Near quantum-limited amplification and conversion based on a voltage-biased Josephson junction

Salha JEBARI, Florian Blanchet, Romain Albert, Dibyendu Hazra, Alexander Grimm, Fabien Portier and Max Hofheinz
Ultra Low Noise Amplification is a must in superconducting qubit experiments
- Qubit read out
- Quantum feedback

Ideal amplifier

- High gain
- Large bandwidth
- High dynamic range
- Low noise
Ideal amplifier

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  - Qubit read out
  - Quantum feedback

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- High dynamic range
- Low noise

**BONUS!** Easy to use
Commercial amplifiers

http://www.lownoisefactory.com

Advantages

❖ Simple to use
❖ Large bandwidth
❖ High dynamic range
❖ High gain

Disadvantages

❖ High noise
   2K at 6 GHz = 10 photons of noise
❖ Power dissipation

http://www.caltechmicrowave.org
Commercial amplifiers

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- Large bandwidth
- High dynamic range
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- High noise
- 2K at 6 GHz = 10 photons of noise
- Power dissipation

Parametric amplifier: new type of amplifier that can amplify without or with very low noise
**Parametric amplification**

**Principle of parametric amplification**

\[ \omega_p = \omega_s + \lambda \]

**Why?**

- Any dissipation at a frequency less than \( \frac{k_B T}{\hbar} \) necessarily introduces a noise
- Only parametric amplifier is able to control exactly the origin of frequency dissipation
Parametric amplification

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The challenge

JPA
Low noise

HEMT
Easy to use
The challenge

- JPA
  - Low noise
- HEMT
  - Easy to use

ICTA
- Inelastic Cooper pair Tunneling Amplifier
- SLUG Amplifier
- SJA
- DC biased Josephson junction amplifier
From dynamic Coulomb blockade physics to Josephson *parametric amplifier* physics: Theory, Measurement results with Aluminium (Al) sample

Optimization of parameters of ICTA samples: Niobium Nitride (NbN) sample
Inelastic Cooper pair tunneling
Inelastic Cooper pair tunneling

Non dissipative element
Inelastic Cooper pair tunneling

Non dissipative element

If we work under the gap

Fun physics
Inelastic Cooper pair tunneling

- A Cooper pair can only tunnel if it can lose its energy 2eV
- No density of states on the other side: No Cooper pair current

G.-L. Ingold and Y. V. Nazarov, Single Charge Tunneling 294, 21 (1992)
Inelastic Cooper pair tunneling

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- One or several modes can absorb it as photons

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Inelastic Cooper pair tunneling

Resonance condition

\[ 2eV \approx \hbar \omega_a + \hbar \omega_b \]
Inelastic Cooper pair Tunneling Amplifier: ICTA

Resonance condition

\[ 2eV \approx h\omega_a + h\omega_b \]

- Use parametric down-conversion process
- Send signal at one of the modes
- Process accelerated due to stimulated emission
- Quantum limited amplification
2eV \approx \hbar \omega_a + \hbar \omega_b

\rho_{a,b} = \sqrt{\frac{\pi Z_{a,b}}{\hbar / 4e^2}}

H_{sys} \approx \hbar \omega_a a^+ a + \hbar \omega_b b^+ b - E_J \cos\left(\frac{2eV t}{\hbar} + \rho_a (a^+ a) + \rho_b (b^+ b)\right)

Isolated resonators a,b

Josephson junction energy

Kirchhoff’s law
ICTA theory

\[ H_{\text{sys}} \approx \hbar \omega_a a^+ a + \hbar \omega_b b^+ b - E_J \cos \left( \frac{2eVt}{\hbar} + \rho_a (a^+ + a) + \rho_b (b^+ + b) \right) \]

Isolated resonators a,b

Resonance condition

\[ 2eV \approx \hbar \omega_a + \hbar \omega_b \]

Josephson junction energy

Rotating wave approximation

\[ |\omega_j - \omega_a - \omega_b| < E_J \]

\[ \rho_{a,b} = \sqrt{\frac{\pi Z_{a,b}}{\hbar / 4e^2}} \]
$\rho_{a,b} = \sqrt{\frac{\pi Z_{a,b}}{\hbar/4e^2}}$

