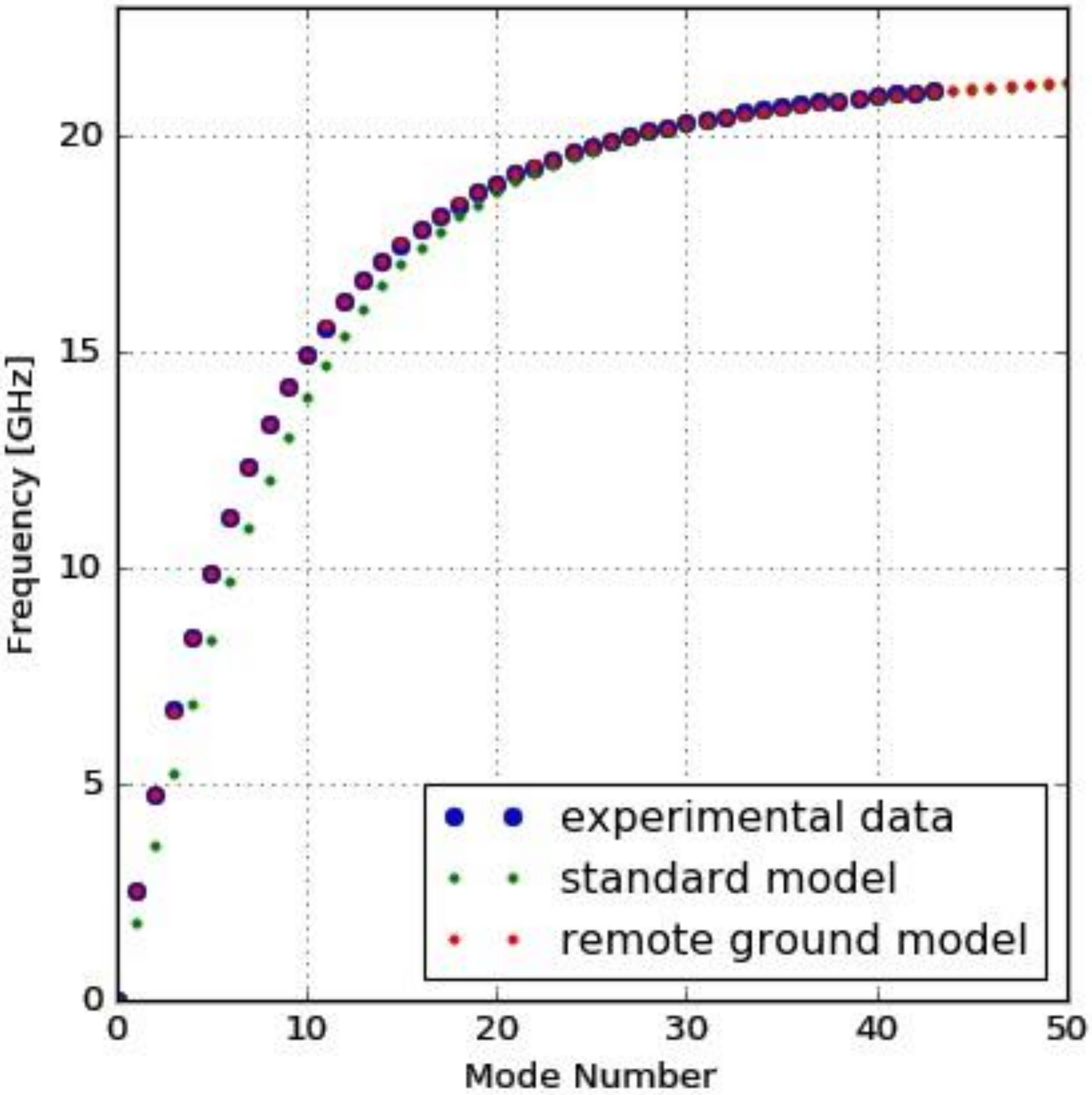
New capacitance matrix

$$V_n = \sum_m \frac{1}{2\pi\varepsilon_0(\varepsilon+1)} Q_m \sum_j \frac{((1-\varepsilon)/(1+\varepsilon))^j}{\sqrt{(n-m)^2 a^2 + (2jd - a_0)^2}} - \frac{((1-\varepsilon)/(1+\varepsilon))^j}{\sqrt{(n-m)^2 a^2 + (2j+2)d)^2}}$$

→

$$V_n = \sum_m C_0^{-1}{}_{nm} Q_m$$

Dispersion: Comparison between theory and experiment for remote ground model



$$\hat{C}^{-1/2}\hat{L}^{-1}\hat{C}^{-1/2}\vec{\psi}_k = \omega_k^2\vec{\psi}_k$$

Perfect agreement !

New fitting parameters: L and a_0

Number of fitting parameters is the same !

Engineering of a controlled electromagnetic environment

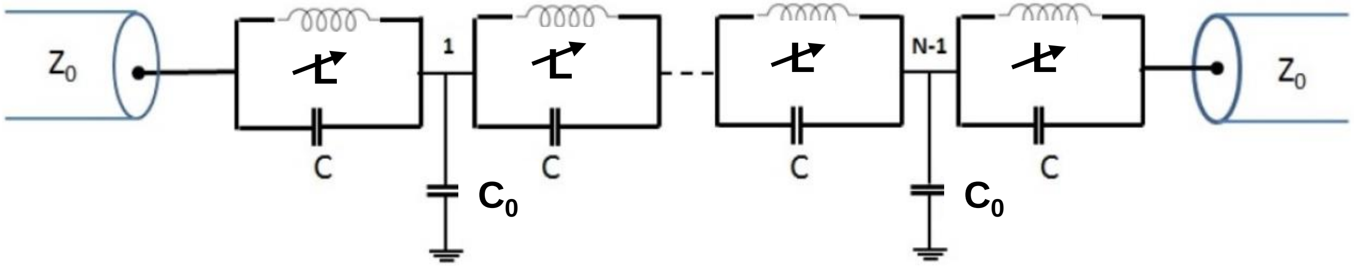
Outline

1) Linear effects: Dispersion of propagation modes in a Josephson junction chain

2) Non-linear effects: Self- and Cross Kerr effects in a Josephson junction chain

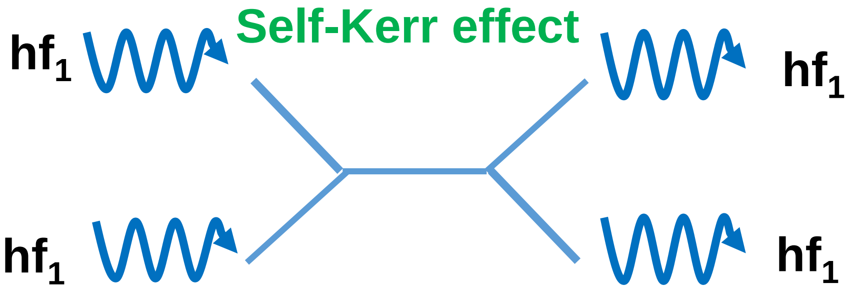
3) Strong non-linear effects: quantum phase-slips

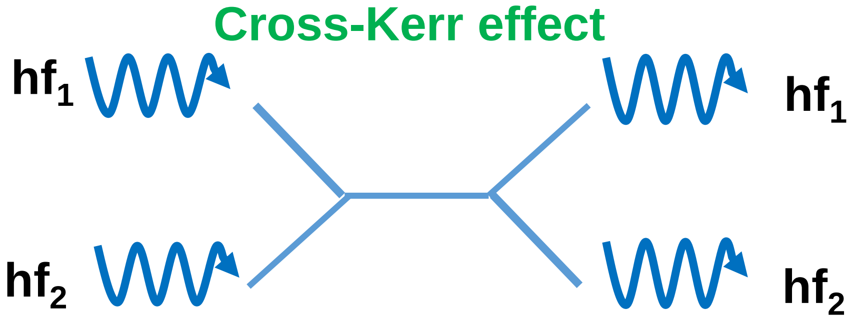
Photon interaction due to non-linear effects in a Josephson junction chain

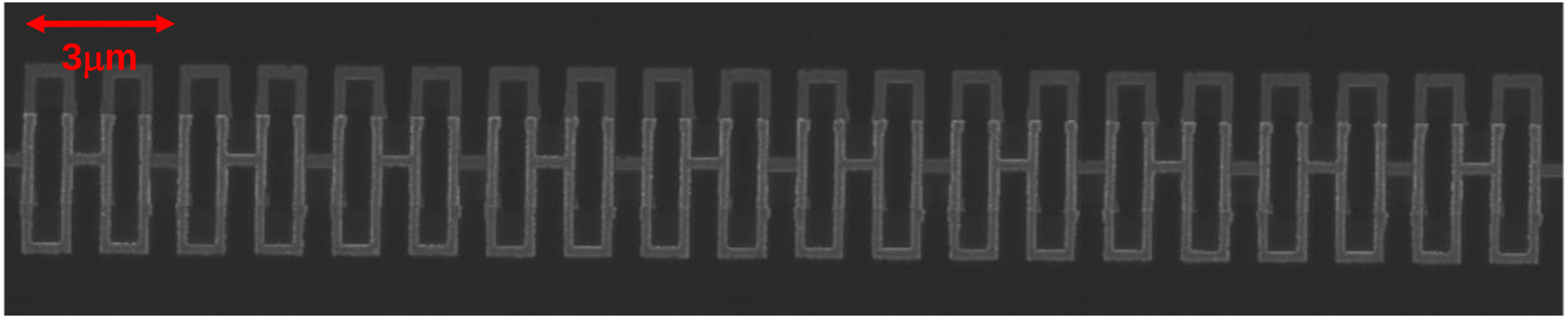


$$L = \frac{\hbar}{2eI_c}\frac{1}{\cos(\varphi)} = \frac{\hbar}{2eI_c}\frac{1}{1-\frac{(I/I_c)^2}{2}+\dots}$$

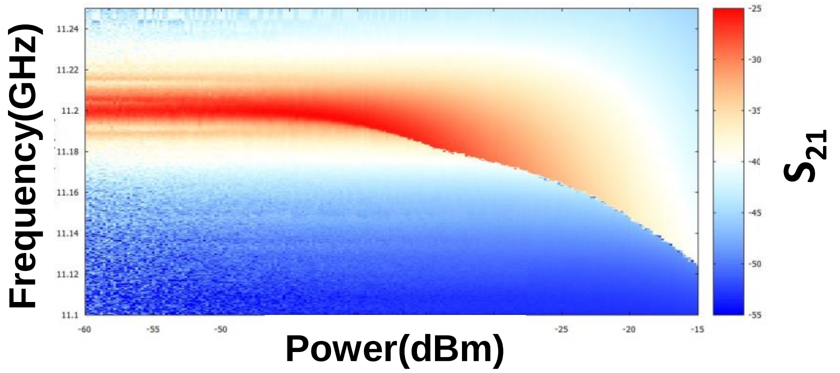
$$H = \sum_k \hbar\omega_k + \sum_{k_1,k_2,k_3,k_4} K_{k_1,k_2,k_3,k_4}\left[4a^+_{k_1}a^+_{k_2}a^+_{k_3}a_{k_4} + 4a^+_{k_1}a_{k_2}a_{k_3}a_{k_4} + 6a^+_{k_1}a^+_{k_2}a_{k_3}a_{k_4} + (6a^+_{k_1}a^+_{k_2} + 6a_{k_1}a_{k_2} + 12a^+_{k_1}a_{k_2})\delta_{k_3,k_4}\right]$$





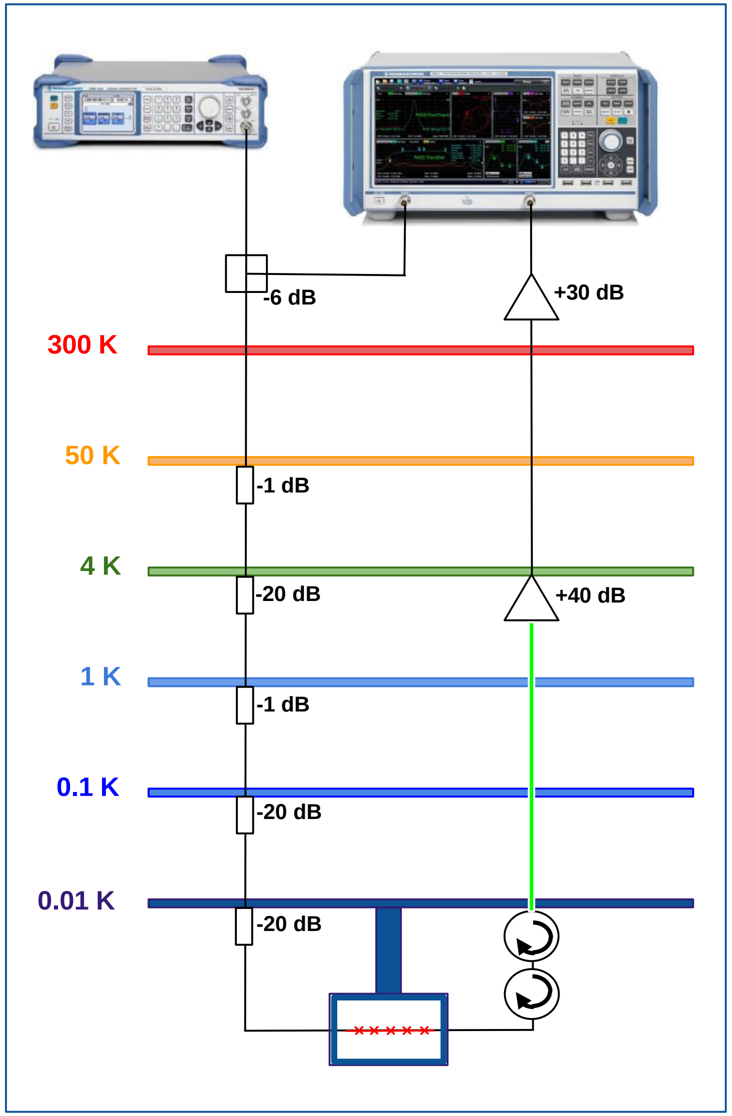


Measured self-and cross Kerr-effect

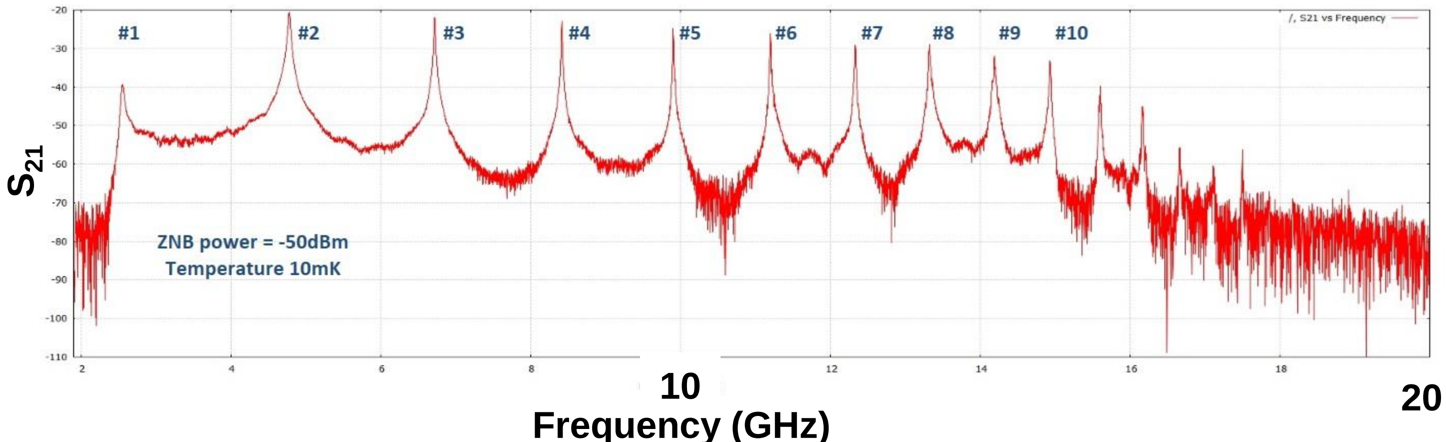


Self-Kerr effect

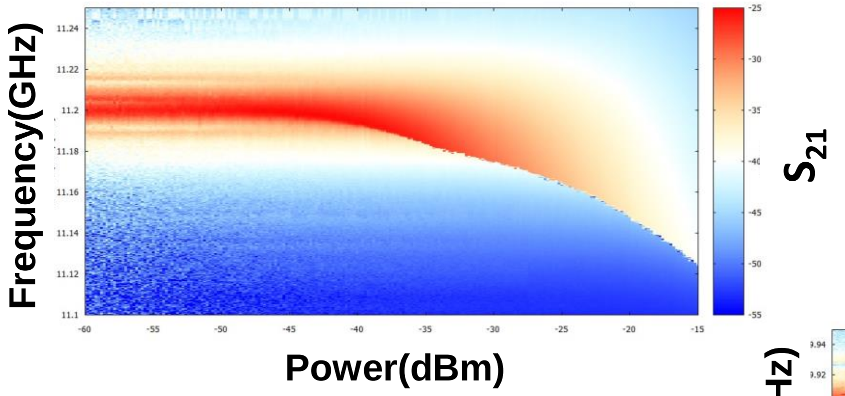
Variable power is applied to the pumping mode



