Cavity quantum electrodynamics with carbon nanotubes: from atomic-like systems to condensed matter

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Theory: A. Cottet, M.-S. Choi, M. Lee, B. Douçot

Atomic Cavity QED

\[ \hat{H}_{eff} = \hbar \nu_{01} \hat{S}_z / 2 + \hbar \omega_r a^\dagger a + \hbar g (a^\dagger \hat{S}^- + a \hat{S}^+) \]

Two level system  Cavity photons  Coupling term

Strong coupling regime = $g >$ dissipation of photon, atom

Circuit QED


• Superconducting coplanar waveguide cavity (CPW).

• Fundamental frequency in the microwave range (6-7GHz). Q~15000

• Measurements carried out at ~20 mK-250mK
Single quantum dots (one localized level): artificial atom

Double quantum dots (two coupled localized levels): artificial molecule

Coupled weakly or strongly to fermionic leads (closed or open quantum systems)

Closed system
Quantum information

Open system
Impurity physics: condensed matter
Electric coupling to the charge density of the conductors (from gauge invariance)

Internal transitions and transitions to leads possible (cavity to circuit QED)

Large dipoles (g ~10-100 MHz) due to mesoscopic scale

Closed system
Quantum information

Open system
Impurity physics: condensed matter

Light-matter coupling

\[ \hat{\mathcal{V}} = -e \int d^3r V_\perp(\vec{r}) \hat{\psi}^\dagger(\vec{r}) \hat{\psi}(\vec{r}) \]

\[ \hat{H}_{\text{tot}} = \int d^3r \hat{\psi}^\dagger(r) \tilde{h}_T(\vec{r}) \hat{\psi}(\vec{r}) + \hat{H}_\text{Coul} + \hbar \omega_0 \hat{a}^\dagger \hat{a} + \hat{\mathcal{V}}(\hat{a} + \hat{a}^\dagger) + (\hat{\mathcal{V}}^2 / \hbar \omega_0) \]

A. Cottet, T. Kontos and B. Douçot, PRB’15
Hybrid circuit QED with quantum dots

A. Blais et al., PRA (2004)

Transfer ideas of cQED to probe/manipulate micro/macroscopic quantum states of hybrid mesoscopic circuits (e.g. quantum dot circuits)

Can one achieve the strong coupling with quantum dot circuits?

Can one use that architecture for condensed matter questions?

Resonator response: susceptibility I

\[
\frac{b_t}{b_{in}} = (A_0 + \Delta A) e^{i(\varphi_0 + \Delta \varphi)} = \frac{t_0}{\omega_{RF} - \omega_0 - i\Lambda_0 - g^2 \chi(\omega_{RF})}
\]

Cavity transmission

DQD charge susceptibility

\[
\chi(\omega) = \frac{1}{\omega - \omega_{DQD} - i\Gamma_2}
\]

The cavity provides a cut of the hybrid DQD spectrum at \( \omega_0 = \omega_{DQD} \)

see also: Frey et al. PRL (2012), Peterson et al. Nature (2012), Toida et al. PRL (2013), ...
Resonator response: susceptibility II

The cavity provides a direct compressibility measurement.

Bruhat, Viennot, Dartailh, Desjardins, Kontos & Cottet, PRX 6, 021014 (2016)
Can one achieve the strong coupling with quantum dot circuits?

L. E. Bruhat et al. submitted ‘16
Physics of Cooper pair splitting studied by transport so far (CNT, SC nanowires, graphene).
Interfacing dots and superconductors