$2eV \simeq \hbar \omega_a + \hbar \omega_b$

$I_{\text{res}} = \hbar \omega_a a^+ a + \hbar \omega_b b^+ b - E_J \cos\left(\frac{2eVt}{\hbar} + \rho_a (a^+ a) + \rho_b (b^+ b)\right)$

Resonance condition

$2eV \simeq \hbar \omega_a + \hbar \omega_b$

Rotating wave approximation

$|\omega_J - \omega_a - \omega_b| < E_J$

$H_{\text{sys}} = \hbar \omega_a a^+ a + \hbar \omega_b b^+ b + \hbar \lambda (a^+ b^+ e^{-i\omega_J t} + h.c.)$

Isolated resonators $a,b$

Coupling term

$\lambda = E_J \frac{\rho_a \rho_b}{2\hbar}$
ICTA theory: scattering matrix

\( \gamma^a, \gamma^b \): Damping terms

\( \lambda \): Coupling frequency

\( \omega_a \approx \omega_s \): Signal frequency

\( \omega_b \approx \omega_i \): Idler frequency
ICTA theory: scattering matrix

\( \gamma^a, \gamma^b \) : Damping terms

\( \lambda \) : Coupling frequency

\( \omega_a \approx \omega_s \) : Signal frequency

\( \omega_b \approx \omega_i \) : Idler frequency
ICTA gain

\[ r^2 = G = \left( \frac{1 + \xi^2}{1 - \xi^2} \right)^2 \]

\[ \xi = \frac{E_f \pi}{R_Q h \sqrt{\gamma_a \gamma_b}} \sqrt{\frac{Z_a Z_b}{\gamma^2}} \]

\( \gamma \) : damping term

\[ r^2 = G = \left( \frac{1 + \xi^2}{1 - \xi^2} \right)^2 \]

\[ \xi = \frac{E_J \pi}{R_q \hbar \sqrt{\gamma_a \gamma_b}} \sqrt{Z_a Z_b} \]

**Our case**

\[ \xi \propto E_J \]

\[ \xi \propto \frac{1}{E_J} \]
Sample

Fabricated in Quantronic group
CEA Saclay
Experimental setup
Experimental setup
Experimental setup

On chip
VNA measurement
**Conversion with gain: amplification**

**Measurement @**

\[ I_c = 17.5 \text{ nA} \quad \text{Signal power} = -125 \text{ dBm} \]
**Conversion with gain: amplification**

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$I_c = 17.5$ nA  \hspace{0.5cm} \text{Signal power} = -125$ dBm

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Conversion with gain: amplification

Measurement @

\[ I_c = 17.5 \text{ nA} \quad \text{Signal power} = -125 \text{ dBm} \]

\[ G < 0 \text{ dB} \]

To eliminate

\[ G > 0 \text{ dB} \]

To preserve
Measurement of higher order terms

\[ H_{\text{sys}} \approx \hbar \omega_a a^+ a + \hbar \omega_b b^+ b - E_f \cos \left( \frac{2eVt}{\hbar} \right) + \rho_a (a^+ a) + \rho_b (b^+ b) \]

\[ H_{\text{sys}} = \hbar \omega_a a^+ a + \hbar \omega_b b^+ b + \hbar \lambda (a^+ b^+ e^{-i\omega_j t} + h.c.) \]
Measurement of higher order terms

\[ H_{sys} \approx \hbar \omega_a a^+ a + \hbar \omega_b b^+ b - E_J \cos \left( \frac{2eVt}{\hbar} \right) + \rho_a (a^+ + a) + \rho_b (b^+ + b) \]

\[ H_{sys} \approx \hbar \omega_a a^+ a + \hbar \omega_b b^+ b - \frac{E_J}{2} \left\{ e^{-i\omega_j t} e^{-\frac{i}{2}\rho_a} e^{-\frac{i}{2}\rho_b} \right\} \left\{ \sum_{s=0}^{\infty} \left( -i \rho_a a^+ \right)^s \sum_{m=0}^{\infty} \frac{1}{m!} \left( -i \rho_b b^+ \right)^m \sum_{q=0}^{\infty} \frac{1}{q!} \left( -i \rho_a b \right)^q \right\} + h.c. \]

\[ H = \ldots \alpha (a^+)^2 b e^{-i(\omega_j - 2\omega_a + \omega_b)} + \ldots + \beta ab (c^+)^3 e^{-i(\omega_j + \omega_b - 3\omega_c)} + \ldots \]

\[ H_{sys} = \hbar \omega_a a^+ a + \hbar \omega_b b^+ b + \hbar \lambda (a^+ b^+ e^{-i\omega_j t} + h.c.) \]
Measurement of higher order terms

\[ H_{sys} \approx \hbar \omega_a a^+ a + \hbar \omega_b b^+ b - E_J \cos(\frac{2eVt}{\hbar} + \rho_a (a^+ + a) + \rho_b (b^+ + b)) \]