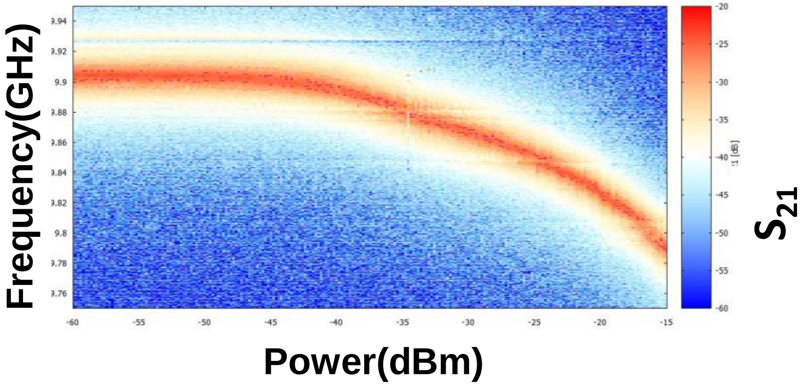
Pump and Measure mode 4



Measured self-and cross Kerr-effect



Variable power is applied to the pumping mode

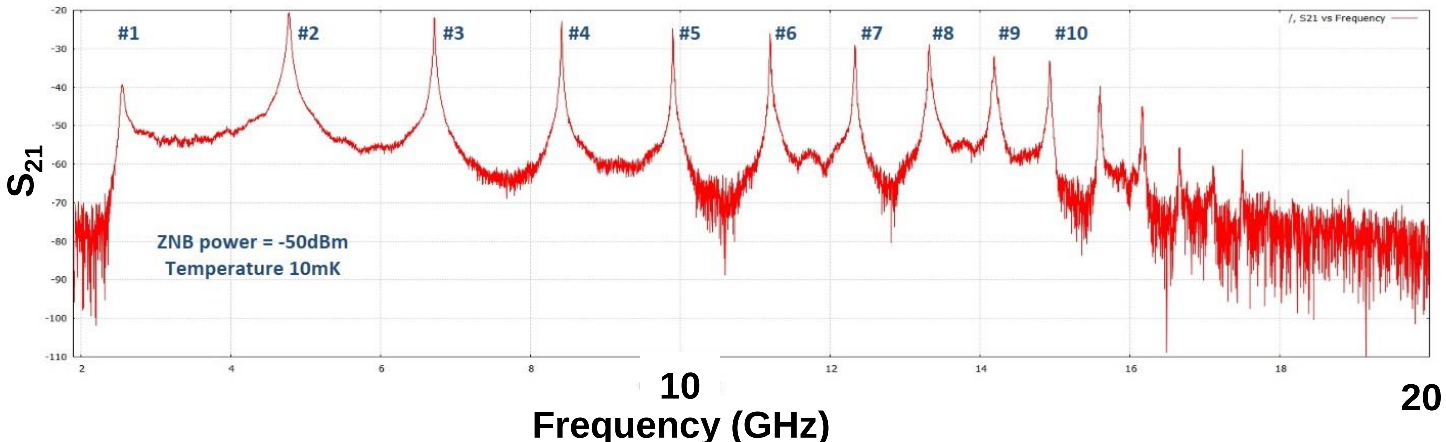


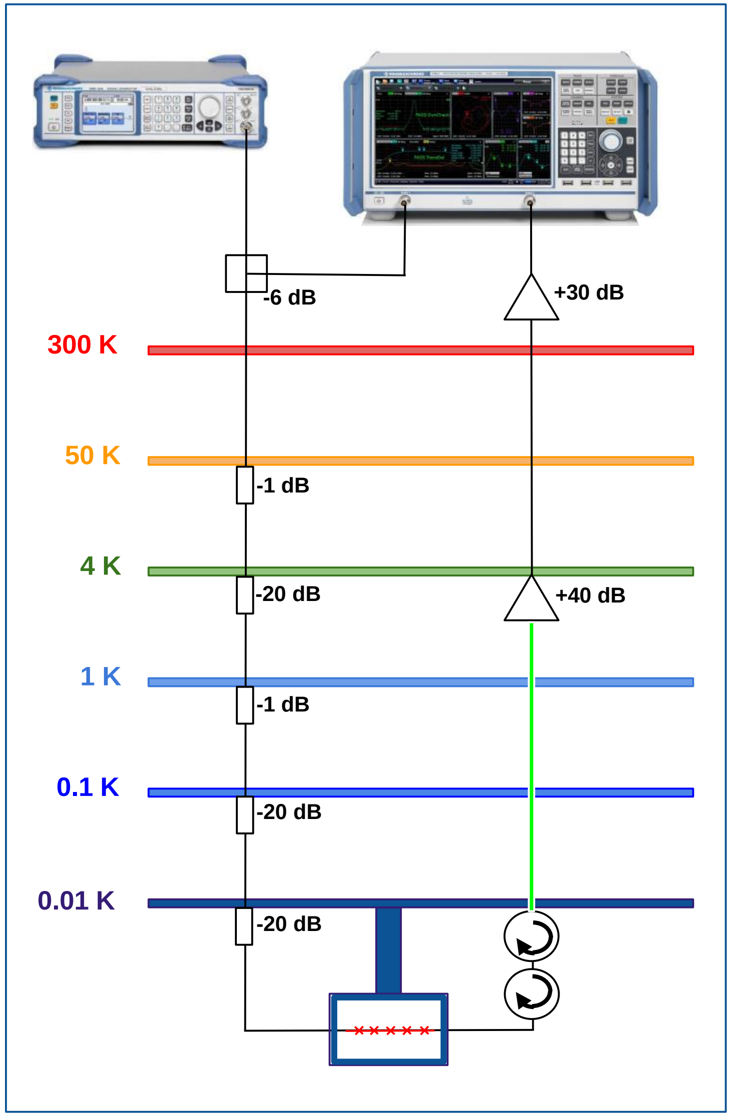
Cross-Kerr effect

…while the probing mode is fed with constant power

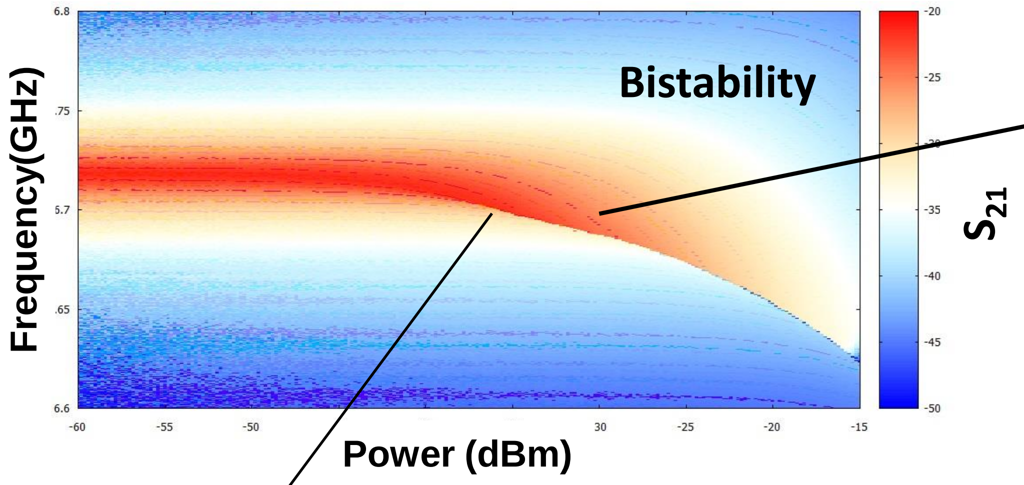
Pump mode 4

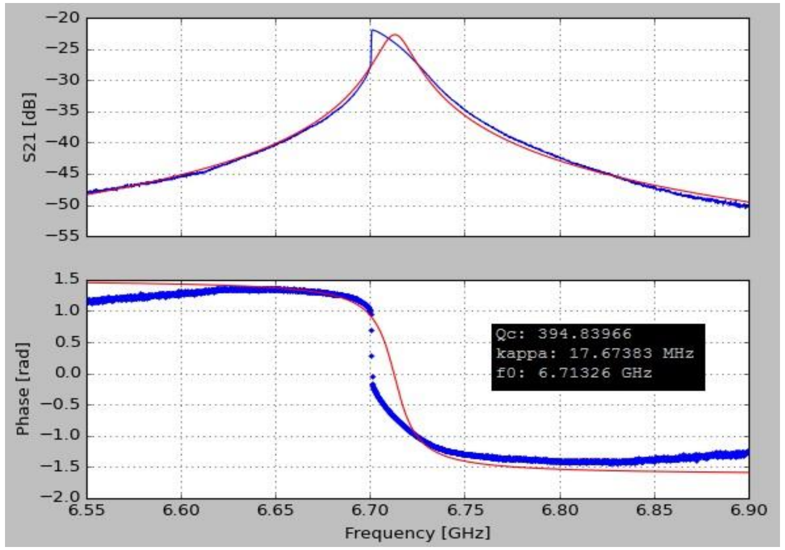
Measure mode 7

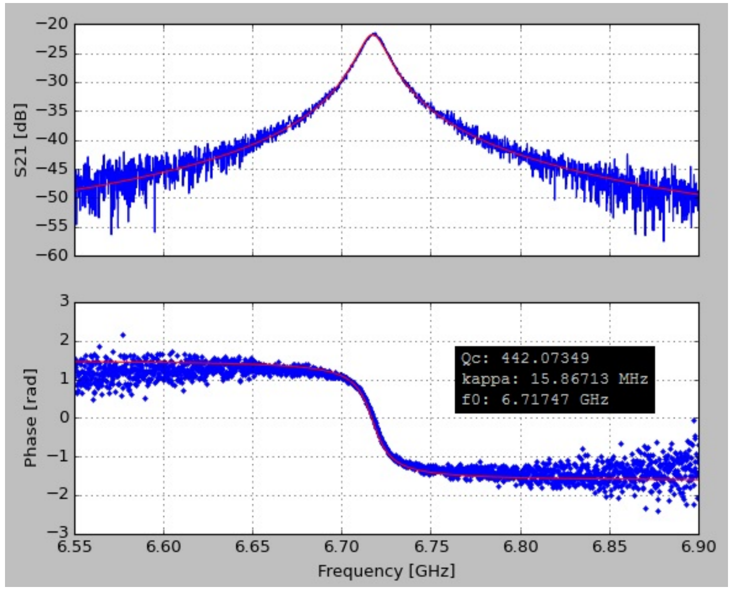


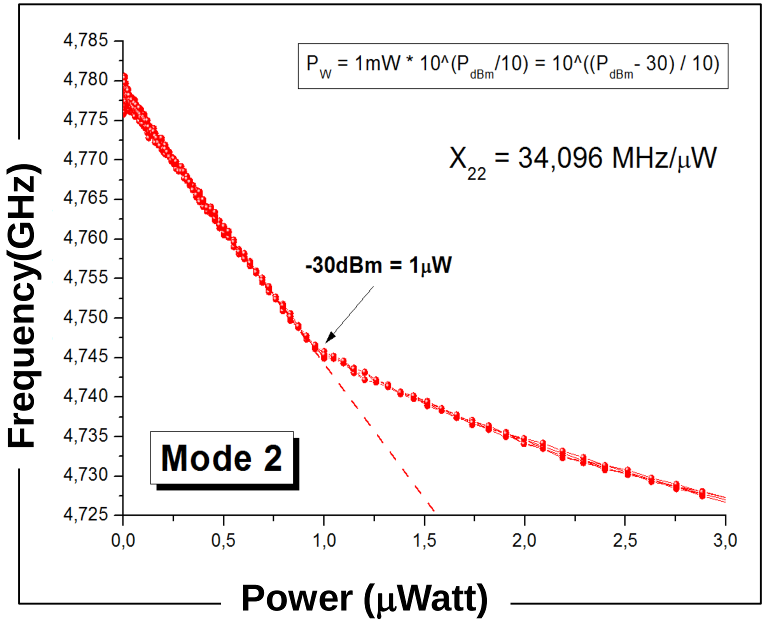


Weak non-linearity



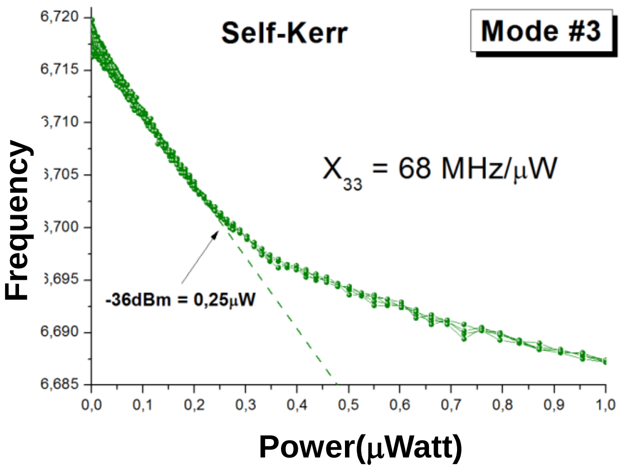




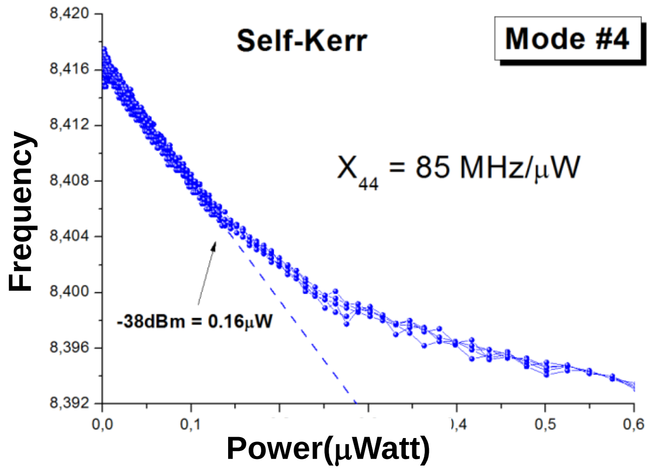


Measurement of Self-and Cross Kerr effects for 8 modes

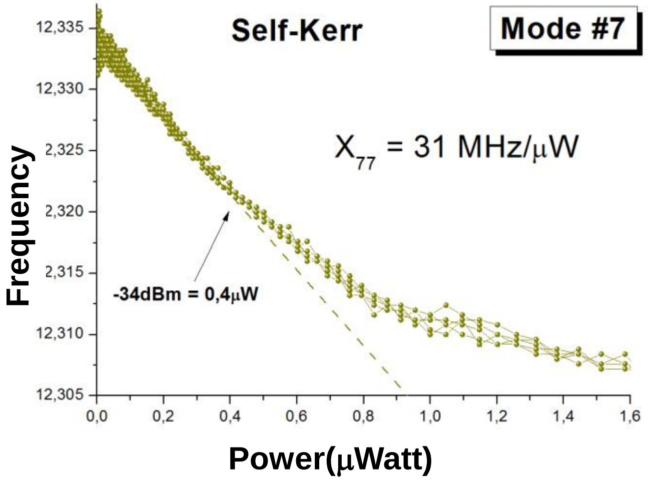
Self-Kerr effect



Measure mode 3, pump mode 3

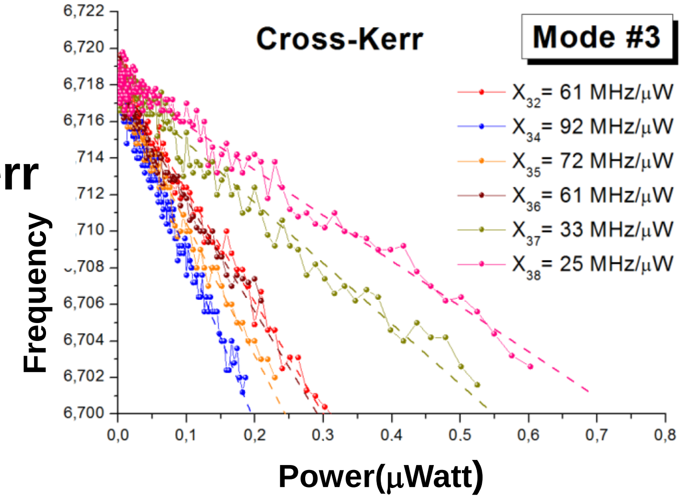


Measure mode 4, pump mode 4

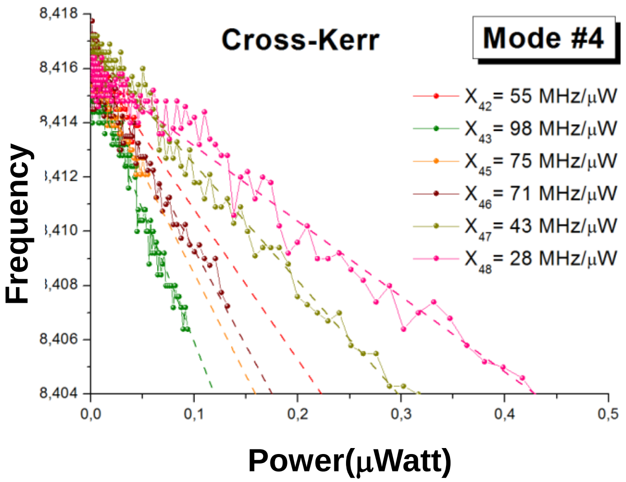


Measure mode 7, pump mode 7

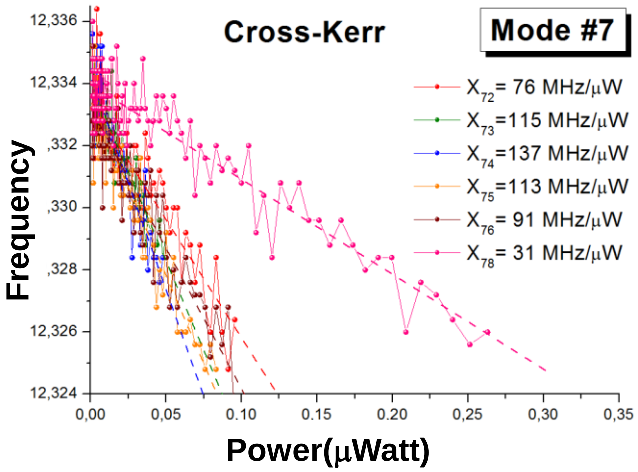
Cross-Kerr effect



Measure mode 3, pump mode 2,4,5,6,7,8



Measure mode 4, pump mode 2,3,5,6,7,8



Measure mode 7, pump mode 2,3,4,5,6,8

Deduce frequency shifts X_{jk} of mode j as function of applied power on mode k

Y. Krupko et al, preprint in preparation

Theory: Self-and Cross Kerr effect as a weak non-linearity

$$E_J \cos(\varphi) \approx E_J (1 - \frac{\varphi^2}{2} + \frac{\varphi^4}{24})$$

$$\hat{H} = \sum_k \hbar \omega_k' \hat{a}_k^+ \hat{a}_k - \sum_k \frac{\hbar}{2} K_{kk} \hat{a}_k^+ \hat{a}_k \hat{a}_k^+ \hat{a}_k - \sum_{\substack{j,k \\ (j \neq k)}} \frac{\hbar}{2} K_{jk} \hat{a}_j^+ \hat{a}_j \hat{a}_k^+ \hat{a}_k - \ldots$$

$$\hat{H} = \sum_{k,j} \hbar (\omega_k' - \frac{1}{2} K_{kk} n_k - K_{jk} n_j) \hat{a}_k^+ \hat{a}_k$$

Self-Kerr **Cross-Kerr**

$$\omega_k' = \omega_k - K_{kk}/2 - \sum_p K_{kp}/2$$

$$K_{kk} = \frac{2\hbar \pi^4 E_J \eta_{kkkk}}{\Phi_0^4 C^2 \omega_k^2}$$

$$K_{jk} = \frac{4\hbar \pi^4 E_J \eta_{jjkk}}{\Phi_0^4 C^2 \omega_j \omega_k}$$

$$\psi_j, \psi_k \rightarrow \eta_{jjjj}, \eta_{jjkk}$$

$$\hat{C}^{-1/2} \hat{L}^{-1} \hat{C}^{-1/2} \vec{\psi}_k = \omega_k^2 \vec{\psi}_k$$

Frequency shifts of propagating modes with increasing power

T. Weissl et al, Phys. Rev. B, 92,104508 (2015)

Comparison between theory and experiment for Self- and Cross Kerr effects

Experimental matrix of Kerr frequency shifts Xjk in MHz/µW

Xjk	j = 2	j = 3	j = 4	j = 5	j = 6	j = 7	j = 8
k = 2	34	61	55	74	59	76	64
k = 3	64	68	98	105	91	115	91
k = 4	56	92	85	124	103	137	98
k = 5	42	72	75	99	85	113	58
k = 6	43	61	71	99	66	91	41
k = 7	24	33	43	54	40	31	32
k = 8	18	25	28	31	19	31	28

$$\hat{H} = \sum_k \hbar(\omega_k' - \frac{1}{2} K_{kk} n_k - K_{jk} n_j) \hat{a}_k^+ \hat{a}_k$$

$$n_k = A_k(\omega) P_k$$

$$X_{jk} = A_j K_{jk}$$

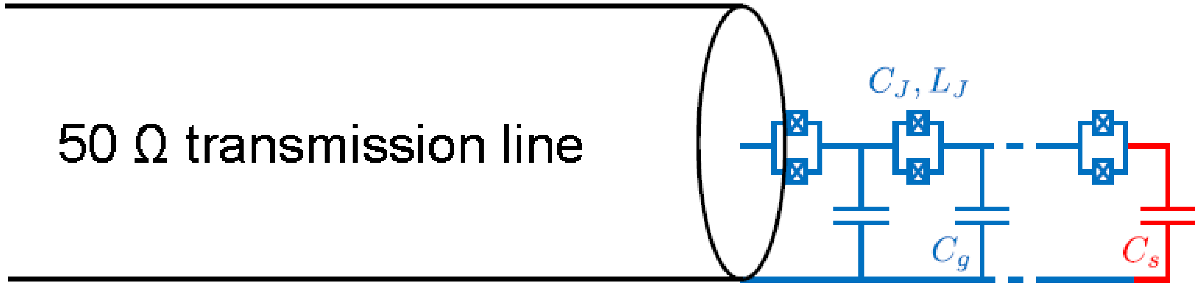
$$K_{j2} = K_{2j}$$

→

K_{jk}/K_{22}	j = 2	j = 3	j = 4	j = 5	j = 6	j = 7	j = 8
k = 2	1,00	1,79	1,62	2,18	1,74	2,23	1,88
k = 3	1,79	1,89	2,71	2,92	2,52	3,20	2,51
k = 4	1,62	2,65	2,45	3,57	2,94	3,95	2,81
k = 5	2,18	3,71	3,85	5,07	4,36	5,83	2,96
k = 6	1,74	2,48	2,86	4,00	2,67	3,67	1,68
k = 7	2,23	3,02	3,92	4,92	3,69	2,82	2,90
k = 8	1,88	2,60	2,91	3,26	1,94	3,20	2,91

Experimental Matrix K_{jk}/K_{22} is symmetric within 5%.
Up to k=4 very good agreement between experiment and theory.
For larger mode numbers increasing disagreement.