From Schrieffer-Wolf transformation
Nigg et al. PRB’15

coherent Cooper pair splitting $t_{eh}$

$|0\rangle \langle S| + \text{h.c.}$

$d$ $\rightarrow$ $d_S + d_R$

$\Gamma_S \rightarrow \Gamma_{S_L} + \Gamma_{S_R}$

$t_{eh} \sim -\pi \sqrt{\Gamma_{S_L} \Gamma_{S_R}} \cos(k_F \delta x) e^{-\frac{|\delta x|}{\xi_0}} f \left(\frac{\epsilon_{\Sigma}}{\Delta}, \frac{\epsilon_{\delta}}{\Delta}\right) + \frac{2t}{\Delta} (\Gamma_{S_L} + \Gamma_{S_R})$

induced interdot hopping $t_{ee}^{ind}$

$|B\rangle \langle AB| + \text{h.c.}$

$t_{ee}^{ind} \sim -\pi \sqrt{\Gamma_{S_L} \Gamma_{S_R}} \cos(k_F \delta x) e^{-\frac{|\delta x|}{\xi_0}} g \left(\frac{\epsilon_{\Sigma}}{\Delta}, \frac{\epsilon_{\delta}}{\Delta}\right) + \frac{2t}{\Delta} (\Gamma_{S_R} - \Gamma_{S_L}) h \left(\frac{\epsilon_{\Sigma}}{\Delta}, \frac{\epsilon_{\delta}}{\Delta}\right)$

$\epsilon_{\Sigma} = \epsilon_{R} + \epsilon_{L} + U_m$

$\epsilon_{\delta} = \epsilon_{R} - \epsilon_{L}$

$t_{eh}$ and $t_{ee}^{ind}$ both depend on $\epsilon_{\Sigma}$ and $\epsilon_{\delta}$ as a result of Cooper pair splitting

Measurement setup

- Mirror
- Wide band RF line
- DC gate voltages
- DC current
- Microwave cavity transmission readout
- BP
- Res.
- 20mK
- 4.2K
- 300K
- fR
- HEMT
- (Tn ~4K)
Cooper pair splitter in a cavity

$\text{Nb resonator parameters :}$

$Q \approx 10000$

$f_c = 6.6480 \text{ GHz}$

$\epsilon_L + \epsilon_R \propto \alpha V_{gL} + \beta V_{gR} + V_{ac}(a + a^\dagger)$

Measurements :
- Currents : $I_L, I_R$
- Resonator transmission : amplitude and phase
Coupling scheme vs dephasing

Renormalization of L/R hoping from elastic cotunnelling through superconductor

Induce large transverse coupling via symmetric coupling to em field

Lower charge noise than in other setups

\[ \Gamma_2^* \sim \frac{\partial^2 E}{\partial e_\delta^2} \frac{E_c^2}{2\omega_c} \left(\frac{\delta n}{2}\right)^2 \sim 10^{-8} \text{ in CNT} \]

For \( E_c \approx 10 \text{ meV} \), \( \Gamma_2 \approx 550\text{MHz} \)

See JJ Viennot et al, PRB 89, 165404 (2014)

Here \( E_c \approx 1 \text{ meV} \), \( \Gamma_2 \approx 5\text{MHz} \)
Transport measurement

$\epsilon_\Sigma = \epsilon_L + \epsilon_R$

$\epsilon_\delta = \epsilon_R - \epsilon_L$

Double dot stability diagram:
2 spatially separated orbitals

$V_{sd} = -0.16 \text{ mV}$

$\Delta = 150 \mu\text{V}$

Superconducting gap
- Resonant interaction between cavity and hybrid superconducting-quantum circuit
- Internal transitions depend both on $\epsilon_{\Sigma}$ and $\epsilon_{\delta}$. Distortions in detuning (different from variable barrier see A. Stockklauser et al. PRL’15)
Cavity measurement

- Resonant interaction between cavity and hybrid superconducting-quantum circuit
- Internal transitions depend both on $\epsilon_\Sigma$ and $\epsilon_\delta$!
- Quantitative agreement with S-induced low energy spectrum.

\[ \sqrt{(\epsilon_\delta^2 + 4t^2)Z(\epsilon_\Sigma, \epsilon_\delta)^2 + 4t_{ee}^{ind}(\epsilon_\Sigma, \epsilon_\delta)^2} \]

$\Gamma_{SL}=800\text{MHz}, \Gamma_{SR}=400\text{MHz}$

$2t=6.3\text{ GHz}, t_{eh}^0=400\text{MHz}$
Engineering strong coupling with hybrid S/dot circuit

- Largest Vacuum Rabi splitting about 10 MHz ~ 3 line widths
- Power dependence consistent with saturation of B/AB transitions
- Quantitative agreement with theory + 2 B/AB transitions
- First observation of strong coupling between a quantum dot circuit and a microwave cavity
First observation of strong coupling between a quantum dot circuit and cavity photons (could be generalized to many other setups RX qubits,...)

