Squeezing by a quantum conductor

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GDR Mesoscopic Quantum Physics, Aussois, 2016
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Parametric excitation of a swing

- Child stands up at zero angle and squats at maximum angle
- Frequency of pumping is twice the oscillator frequency \( \omega_P = 2\omega_0 \)
- In-phase noise is amplified, out-of-phase noise is reduced

\[ \Rightarrow \text{Squeezing of noise} \]
Observation of Squeezing in the Electron Quantum Shot Noise of a Tunnel Junction

Gabriel Gasse, Christian Lupien, and Bertrand Reulet
Département de Physique, Université de Sherbrooke, Sherbrooke, Québec, Canada J1K 2R1
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We report the measurement of the fluctuations of the two quadratures of the electromagnetic field generated by a quantum conductor, a dc- and ac-biased tunnel junction placed at very low temperature. We observe that the variance of the fluctuations on one quadrature can go below that of vacuum, i.e., that the radiated field is squeezed. This demonstrates the quantum nature of the radiated electromagnetic field.

Radiation of a tunnel junction
I. Squeezed light with tunnel junction

II. Dynamical Coulomb Blockade
Squeezed light with tunnel junction
Hamiltonian vs dissipative squeezing

- **Hamiltonian squeezing**

\[
\frac{H}{\hbar \omega} = \frac{X^2}{2} + \left[1 + \varepsilon \sin(2\omega t)\right] \frac{P^2}{2} \approx a^+ a - \frac{i \varepsilon}{4} \left(a^2 e^{2i\omega t} - h.c.\right)
\]

- **Example:** Josephson parametric Amplifier (JPA), limited to half-squeezing for cavity mode

\[
\Delta X^2 \geq 1/2
\]
Differential capacitance

Josephson parametric amplifier (JPA)

Mallet et al., PRL 2011
Hamiltonian vs dissipative squeezing

**Hamiltonian squeezing**

\[
\frac{H}{\hbar\omega} = \frac{X^2}{2} + \left[1 + \epsilon \sin(2\omega t)\right]\frac{P^2}{2} \approx a^+ a - \frac{i \epsilon}{4} \left( a^2 e^{2i\omega t} - h.c. \right)
\]

Example: Josephson parametric Amplifier (JPA), limited to half-squeezing for cavity mode

\[
\Delta X^2 \geq 1/2
\]

**Dissipative squeezing**

Environment noise engineered to squeeze

Example: time-dependent damping rate

\[
\lambda(t) = \sqrt{\kappa_c} \left( 1 + \lambda_1 e^{2i\omega t} \right)
\]

\[
\Delta X^2 = \frac{1 - \lambda_1}{1 + \lambda_1}
\]

Kronwald, Marquardt, Clerk, PRA 2014
Didier, Qassemi, Blais, PRA 2014
Coupling electrons and photons

- Hamiltonian for tunneling

\[ H_T = \sum_{k,q} t \, c_{L,k}^+ c_{R,q} \Lambda + h.c. \]

\[ \Lambda = e^{i\Phi} \quad [\Lambda, Q] = Q \]

\( \Lambda \) transfers one electronic charge through the junction

- Gauge transform (cavity)

\[ \Phi = i\lambda(a^+ - a) \]

\[ H_T = \sum_{k,q} t \, c_{L,k}^+ c_{R,q} + h.c. + H_C \]

\[ H_C = (a + a^+)\left(\lambda_L \hat{N}_L + \lambda_R \hat{N}_R\right) \]

Gives a capacitive coupling

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Udson, Mora PRB 2016  Dmytruk, Trif, Mora, Simon, PRB 2015
(LC) Harmonic oscillator

\[
\Phi = i\lambda(a^+ - a)
\]

Derivation of a Quantum Langevin equation \((a, I)\) are quantum operators

\[
\dot{a} + i\omega_0a + \frac{\kappa}{2}a = \lambda I
\]

\[
\kappa = \lambda^2[S_0(\omega_0) - S_0(-\omega_0)]
\]

Fluctuation-dissipation theorem

The noise of the current \(I\) both damps and excites the LC resonator
Equilibrium noise

\[ \dot{a} + i \omega_0 a + \frac{\kappa}{2} a = \lambda I \]

\[ \langle I(\omega_1)I(\omega_2) \rangle = S_0(\omega_1) 2\pi \delta(\omega_1 + \omega_2) \]

\( S_0 \) is absorption noise for positive frequency and emission noise for negative.