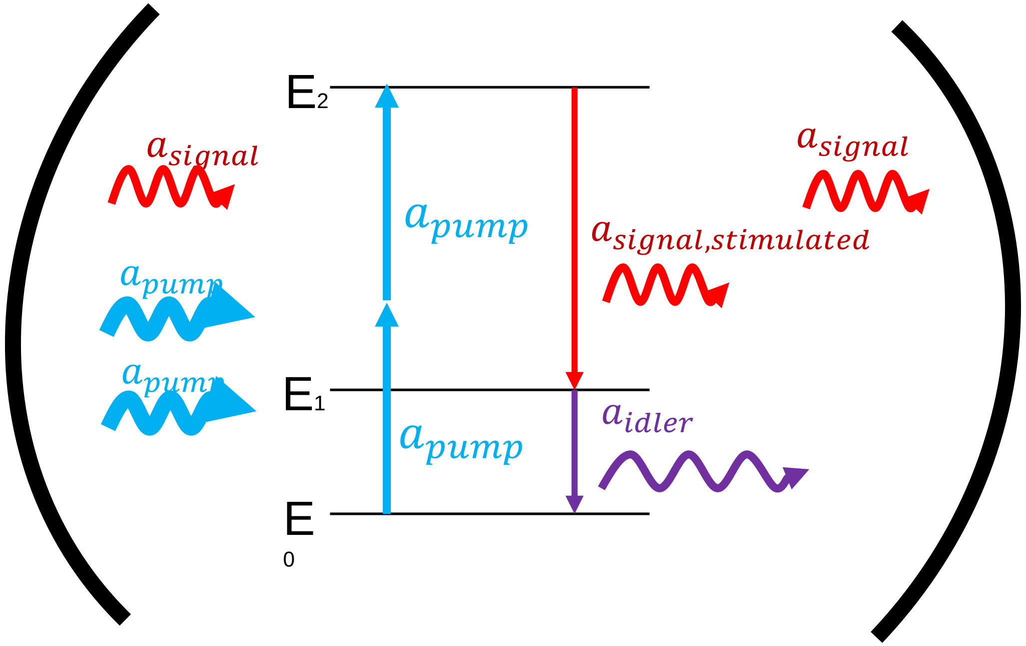
From Josephson parametric amplifier towards a Traveling Wave parametric amplifier



Nicolas Roch

Luca Planat
PhD-student

$$\hat{H} = \hbar\omega_p \hat{a}^+\hat{a} - \frac{\hbar}{2}K\hat{a}^+_{signal}\hat{a}_{pump}\hat{a}^+_{idler}\hat{a}_{pump} + \cdots$$



Stimulated emission of a photon amplified in a cavity

Outline

1) Linear effects: Dispersion of propagation modes in a Josephson junction chain

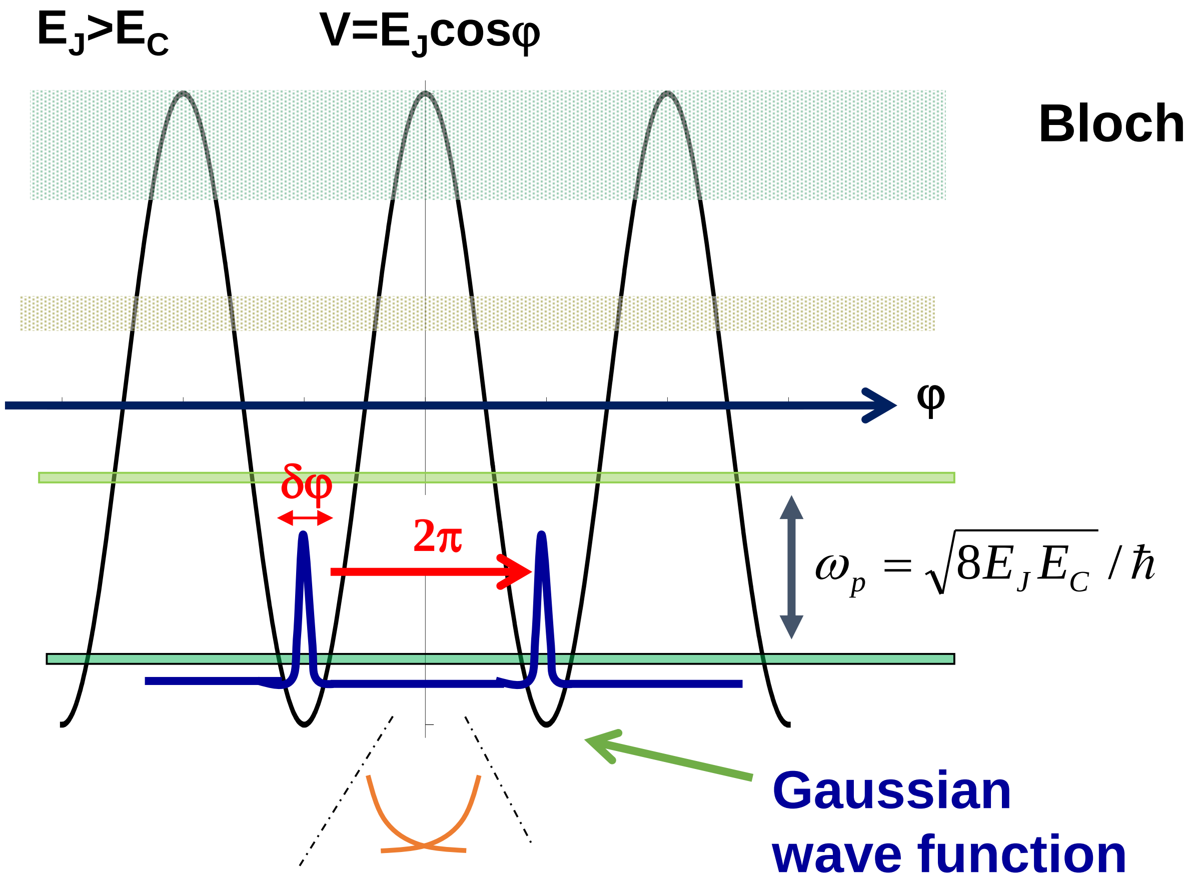
2) Non-linear effects: Self- and Cross Kerr effects in a Josephson junction chain

3) Strong non-linear effects: quantum phase-slips

Quantum phase-slip

$$H_{dual} = \frac{Q^2}{2C} - E_J \cos(\varphi)$$

Schrödinger equation: $$\frac{d^2\psi}{d(\varphi/2)^2} + \left(\frac{E}{E_C} + \frac{E_J}{E_C}\cos\varphi\right)\psi = 0$$



Bloch waves:

$$\psi_{n,q}(\varphi) = e^{iq\varphi}u_n(\varphi)$$
$$u_n(\varphi + 2\pi) = u_n(\varphi)$$

Lowest Bloch state:

$$\psi_q(\varphi) = \sum_m e^{i\hat{q}2\pi m} W_0(\varphi - 2\pi m)$$

$$W_0(\varphi) = \sqrt{\frac{1}{2\pi\delta\varphi}}\, e^{-\frac{1}{2}\frac{\varphi^2}{\delta\varphi^2}}$$

$$\delta\varphi \sim \sqrt{\frac{E_C}{E_J}}$$

Averin, Likharev, Zorin (1985)

Exponentially small overlap of Gaussian tails →

Quantum Phase-Slip amplitude $$\upsilon_{QPS} \approx \left(E_J^3 E_C\right)^{1/4} \exp\left(-\sqrt{8E_J/E_C}\right)$$

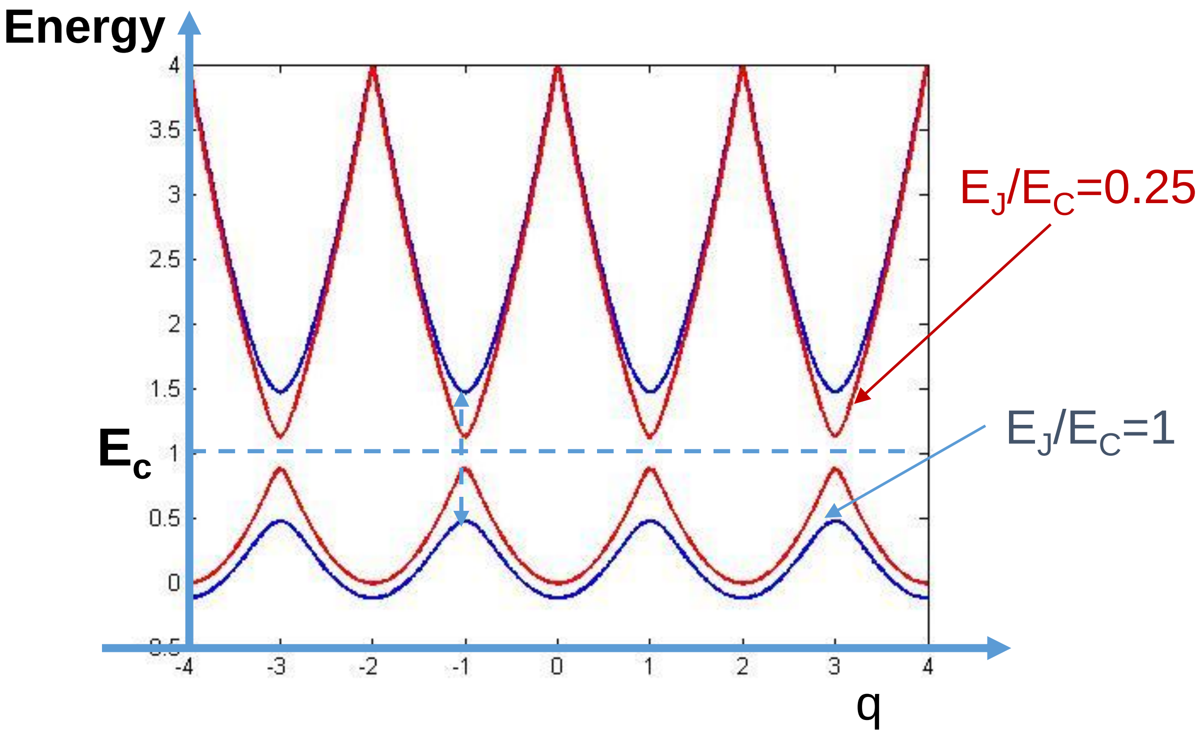
Realisation of the phase-slip non-linearity with a small Josephson junction

Energy spectrum of the junction consists of Bloch bands

Lowest Bloch band:

$$E_0(\hat{q}) = \sum_{k=1}^{\infty} U_k \cos(k\pi\hat{q}/e)$$

$$U_1 = \upsilon_{QPS} \approx \left(E_J^3 E_C\right)^{1/4} \exp\left(-\sqrt{8E_J/E_C}\right)$$



For intermediate values of E_J/E_c:

$$H = \frac{Q^2}{2C} - E_J \cos\varphi \quad \longrightarrow \quad H = \upsilon_{QPS} \cos\left(\frac{\pi q}{e}\right)$$

Averin, Likharev, Zorin (1985)

Ordinary Josephson junction to Dual Josephson junction

Ordinary Josephson junction

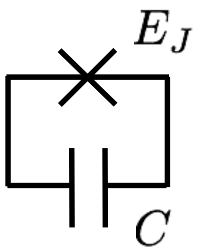
$$H_{dual} = \frac{Q^2}{2C} - E_J \cos(\varphi)$$

Coherent Cooper pair tunneling

Josephson Relations

$$I_J = I_c \sin(\varphi)$$

$$V_J = \frac{\hbar}{2e}\frac{d\varphi}{dt}$$



Quantum phase-slip junction

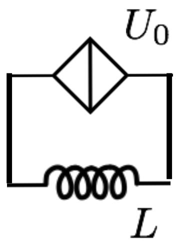
$$H_{dual} = \frac{\Phi_0^2}{2L}\varphi^2 - \upsilon_{QPS}\cos\left(\frac{\pi}{e}q\right)$$

Coherent quantum phase-slips

Dual Josephson relations

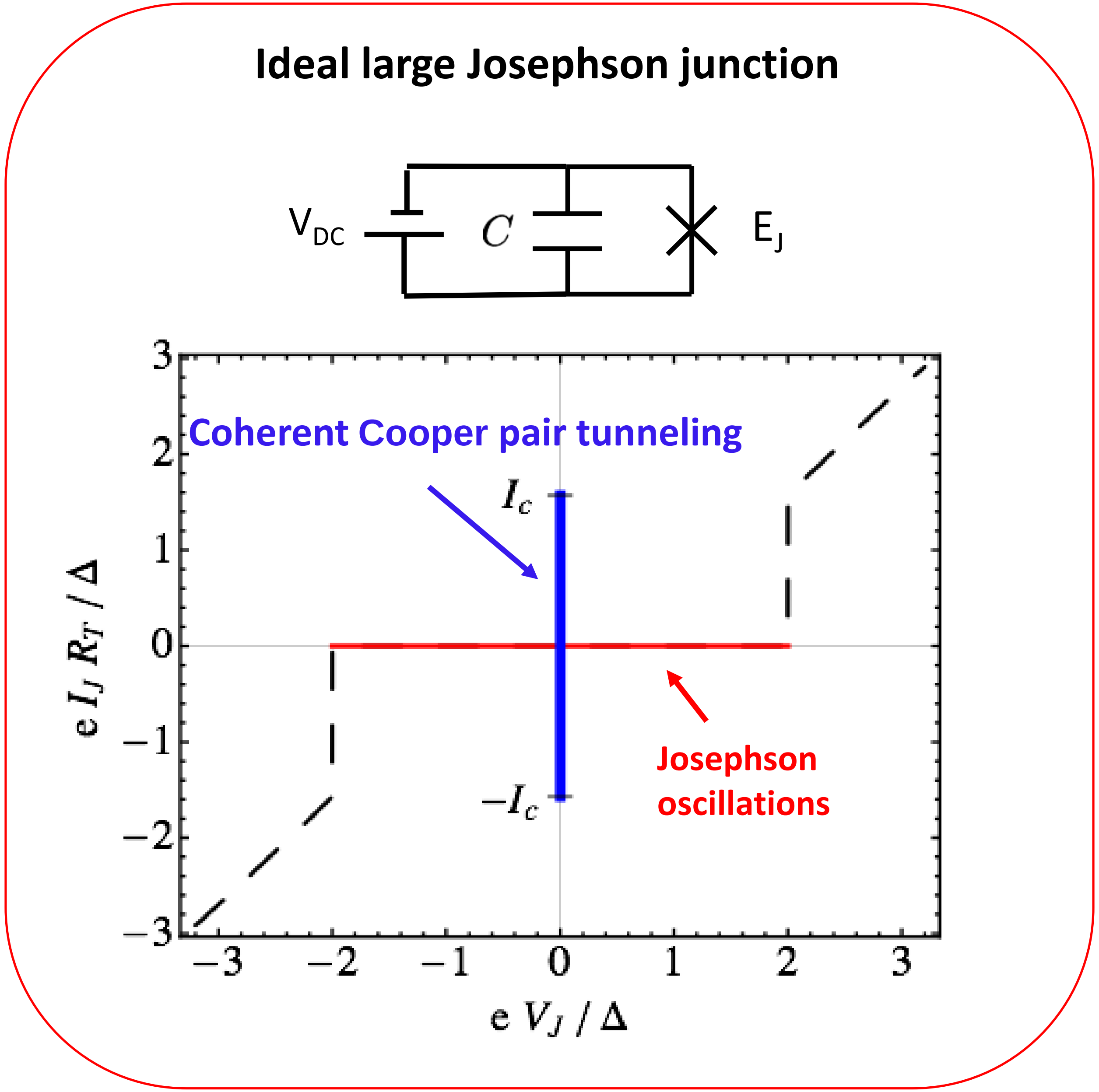
$$V_J = V_c \sin\left(\frac{\pi}{e}q\right)$$