Cavity QED with a hybrid superconductor-quantum dot circuit

Study of Cooper pair splitting physics
($t^0_{eh} \sim 400\text{MHz}$)

L. E. Bruhat et al. submitted ‘16

Perspectives

Ultra-long distance coupling of double quantum dots. (G. Burkard, A. Imamoglu PRB’06)

Entanglement in condensed matter (Cooper pair splitting)
Can one use that architecture for condensed matter questions?

M.M. Desjardins et al. submitted ‘16
Kondo physics in alloys


- Resistance of a metal usually decreases as temperature lowered.
- Increase of resistance in some magnetic alloys even though tiny amount of magnetic impurities added!
- Discovered as early as in the 1930’s
Anderson model for a magnetic impurity

- Simplest model of a magnetic impurity... $U$ favors magnetic moment.
- Can be mapped onto Kondo (spin) problem at low energy (Schrieffer-Wolf)
- «Simplest» many body problem (energy level with coulomb +Fermi sea)
- Physics relevant for many condensed matter systems (« test bench »)
Virtual processes and Kondo/AS resonance

- Virtual processes quantum mechanically allowed at equilibrium
- Decoupling of spin and charge degrees of freedom (not usual resonant level)

Contribute to current through impurity although charge frozen
The experimental setup

Carbon nanotube based quantum dot

Stamped single wall carbon nanotubes in high finesse Nb microwave cavity (Q~15000)

Cavity measures directly the compressibility of the electronic system

Simultaneous measurement of conductance and compressibility

\[ \chi = \frac{\delta N}{\delta \mu} \]
Electron-photon coupling calibration

Coulomb blockade: $U \sim 3$ meV and $\Gamma \sim 0.7$ meV

Compressibility contrast $\sim 1000$ eV$^{-1}$
Piece of metal of $1 \mu$m$^3 \sim 10^{10}$ eV$^{-1}$

Charge sensitivity $\sim 2 \times 10^{-4}$ e

- Coulomb peaks visible both in conductance and phase (both measure the same physics)
- Amplitude of phase contrast allows to measure $g \sim 100$ MHz

Electronic and photonic systems are well coupled: what about in the Kondo regime?
Phase and conductance do not measure the same physics!

Kondo resonance is ‘transparent’ to photons while charge peaks visible. Zero charge susceptibility and shifts well reproduced by NRG (M. Lee, M.-S. Choi)

Illustrates the separation of spin and charge degrees of freedom in a Kondo system.

M.M. Desjardins et al. submitted ‘16
Phase and conductance do not have the same temperature dependence.

G evolves on temperature scale given by $T_K$ whereas phase on temperature scale given by $\Gamma$.

Experimental logarithmic slopes are in good agreement with NRG data.

Illustrates the separation of spin and charge degrees of freedom in a Kondo system.

M.M. Desjardins et al. submitted ‘16
Conclusion II

✓ cQED architecture can be used to study condensed matter problems
✓ Large charge-photon coupling
✓ Observation of separation of spin and charge degrees of freedom in a Kondo system.

M.M. Desjardins et al. submitted’16

Perspectives of this setup
• Probe of transport in mesoscopic circuits
  L.E. Bruhat et al., PRX 6, 021014 (2016)
• Quantum quench of Kondo cloud
• Quantum simulation of fermion-boson systems.
  • Probe of Majorana fermions in condensed matter (A. Cottet, T. Kontos and B. Douçot PRB’13, M.C Dartiaiilh et al. submitted)