At equilibrium (no voltage)

\[ S_{0eq}(\omega) = \frac{2}{R_T} \frac{\omega}{1 - e^{-\beta \hbar \omega}} \]

Solution of the Langevin equation

\[ \langle a^+ a \rangle = \frac{\lambda^2 S_0(-\omega_0)}{\kappa} \]
Out-of-equilibrium noise

\[ \dot{a} + i \omega_0 a + \frac{\kappa}{2} a = \lambda I \]

\[
\langle I(\omega_1)I(\omega_2) \rangle = \sum_n S_n(\omega_1) 2\pi \delta(\omega_1 + \omega_2 - 2n\omega_0)
\]

Quadrature Squeezing

\[
\langle a a \rangle = \frac{\lambda^2 S_1(\omega_0)}{\kappa}
\]

Non-stationary terms
Optimizing squeezing

Squeezing depends on photo-assisted properties by the ac modulation or on emission/absorption probabilities $c_n$

$$X_1 = i(a^+ - a)$$

**Single tone**

$$\Delta X_1^2 = 0.618$$

$$\frac{eV_{AC}}{2\hbar \omega_0} = 0.706$$

$$eV = \hbar \omega_0$$

Full agreement with Reulet’s experiment

**With harmonics**

![Graph showing comparison between single tone and harmonics]
Optimal squeezing for a tunnel junction

Optimal squeezing is reached with the pulse shape

\[ V_{opt}(t) = \frac{\hbar}{2e} \sum_{l} \delta \left( t - \frac{l \pi}{\omega_0} \right) \]

\[ \Delta X_1^2 = \frac{4}{\pi^2} = 0.405 \]
Dynamical Coulomb blockade
Input-output formalism describes how the conductor interacts with its electromagnetic circuit environment.

Photon radiation in the transmission line (impedances are not matched)

\[ P(t) = P_{out}(t) - P_{in}(t) = \frac{(1 + n_B(\omega_0))S_I(\omega_0) - n_B(\omega_0)S_I(-\omega_0)}{2C} \]

Lesovik, Loosen, JETP 1997
Dynamical Coulomb blockade: transferred electric charge may excite environmental modes: reduction of current due to elastic processes.
Dynamical Coulomb blockade: transferred electric charge may excite environmental modes: reduction of current due to inelastic processes.

\[ \Lambda = e^{i\Phi} \quad [\Lambda, Q] = Q \]

Probability not to excite environment:

\[ \left| \langle 0 | \Lambda | 0 \rangle \right|^2 \leq 1 \]

| 0 \rangle \quad \text{Ground state (electromagnetic circuit)}
Dynamical Blockade for a tunnel junction

Dynamical Coulomb blockade: transferred electric charge may excite environmental modes: reduction of current because of inelastic processes.

\[ q = -e \quad q = e \]

\[ \Lambda = e^{i\Phi} \quad [\Lambda, Q] = Q \]

Probability not to excite environment:

\[ \left| \langle 0 | \Lambda | 0 \rangle \right|^2 \leq 1 \]

Ground state (electromagnetic circuit):

Inelastic transport encoded in \( P(E) \) function:

\[ Z / \left( \hbar / e^2 \right) \]

\[ E_C = e^2 / 2C \]
We propose a circuit where dynamical Coulomb blockade and squeezed radiation readout are spatially separated.

Impedance matching: \( R_T Z_I = Z_{LC}^2 \)

\[
a_{out,\omega} = \frac{\delta \omega}{\delta \omega + i \kappa} a_{in,\omega} - \sqrt{\frac{R_T}{2\hbar \omega_0}} \frac{i \kappa}{\delta \omega + i \kappa} \hat{I}_\omega
\]
Squeezed radiation from Dynamical Coulomb

Strong DCB, reflected in the noise properties of the junction

With DCB

\[
\begin{align*}
(\omega + i\kappa_+)a_{out,\omega+\omega_0} &= (\omega + i\kappa_-)a_{in,\omega+\omega_0} \\
- i\sqrt{\frac{\omega_0 Z_l}{2\hbar}} \hat{I}_{\omega+\omega_0} - \frac{i Y_1}{2C} (a_{out,\omega_0-\omega} - a_{in,\omega_0-\omega})
\end{align*}
\]

\[
2\kappa_{+/-} = \frac{Y_0}{C} \pm \frac{Z_l}{L} \\
Y_{0/1} \propto S_{0/1}(\omega_0) - S_{0/1}(-\omega_0)
\]
Squeezed radiation from Dynamical Coulomb

- Important **temperature corrections**
- Reflection coefficient is effectively strongly modulated

\[ r = \frac{Z_l - Z_{sys}}{Z_l + Z_{sys}} \]
Conclusions

- A tunnel junction is in principle able to produce squeezed light in a resonator

- Squeezing is improved with concentrated pulses of voltage

- Non-linearities in a tunnel junction under strong Coulomb blockade could be used to achieve a competitive squeezed radiation