$$I_J = \frac{dq}{dt}$$

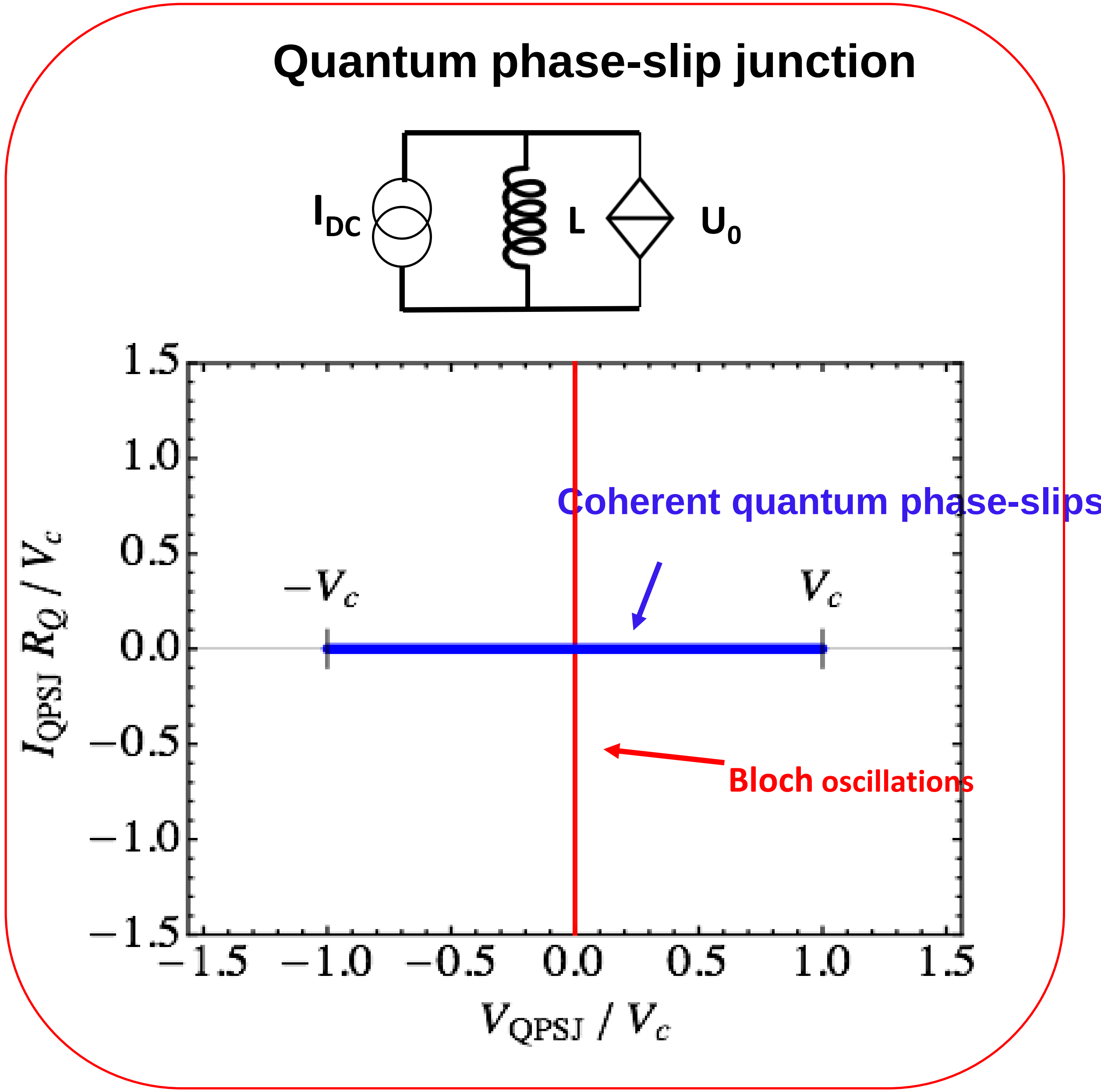


- *Quantum Complementarity for the Superconducting Condensate and the Resulting Electrodynamic Duality, D. B. Haviland et al, Proc. Nobel Symposium on Coherence and Condensation, Physica Scripta T102 , pp. 62 - 68 (2002)*
- *A.D. Zaikin, Journal of Low Temperature Physics, 80, Nos 5/6,(1990)*
- *J. E. Mooij and Y. V. Nazarov, Nat. Phys.(2006)*

Duality



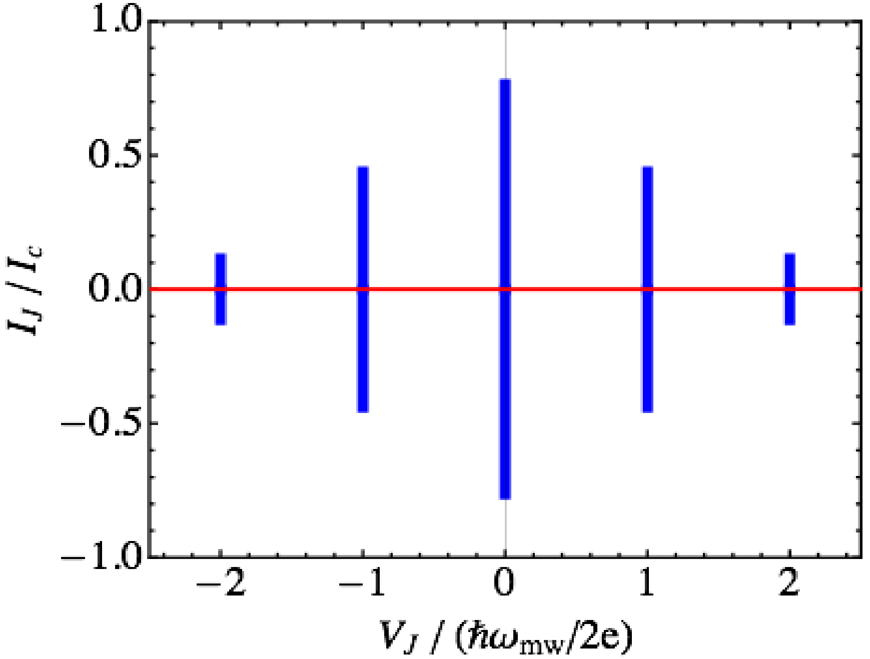
Ideal large Josephson junction
V_DC
C
E_J
Coherent Cooper pair tunneling
I_c
−I_c
Josephson oscillations
e I_J R_T / Δ
e V_J / Δ



Quantum phase-slip junction
I_DC
L
U_0
Coherent quantum phase-slips
−V_c
V_c
Bloch oscillations
I_QPSJ R_Q / V_c
V_QPSJ / V_c

Quantum phase-slip junction under microwave irradiation

Ideal large Josephson Junction

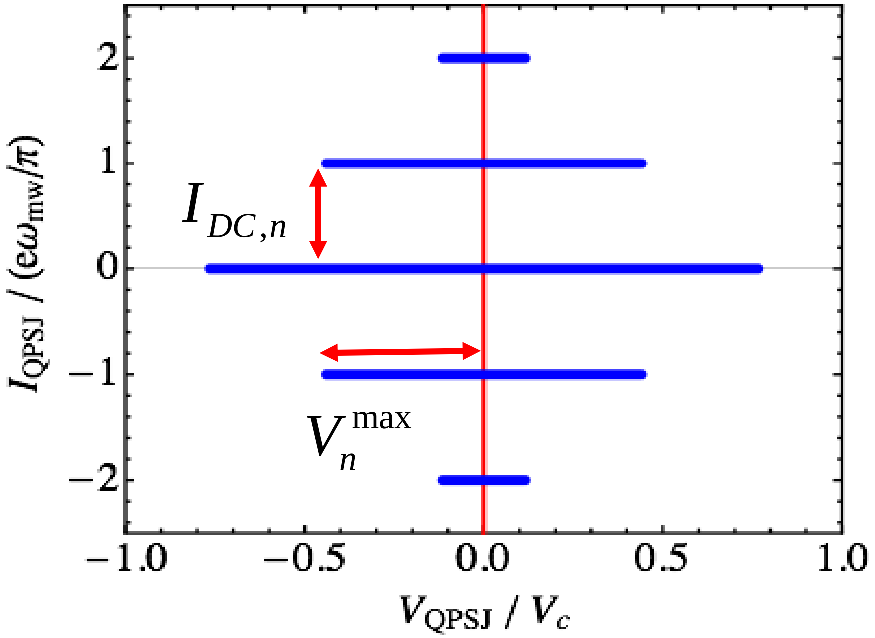


Phase Locking relations

$$V_{DC,n} = n\frac{\hbar\omega_{mw}}{2e}$$

$$I_n^{\max} = I_c J_n\left(\frac{2eV_{mw}}{\hbar\omega_{mw}}\right)\sin(\varphi_0)$$

Quantum Phase-slip junction



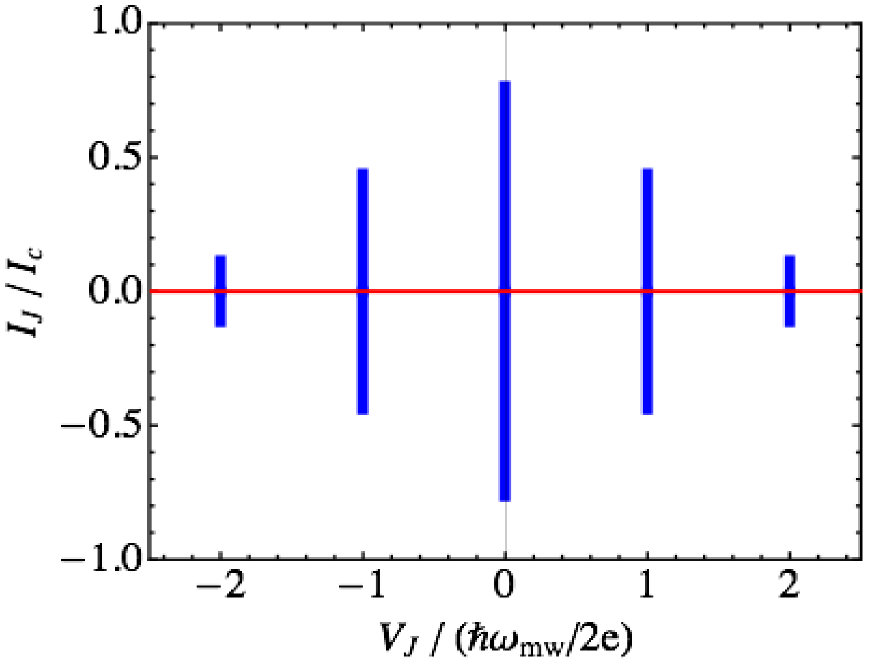
Dual Phase Locking relations

$$I_{DC,n} = n\frac{e\omega_{mw}}{\pi}$$

$$V_n^{\max} = V_c J_n\left(\frac{\pi I_{mw}}{e\omega_{mw}}\right)\sin(\varphi_0)$$

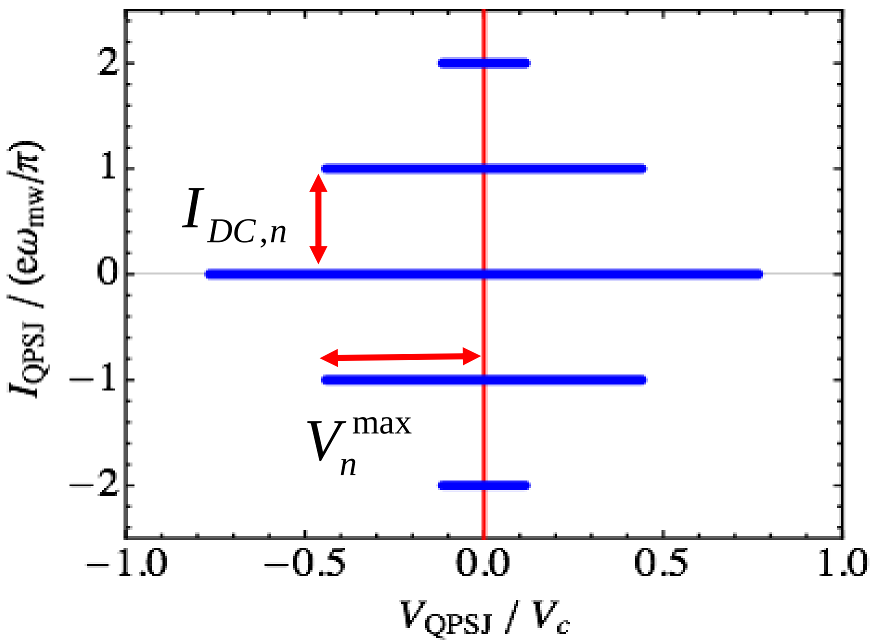
Quantum phase-slip junction under microwave irradiation

Ideal large Josephson Junction



$$\frac{E_J}{E_C} >> 1$$

Quantum Phase-slip junction

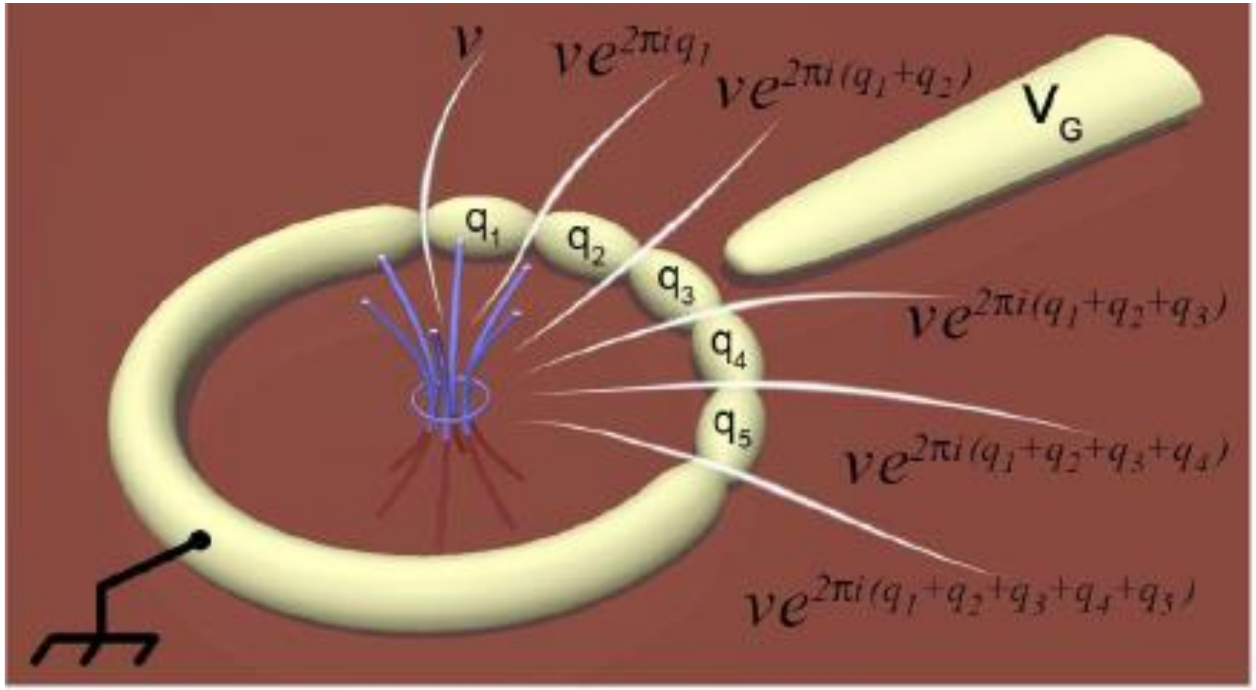


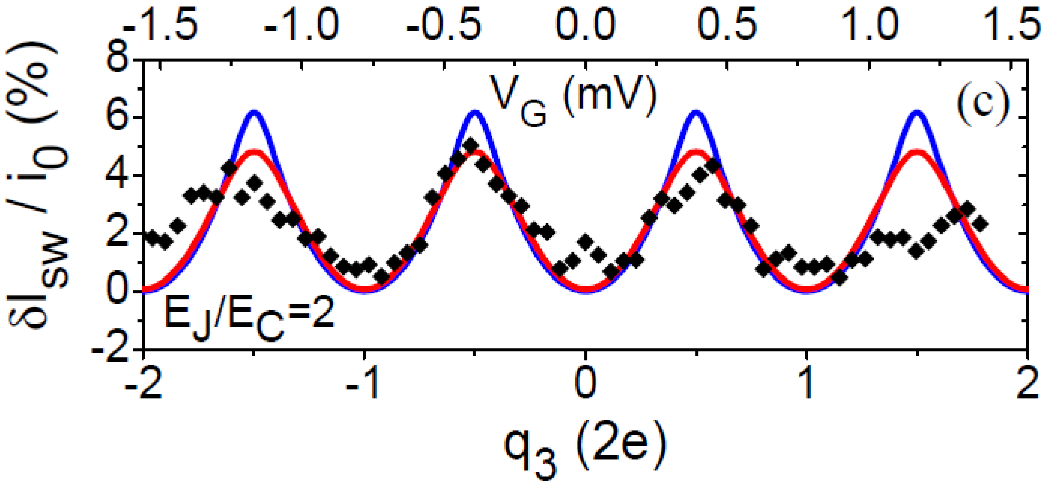
$$\frac{\upsilon_{QPS}}{E_L} >> 1$$

Aharonov Casher effect in a short Josephson junction chain

$$\upsilon_{qps}^{tot} = \sum_i^N \upsilon_i$$

$$\upsilon_j = \upsilon \exp\left[i2\pi \sum_{k=1}^{j-1} \frac{q_k}{2e} \right]$$



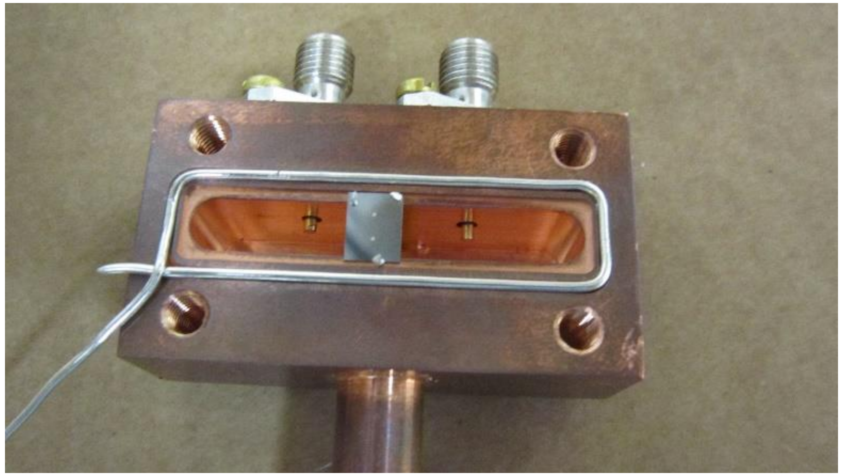


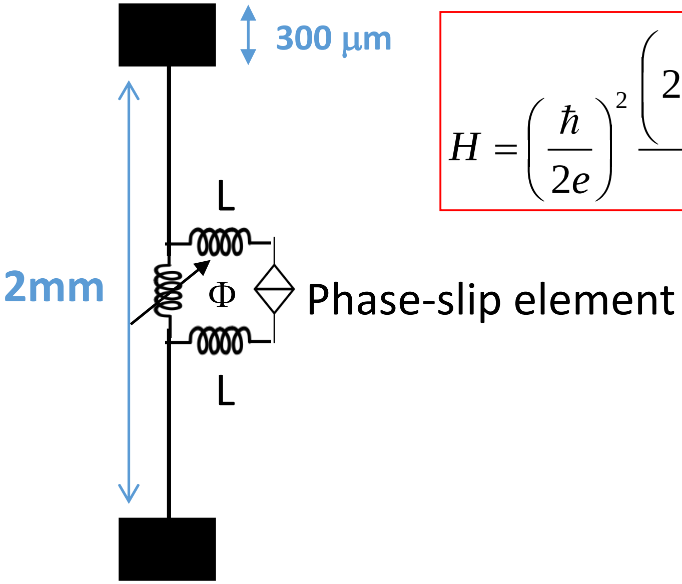
Dual to Aharonov-Bohm effect

I. Pop et al, Nature Physics, Vol 6, 591, (2010)

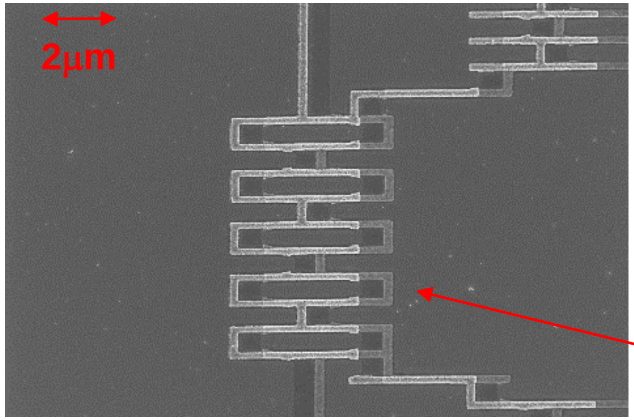
I. Pop et al, Phys. Rev. B (2012)