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High signal power

\[ H_{sys} = \hbar \omega_a a^+ a + \hbar \omega_b b^+ b + \hbar \lambda (a^+ b^+ e^{-i\omega_j t} + h.c.) \]

Low signal power

QUESTION!
Measurement of higher order terms

Measurement @

\[ I_c = 17.5 \text{ nA} \quad \text{Signal power} = -90 \text{ dBm} \]
Measurement of higher order terms

\[ \tilde{I}(2eV) = \delta(0) \sum_{n} \left| J_{n} \left( \frac{2eU}{\hbar \omega_{0}} \right) \right|^{2} I(2eV - n\hbar \omega_{0}) \]

**Measurement @**

\[ I_{c} = 17.5 \text{ nA} \quad \text{Signal power} = -90 \text{ dBm} \]
PSD measurement
Noise measurement

- Gain
- PSD
- Noise

V = 12.25 GHz
Noise measurement

- Gain (dB) vs. Frequency $f$ (GHz)
- Power spectral density (photons) vs. Frequency $f$ (GHz)
- Noise (photons) vs. Frequency $f$ (GHz)

$V = 12.25 \text{ GHz}$
Sample **NOT** designed for amplification:

- 10 dB gain over 280 MHz
- Noise 0.9 photons: $1.8 \times$ quantum limit
Summary

First generation of ICTA Test sample

Sample **NOT** designed for amplification:
- 10 dB gain over 280 MHz
- Noise 0.9 photons: 1.8 * quantum limit

Second generation of ICTA Real sample

Points to optimize
- Eliminate frequency conversion process
- Increase junction size
- Lower resonator quality factor
- Reduce voltage noise
- Idler @ 100 GHz

By using NbN superconductor

Salha Jebari and Max Hofheinz, patent application FR 16 58429
Submitted on 09-09-2016
Outline

Part 1

From dynamic Coulomb blockade physics to Josephson parametric amplifier physics: Theory, Measurement results with Aluminium (Al) sample

Part 2

Optimization of parameters of ICTA samples: Niobium Nitride (NbN) sample
Our ICTA implementation
NbN/MgO/NbN Josephson junction

SEM of SQUID

4K Measurement

<table>
<thead>
<tr>
<th>Size($\mu m^2$)</th>
<th>$I_c$ ($\mu A$)</th>
<th>$V_g$ (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>200</td>
<td>5.10</td>
</tr>
<tr>
<td>0.5</td>
<td>560</td>
<td>5.14</td>
</tr>
<tr>
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</tr>
<tr>
<td>2</td>
<td>1920</td>
<td>5.09</td>
</tr>
</tbody>
</table>
First proof of amplification using NbN samples

Maximum of measured gain is 22.5 dB over 50 MHz

Gain vs. Frequency (GHz)

Similair sample with big Josephson junction

$E_J > E_J > E_J$
First proof of amplification using NbN samples

1 dB compression point: -97 dBm
First proof of amplification using NbN samples

\[ \frac{\lambda}{4} \text{ @ } f_1 \quad \frac{\lambda}{4} \text{ @ } f_2 \quad \frac{\lambda}{4} \text{ @ } f_3 \]

Real \( Z (\Omega) \)

Signal frequency (Hz)
Single Cooper pair photonics group: ICTA project
Conclusions in pictures

- Parametric amplification & Powered by DC voltage
- Close to quantum limit
- High frequency, high temperature
  - Bandwidth ?
  - Saturation ?
Real SEM image

NbN PhEISQS

Salha

Black lines
Where we etch NbN

Single Cooper pair photonics group

20 μm

Signal A = InLens
Mag = 315 X
EHT = 5.00 kV
WD = 3.6 mm
26 Nov 2015 14:23:36
Ultra Plus