Fluxonium qubit

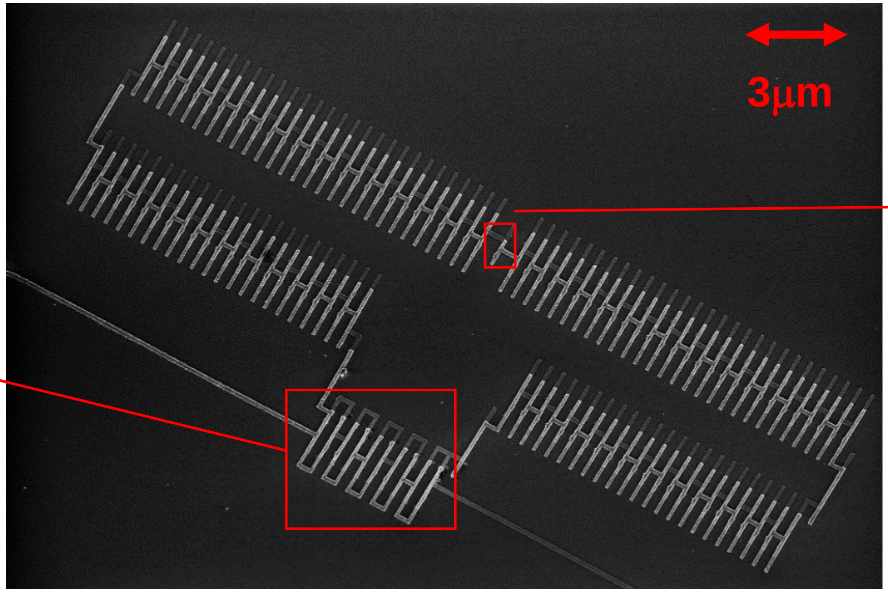


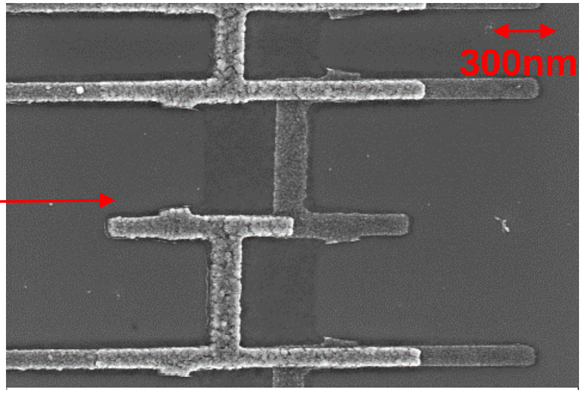


$$H = \left(\frac{\hbar}{2e}\right)^2 \frac{\left(2\pi\frac{\Phi}{\Phi_0} - 2\pi m\right)^2}{2L} - \sum_m \upsilon_{qps}^{tot}\left(|m+1\rangle\langle m| + |m\rangle\langle m+1|\right)$$



SQUID antenna junctions

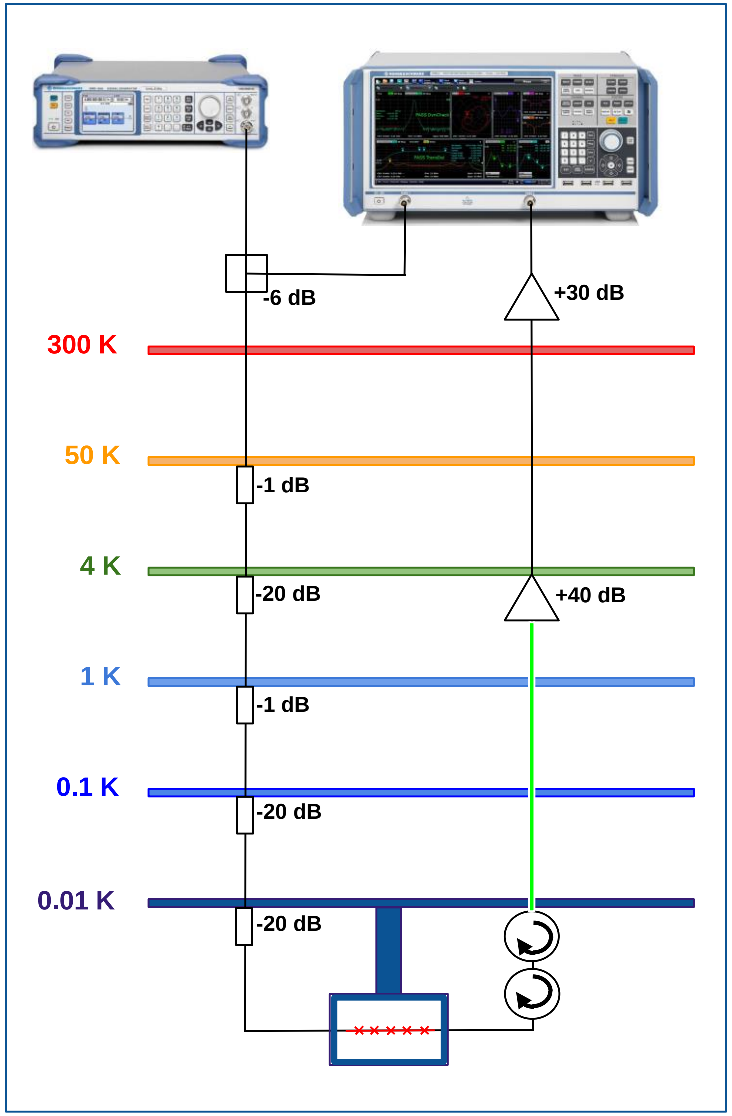


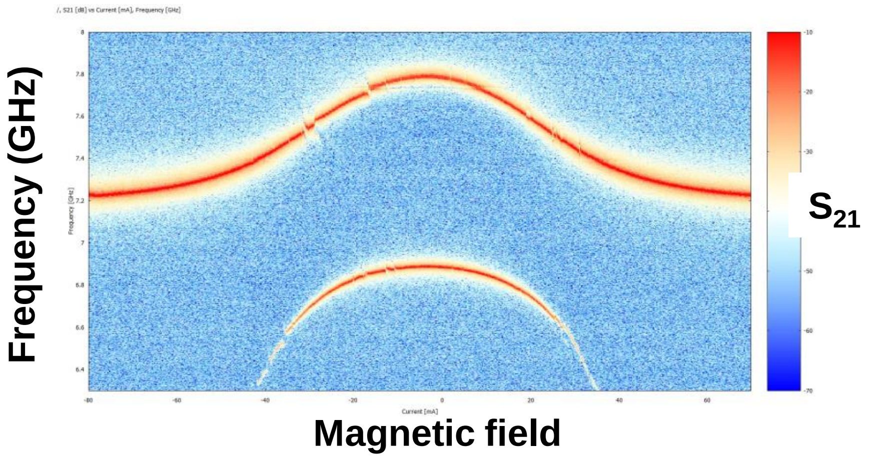


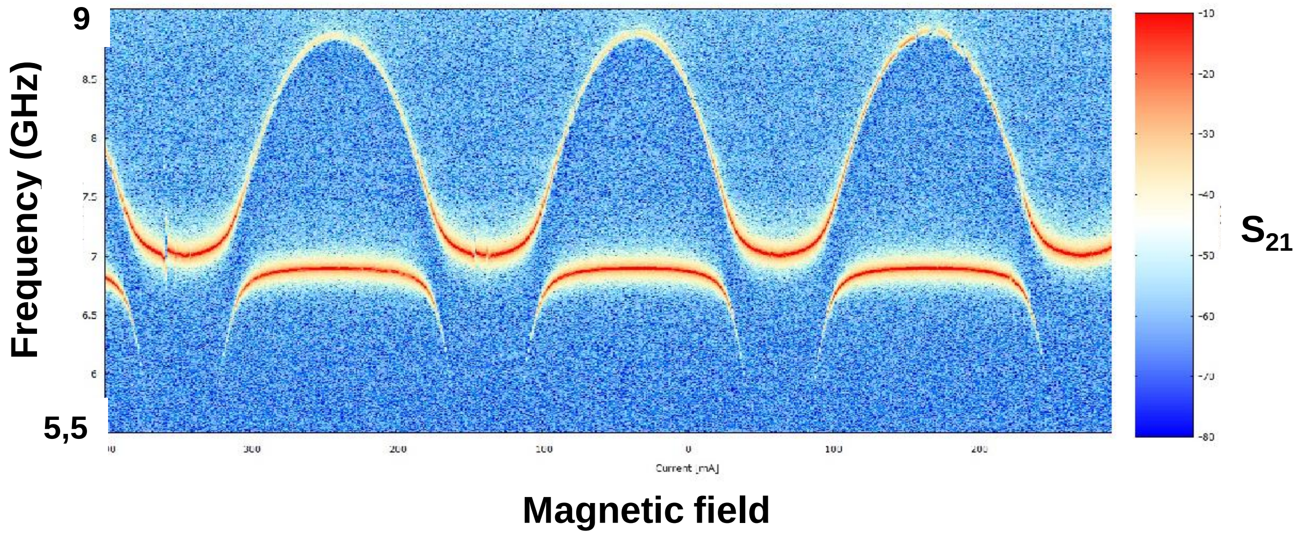
Small junction with Quantum phase-slips

I.M. Pop et al.,Nature 508, 369–372 (2014)

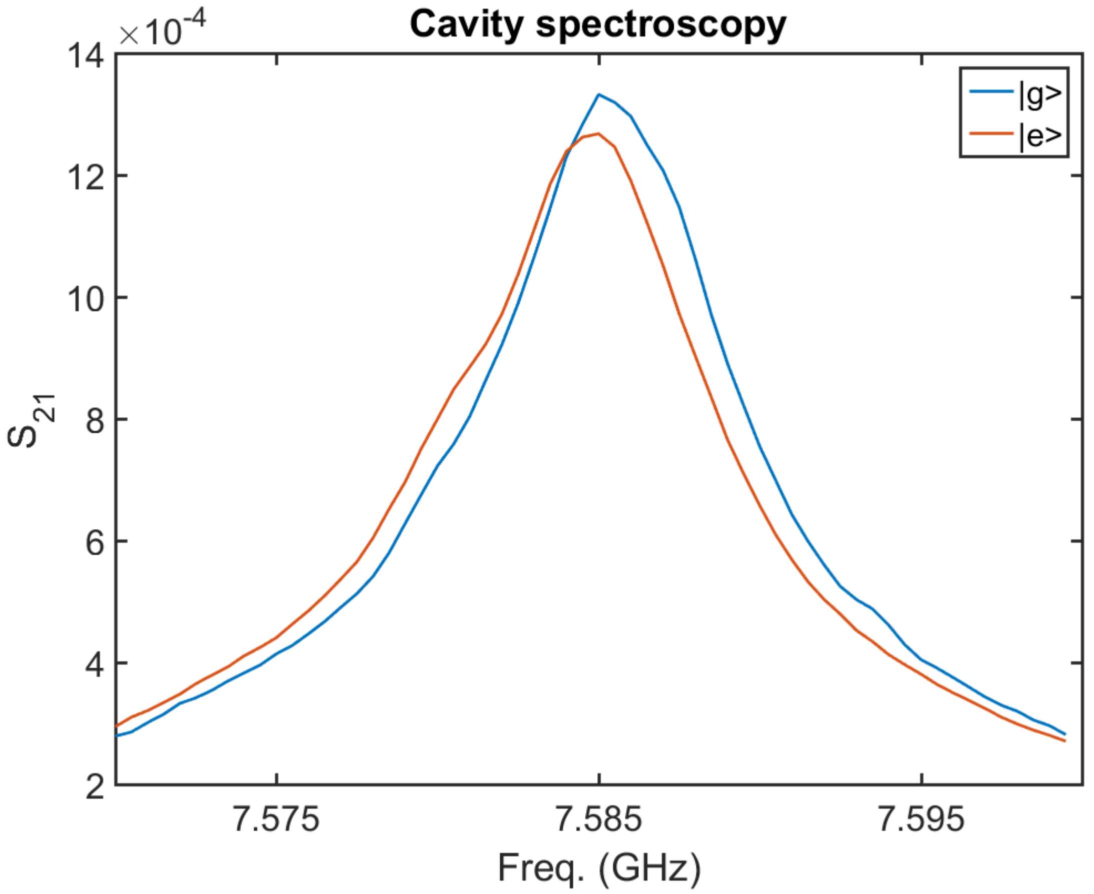
Spectroscopy measurements with VNA



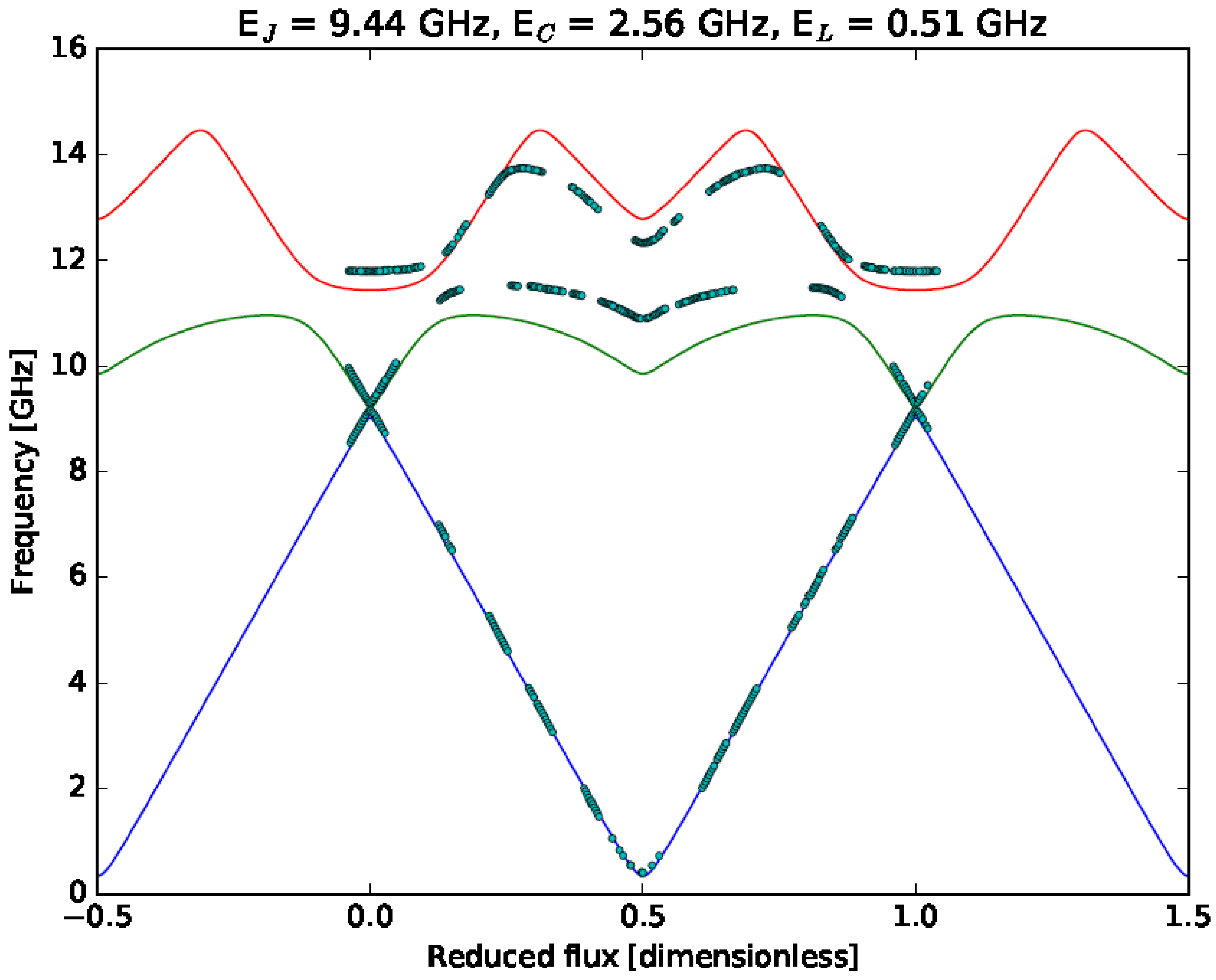




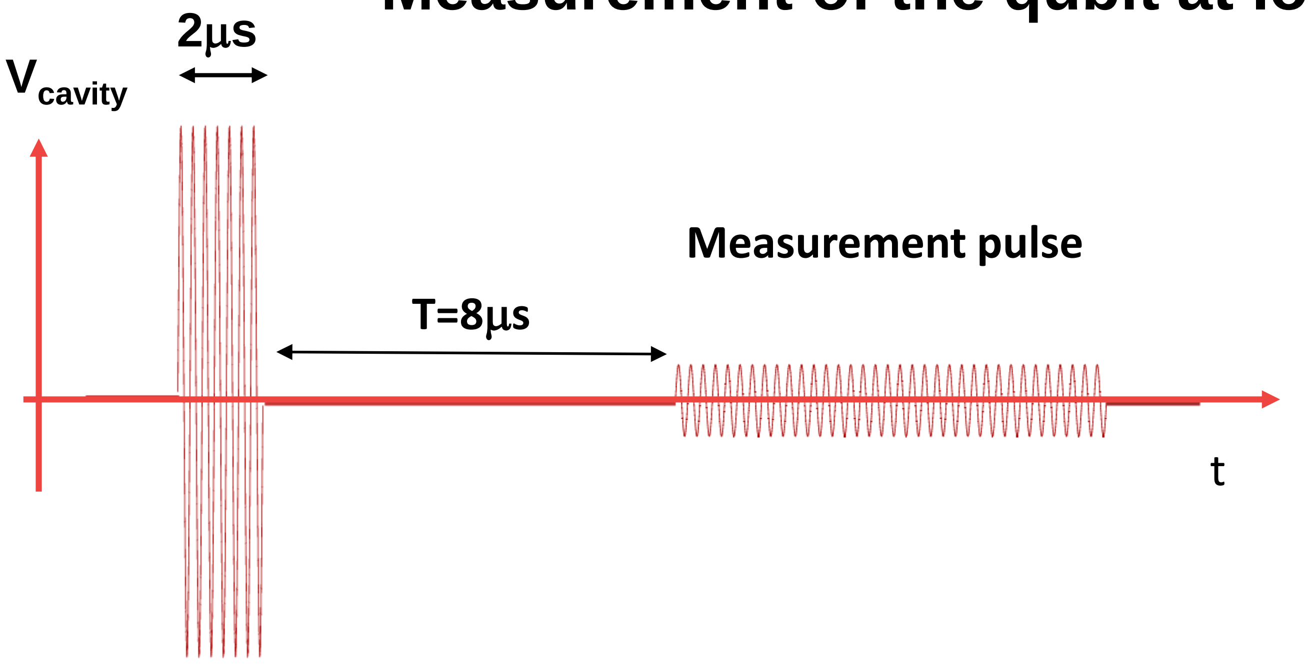
Measurement of the energy spectrum of the qubit

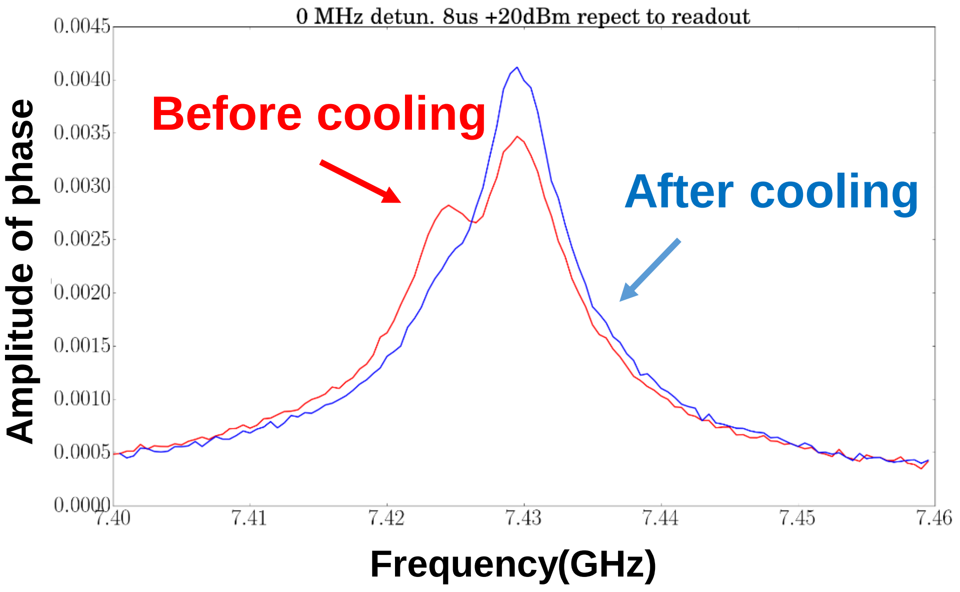


Energy spectrum of the qubit as a function of flux

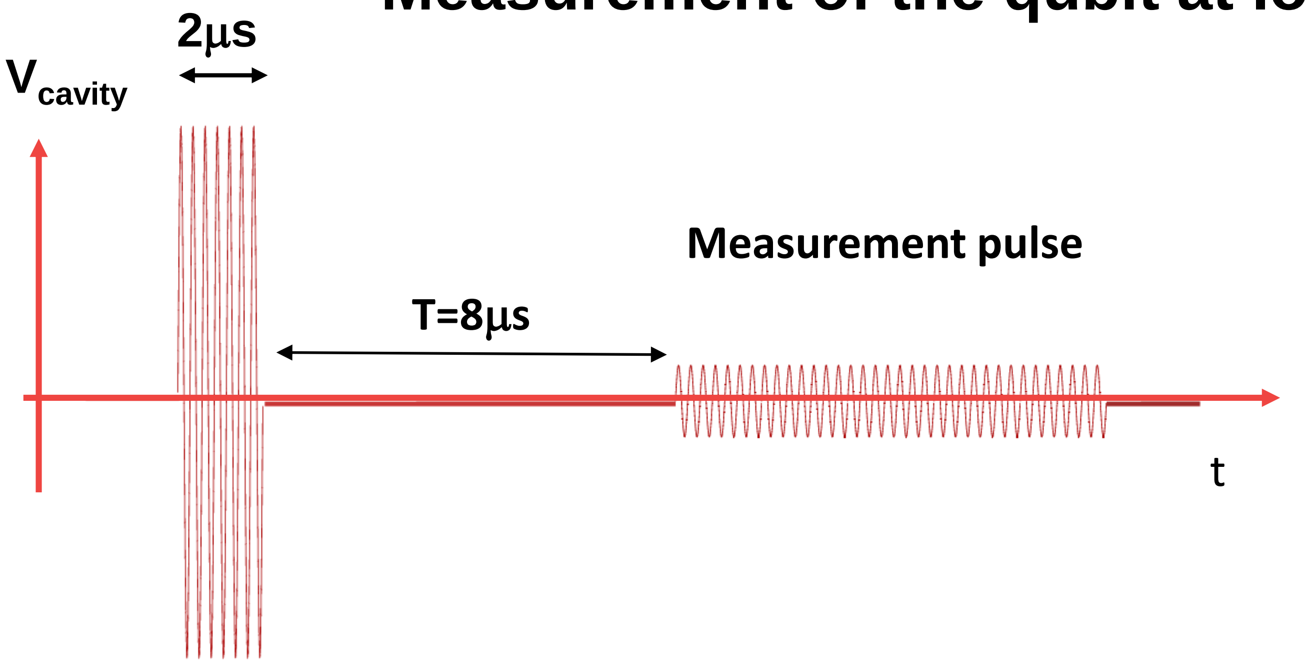


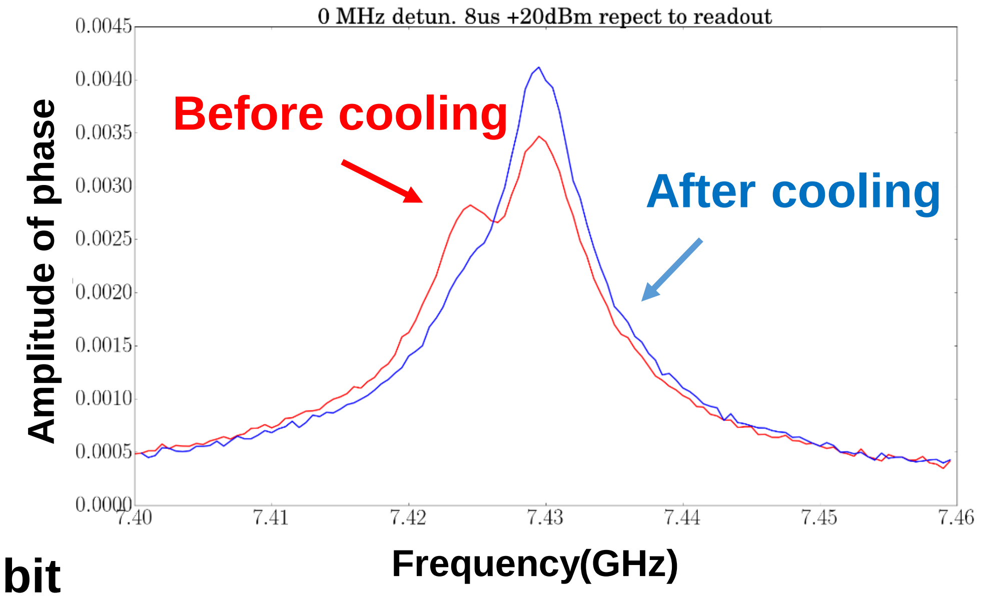
Measurement of the qubit at low frequency:cooling pulses



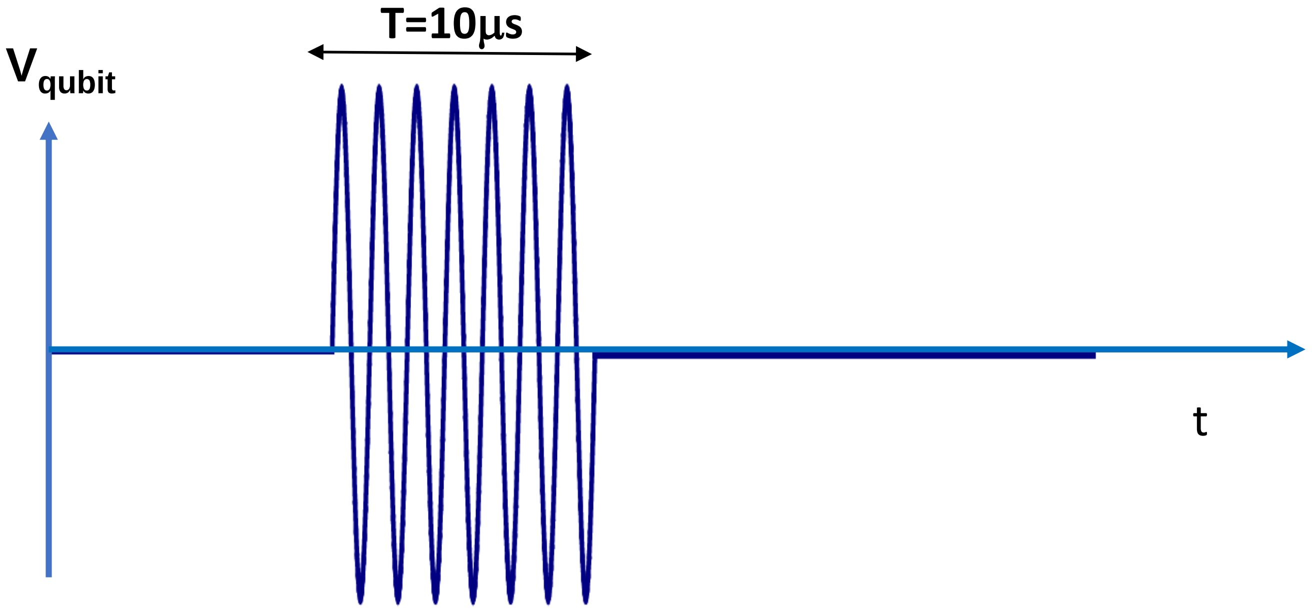


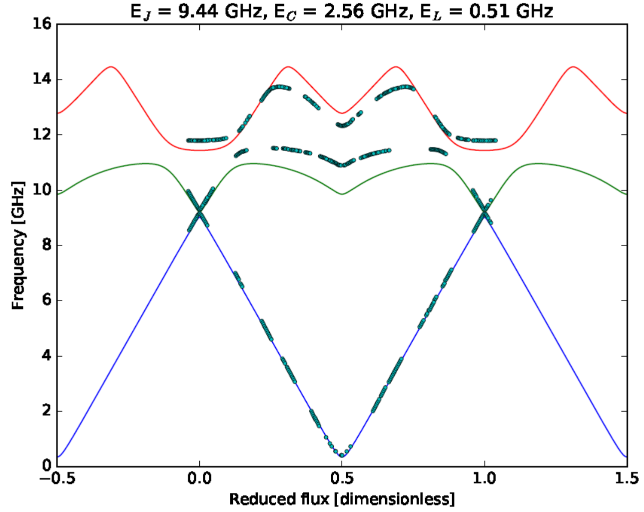
Measurement of the qubit at low frequency:cooling pulses





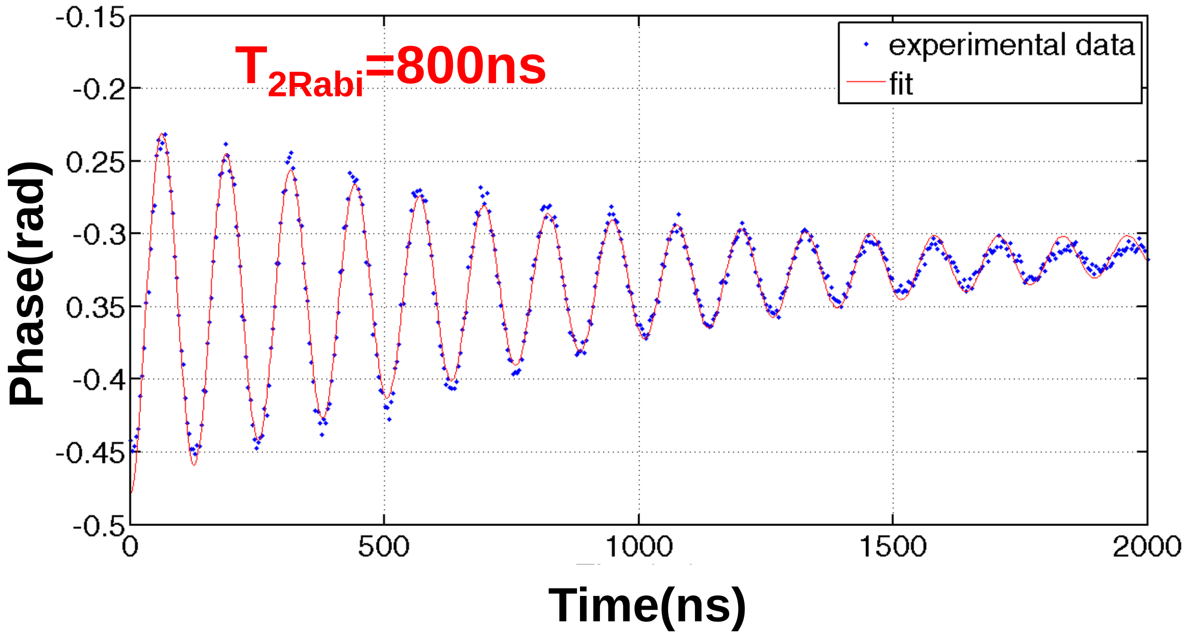
For spectroscopy measurements add manipulation pulse on qubit



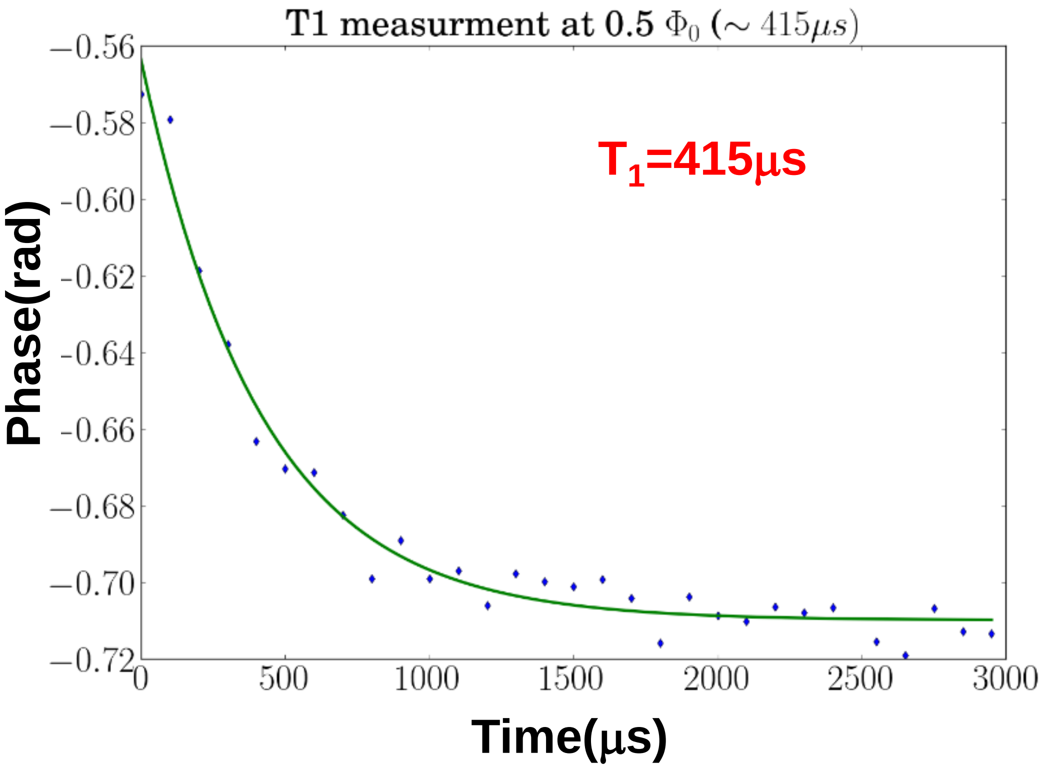


Time dependant measurements

Measurement of Rabi-oscillations at f_{qubit}=2.8GHz

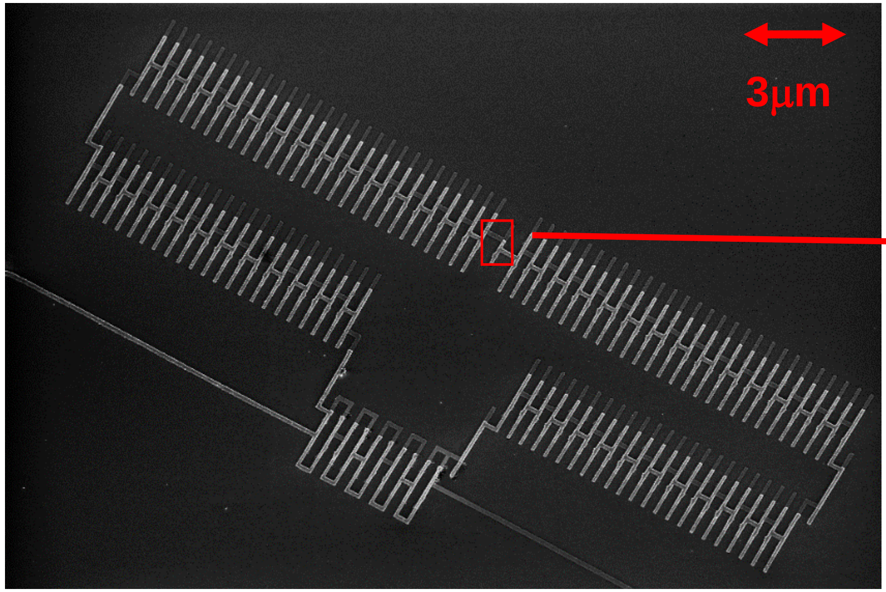


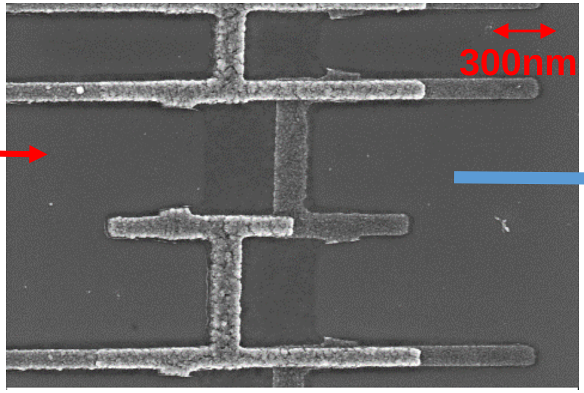
Measurement of relaxation time at $\Phi=\Phi_0/2$



Future experiments

1) Measurement of off-set charge dynamics on coherent quantum phase-slips in a Josephson junction chain





Small junction

Replace single small junction by a chain of small junctions

→ **Study of the coherence of Quantum phase-slips in a Josephson junction chain**

2) Measurement of interaction between chain modes and qubit degrees of freedom

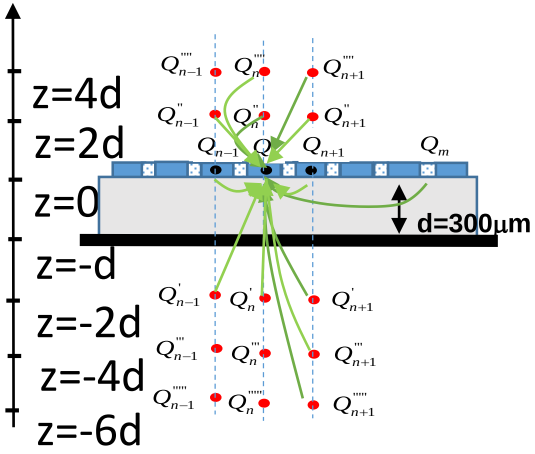
→ **Increase the number of junctions of the inductive chain**

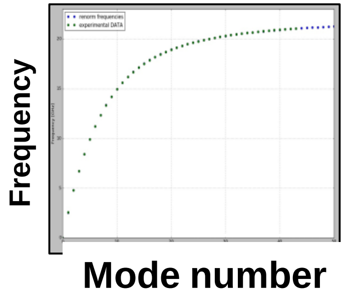
→ **Measurement of revival-effects in the coherent oscillations of the qubit**

G. Rastelli et al, New J. Phys. 17 (2015) 053026
G. Viola and G. Catelani, Phys. Rev. B 92,224511, (2015)

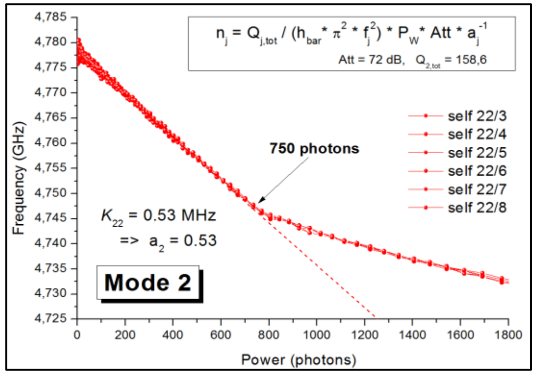
Summary

1) **Dispersion of propagating modes in a Josephson junction chain (Remote ground model)**

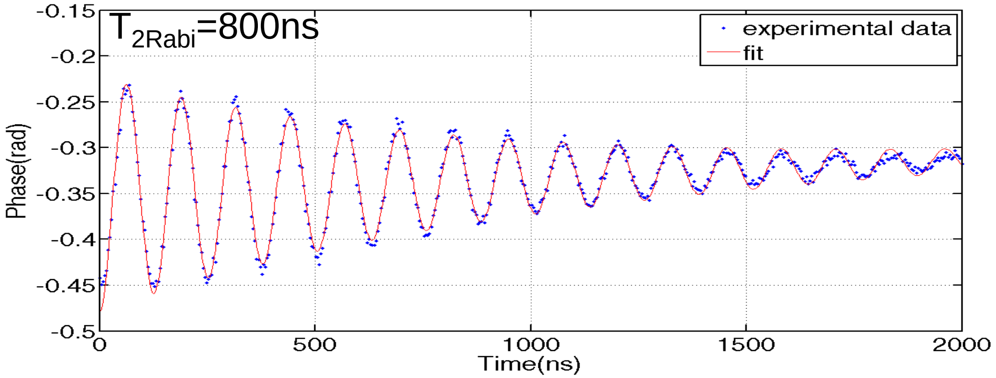




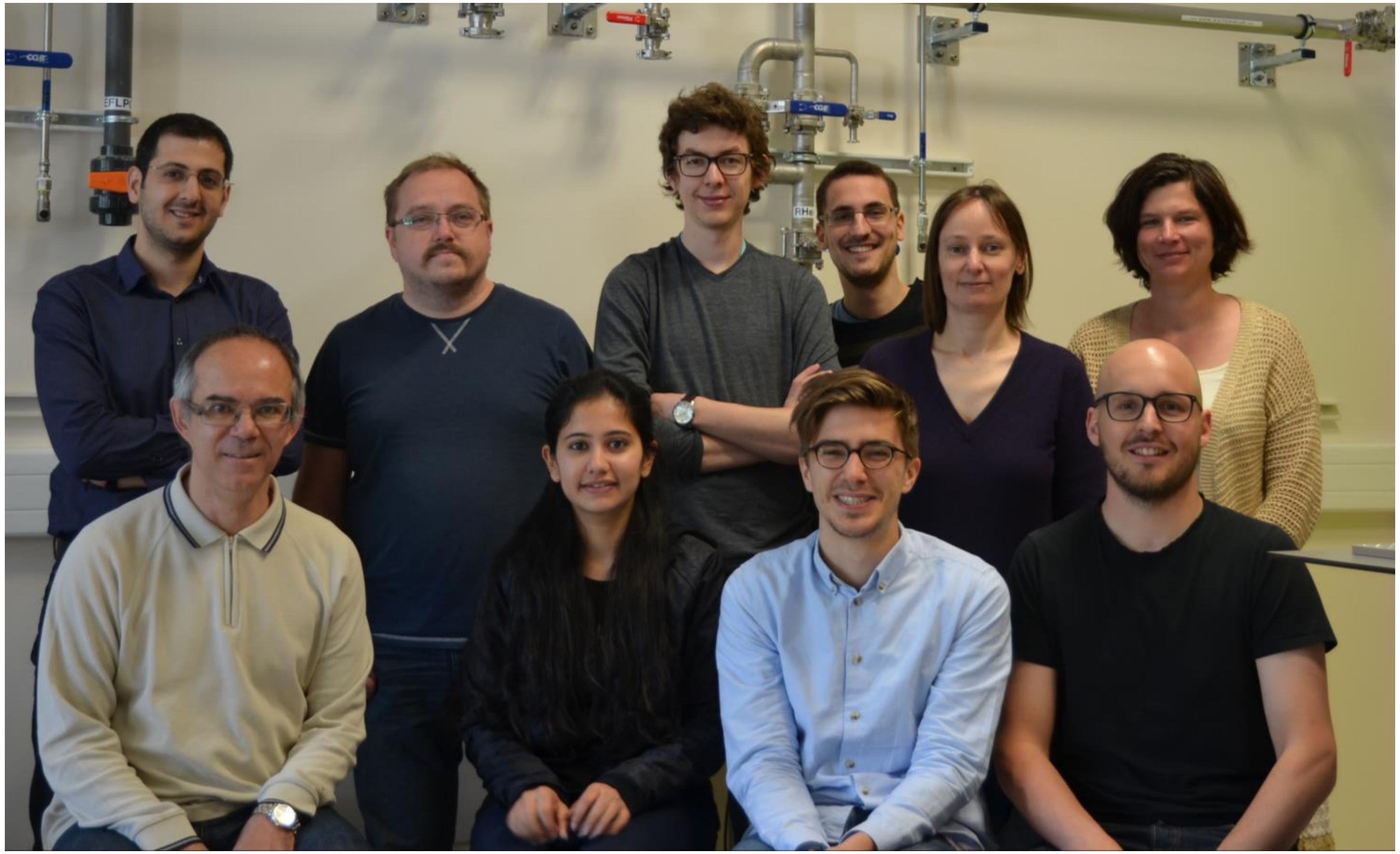
2) **Study of Self-and Cross Kerr effects: Fairely good agreement between theory and experiment**



3) **Quantum phase-slips**
   - **Fluxonium**



Superconducting quantum circuits team at Neel Institute



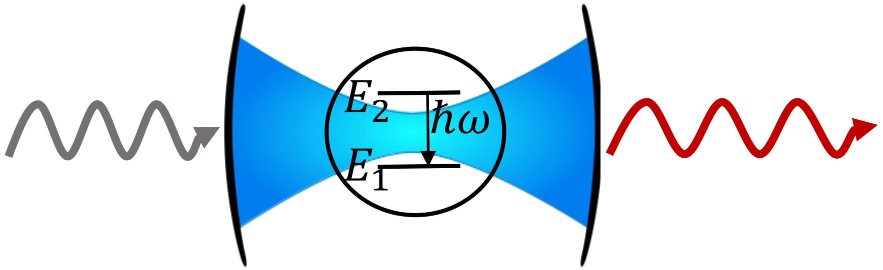
Current group members:

Permanent: Olivier Buisson, Wiebke Guichard, Cécile Naud, Nicolas Roch

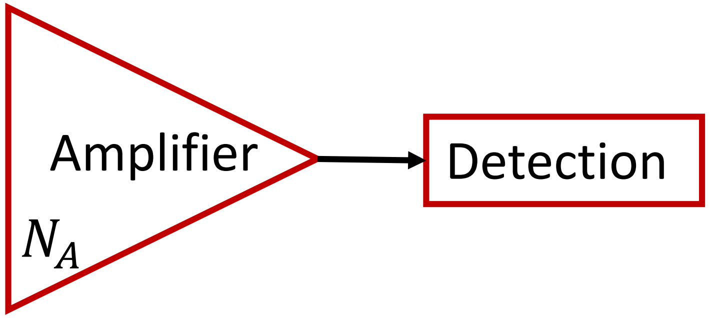
PhD and postdocs: Rémy Dassonneville, Farshad Foroughi, Yuriy Krupko, Luca Planat, Javier Puertas-Martinez

Université Grenoble Alpes, Ville de Grenoble, erc FREQUJOC, CNRS, Néel institut, Agence Nationale de la Recherche ANR

Amplification of a single photon



$k_B T \ll \hbar\omega$



N_A: Added noise

Commercial amplifier: $N_A \approx \mathbf{10\hbar\omega}$
Experimental signal $\approx 1\,\boldsymbol{\hbar\omega}$

Realisation of an amplifier working at the quantum limit of noise: **$N_A = \frac{1}{2}\hbar\omega$**

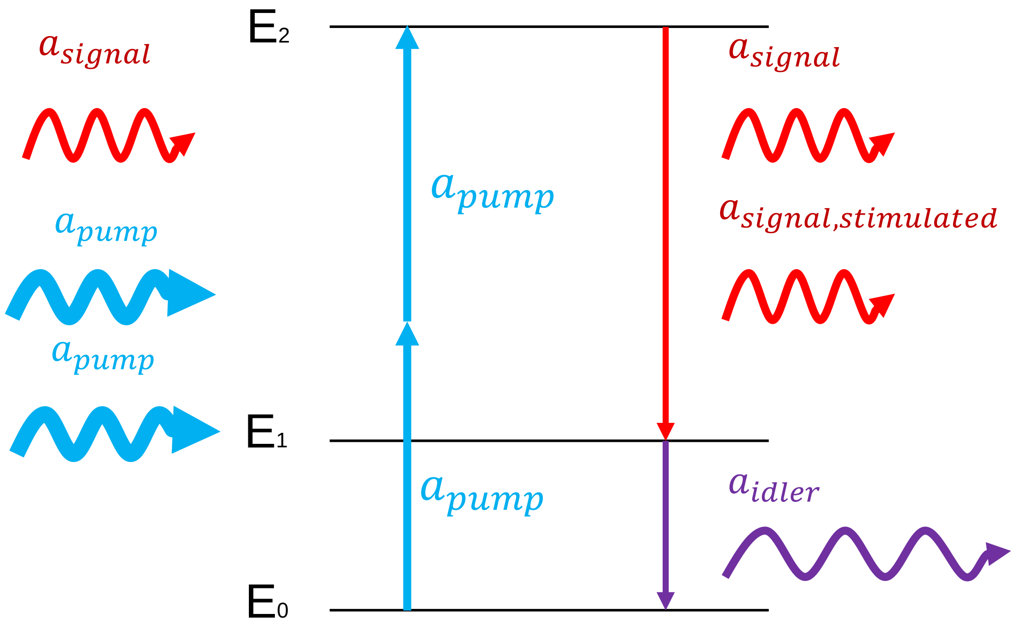
Principal of amplification due to the non-linearity of the Josephson effect

$$\hat{H} = \hbar\omega_p \hat{a}^+ \hat{a} - \frac{\hbar}{2} K \hat{a}^+_{signal} \hat{a}_{pump} \hat{a}^+_{idler} \hat{a}_{pump} + \cdots$$

$$\omega_p = \frac{1}{\sqrt{L_J C}}$$ **Plasma frequency of Josephson junction**

Energy conservation:

$$2\omega_{pump} = \omega_{signal} + \omega_{idler}$$



Principal of amplification due to the non-linearity of the Josephson effect

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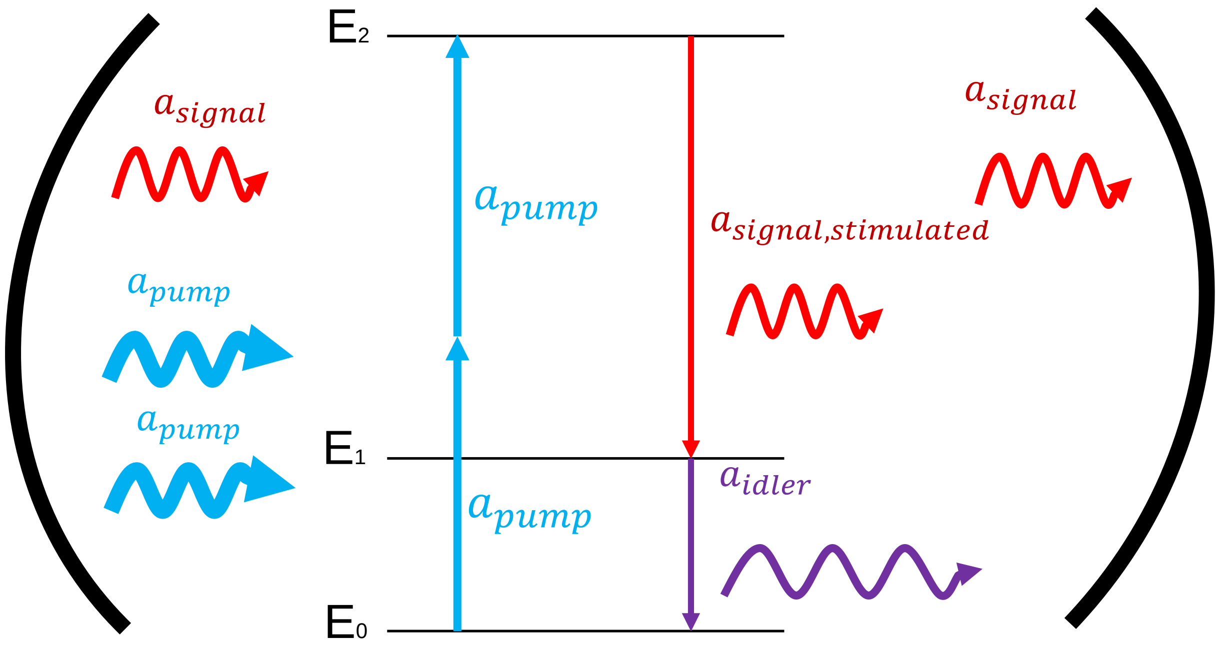
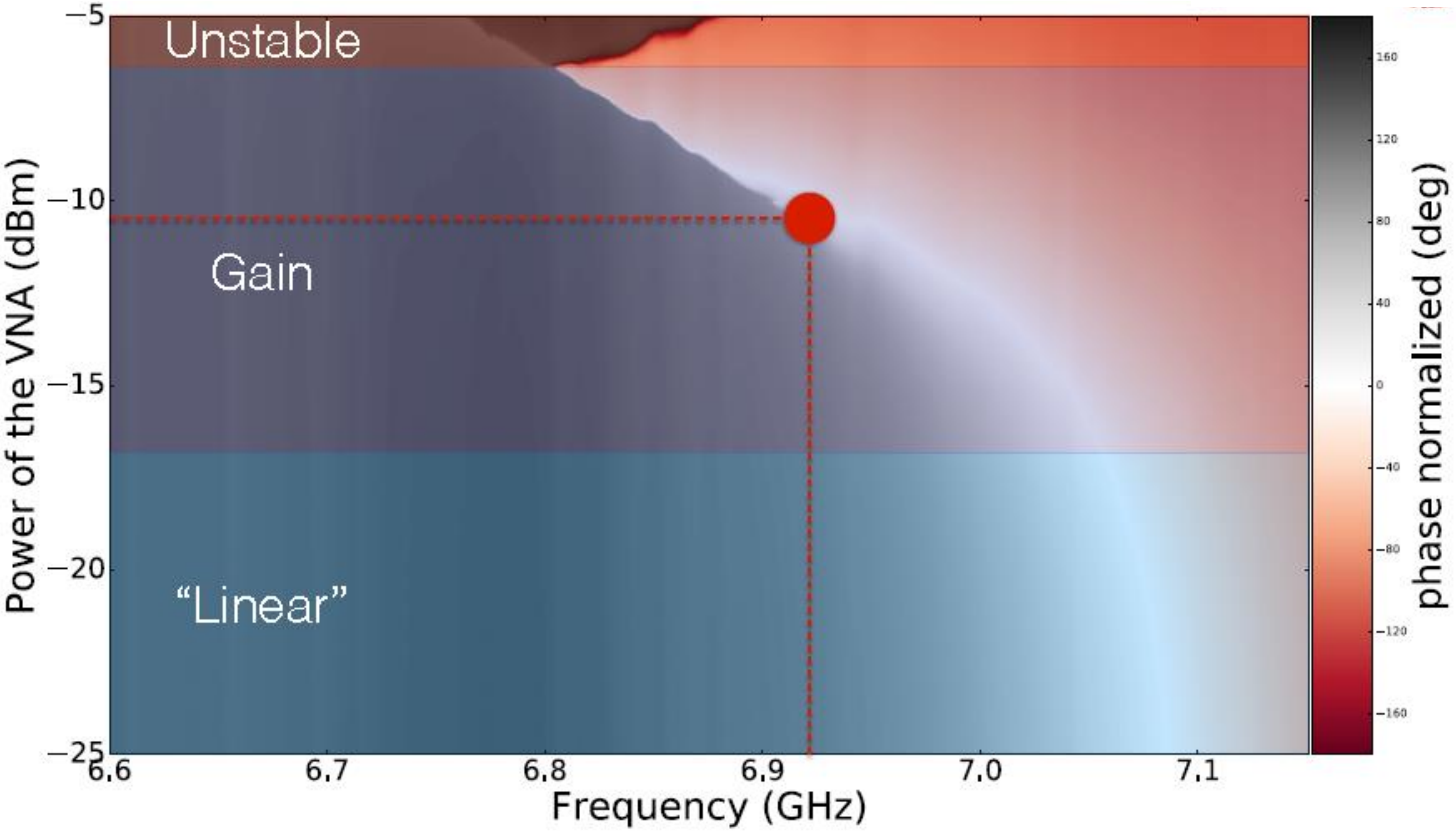
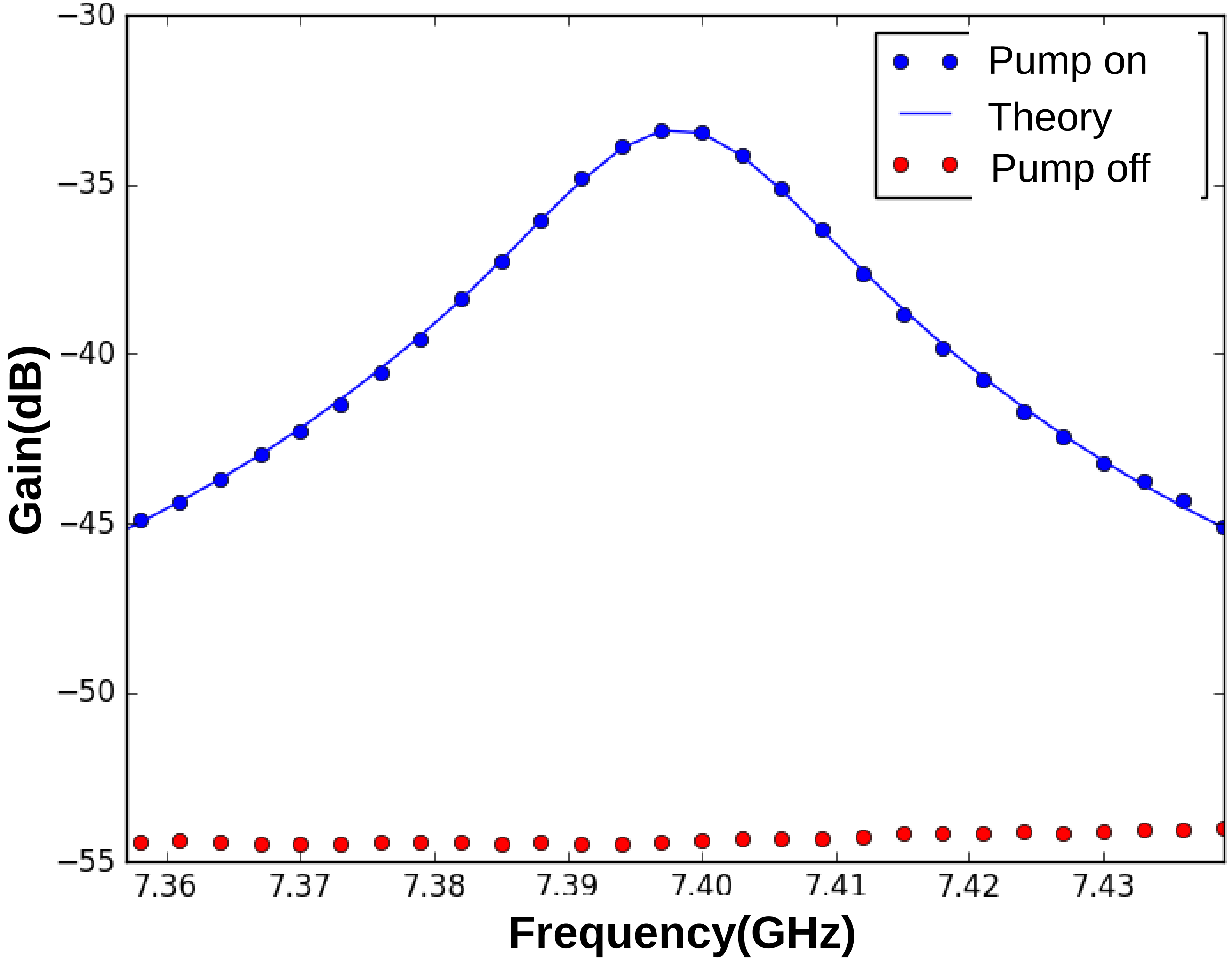


Stimulated emission of a photon amplified in a cavity

Experimental characterisation of the non-linearity



Experimental results of amplification



Figures of merit:

G_{max}=20 dB

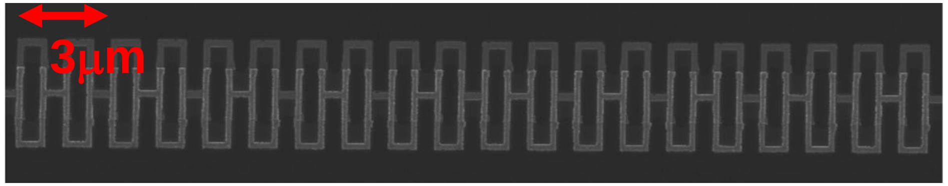
Δf=20 MHz

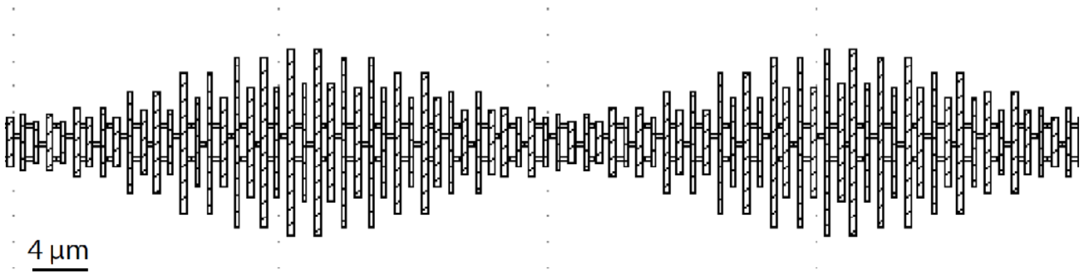
1 dB compression point: P_{sat}=-128dBm

Future Developments

Traveling Wave Parametric amplifie (TWPA) with large band width at the quantum limit of noise

Engineering of the dispersion relation of a Josephson junction chain acting as a metamaterial





Homogeneous chain

Chain where the size of the junctions is modulated

Kerr-effect

$$\eta_{jjkk} = \sum_n \left[\left(\sum_m \left(\sqrt{C}\hat{C}_{n,m}^{-1/2} - \sqrt{C}\hat{C}_{n-1,m}^{-1/2} \right) \psi_{m,j} \right)^2 \cdot \left(\sum_m \left(\sqrt{C}\hat{C}_{n,m}^{-1/2} - \sqrt{C}\hat{C}_{n-1,m}^{-1/2} \right) \psi_{m,k} \right)^2 \right]$$

Dispersion: Comparison between theory and experiment for remote ground model

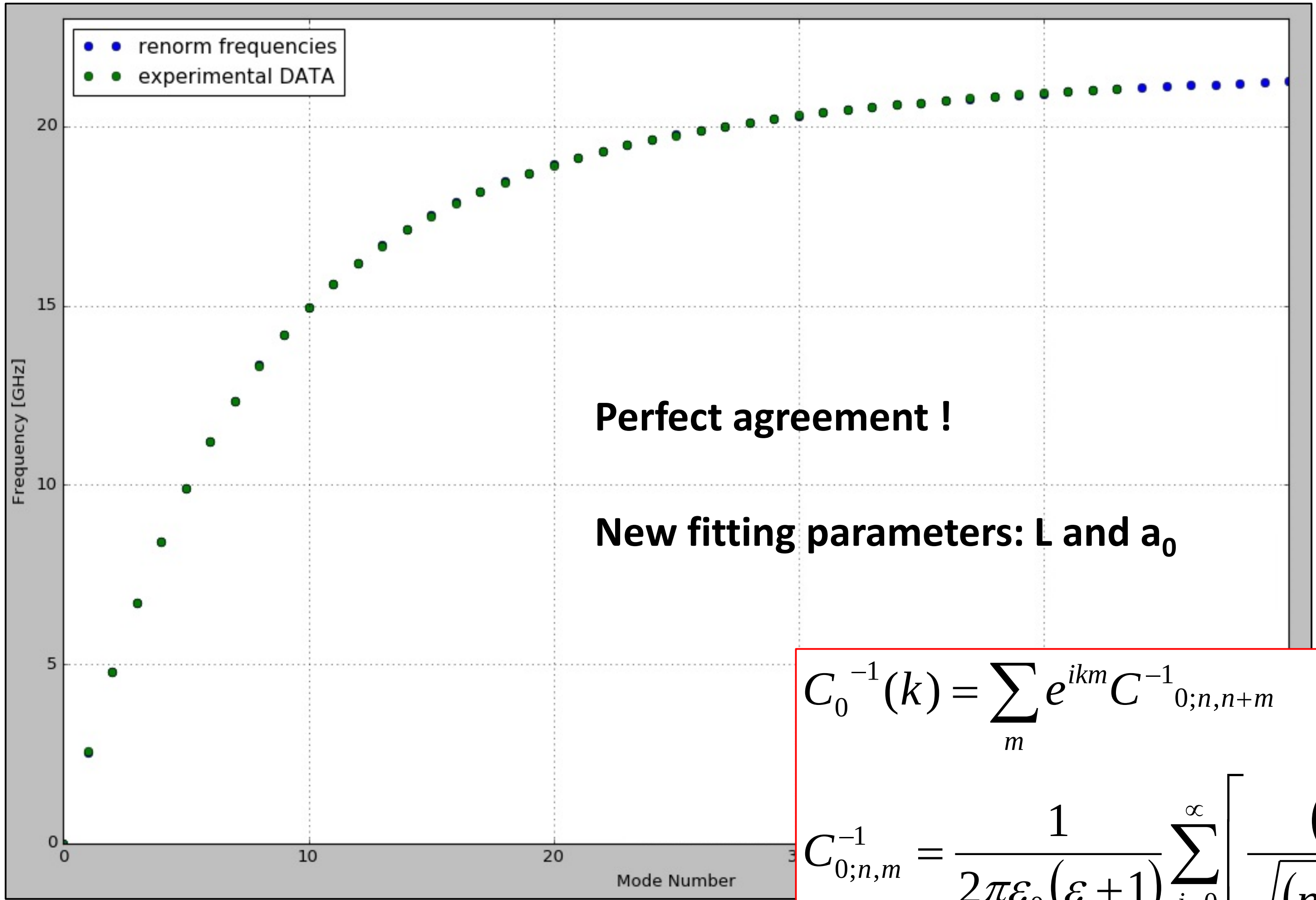
$$Q_n = C(V_n - V_{n-1}) + C(V_n - V_{n+1}) + \tilde{Q}_n$$

Standard model: $$\tilde{Q}_n = C_g V_n$$

Remote ground model

$$V_n = \sum_{m=1} \tilde{Q}_m \frac{1}{2\pi\varepsilon_0(\varepsilon+1)} \sum_{j=0}^{\infty} \left[\frac{((1-\varepsilon)/(1+\varepsilon))^j}{\sqrt{(n-m)^2 a^2 + (2jd - a_0)^2}} - \frac{((1-\varepsilon)/(1+\varepsilon))^j}{\sqrt{(n-m)^2 a^2 + (2j+2)^2 d^2}} \right]$$

Dispersion: Comparison between theory and experiment for remote ground model



Perfect agreement !

New fitting parameters: L and a_0

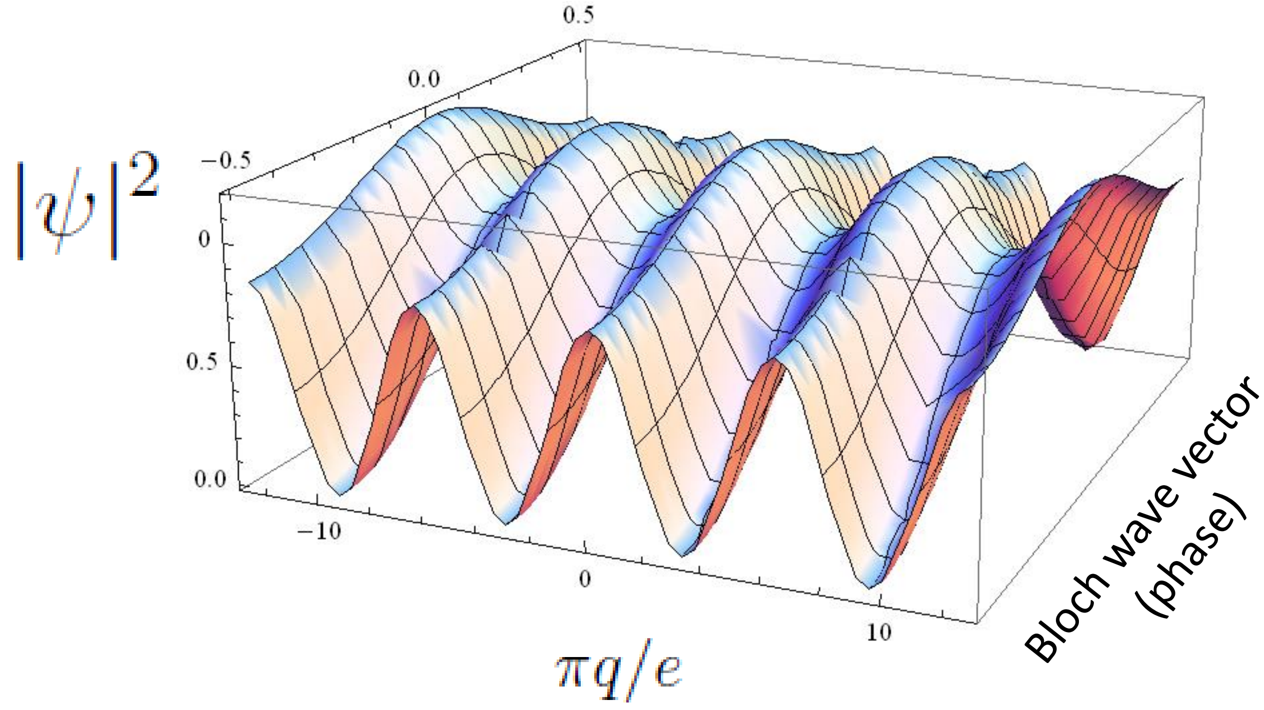
$$\hat{C}^{-1/2}\hat{L}^{-1}\hat{C}^{-1/2}\vec{\psi}_k = \omega_k^2\vec{\psi}_k$$

$$\omega_k = \omega_p\sqrt{\frac{1-\cos k}{1-\cos k + \frac{C_0(k)}{2C}}}$$

$$C_0^{\ -1}(k) = \sum_m e^{ikm} C^{-1}{}_{0;n,n+m}$$

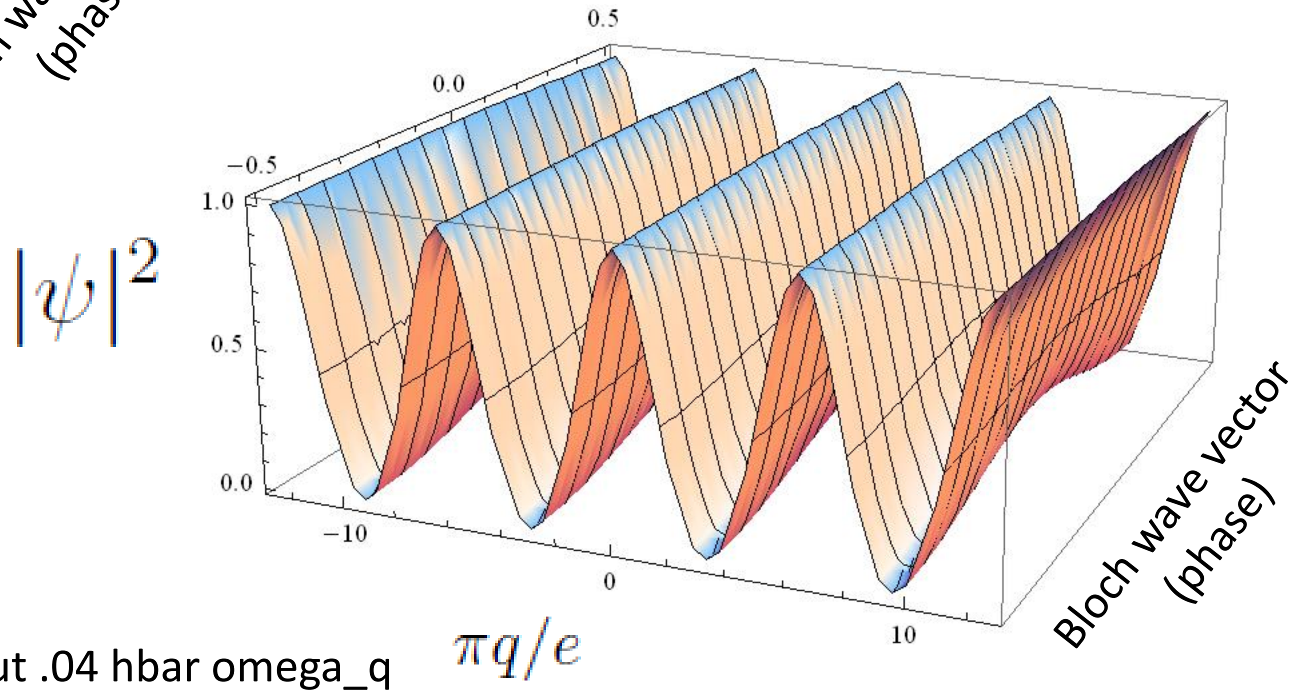
$$C_{0;n,m}^{-1} = \frac{1}{2\pi\varepsilon_0(\varepsilon+1)}\sum_{j=0}^{\infty}\left[\frac{\left((1-\varepsilon)/(1+\varepsilon)\right)^j}{\sqrt{(n-m)^2+(2jd+a)^2}} - \frac{\left((1-\varepsilon)/(1+\varepsilon)\right)^j}{\sqrt{(n-m)^2+((2j+2)d+a)^2}}\right]$$

L = 300 nH, C = 7 fF, hence rho_q = 0.25 (from Thomas)



Bandwidth is about .14 hbar omega_q

Rho_q = 0.5 (from figure 1b)



Bandwidth is about .04 hbar omega_q