Simple Floquet-Wannier-Stark-Andreev Viewpoint and Emergence of Low-Energy Scales in a Voltage-Biased Three-Terminal Josephson Junction

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Collaborators on this Project

Jean-Guy Caputo:
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The one who optimized my codes and
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Kang Yang:
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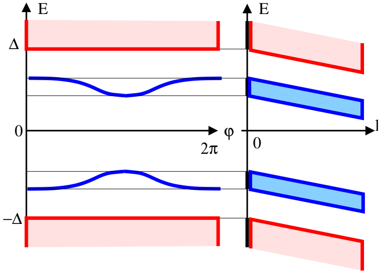
Laboratoire de Physique des Solides, Orsay
The one who made his Master1 Internship at the time where everything was unclear, and who is now making semi-classics.

Benoît Douçot:
Laboratoire de Physique Théorique et des Hautes Energies, UPMC
The one who found interpretation in terms of
Floquet-Wannier-Stark-Andreev ladders.

arXiv:1611.01932, submitted to Phys. Rev. B

Floquet-Wannier-Stark-Andreev Resonances (2 Terminals)

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_a + \hat{\mathcal{H}}_b + \hat{\mathcal{H}}_{a-b} - eV\left(\hat{N}_a - \hat{N}_b\right) \;,\; \left[\hat{N}_a - \hat{N}_b, \frac{\hat{\varphi}_a - \hat{\varphi}_b}{2}\right] = i$$



Two uncoupled FWS-Andreev bands:
$\hat{H}_\pm = E_\pm(\hat{\varphi}) - 2eV\hat{I}$, with
$\hat{I} = (\hat{N}_a - \hat{N}_b)/2$ (auxiliary variable)

Steady state $\Rightarrow$
$\hat{H}_\pm|\psi_\pm\rangle = E_\pm|\psi_\pm\rangle \Rightarrow$

$$\left[E_\pm(\hat{\varphi}) - 2eV\hat{I}\right]|\psi_\pm\rangle = E_\pm|\psi_\pm\rangle$$

with $I = i\partial/\partial\varphi$ (e.g. $[\hat{I}, \hat{\varphi}] = i$)

$\Rightarrow$ **First order differential equation for wave-function**

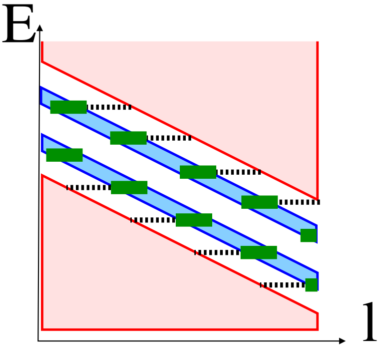
Imposing 2π-periodicity in φ leads to quantized energy levels:

$$\begin{cases} E_j = 2eVj + \langle E\rangle \\ E'_{j'} = 2eVj' - \langle E\rangle \end{cases} , \text{ with } \langle E\rangle = \frac{1}{2\pi}\int_0^{2\pi} E_+(\varphi)d\varphi$$

$\Rightarrow$ **Two Floquet-Wannier-Stark-Andreev ladders**

Floquet-Wannier-Stark-Andreev ladders

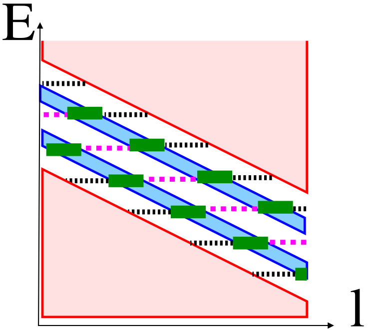
Non-coinciding resonances



- Tunneling between ladders and continua

⇒ Finite width of FWS-Andreev resonances

Coinciding resonances



- Tunneling between ladders and continua
- Inter-ladder tunneling

⇒ Landau-Zener-Stückelberg transitions

Differences between 2 and 3 terminals

- Ladders parameterized by the quartet phase φ_Q
  $\Rightarrow$ Level crossings as a function of φ_Q
- Multiple Andreev Reflections become Phase-sensitive Multiple Andreev reflections $\Rightarrow$ Interference process in the tunnel effect

A single picture for three phenomena:

- Width of resonances
- Landau-Zener-Stückelberg transitions
- Phase-sensitive Multiple Andreev Reflections

We want to suggest new experiments on the spectroscopy of those ladders:

$\Rightarrow$ Variation of the resonance energies with voltage

$\Rightarrow$ Variation of the width with voltage

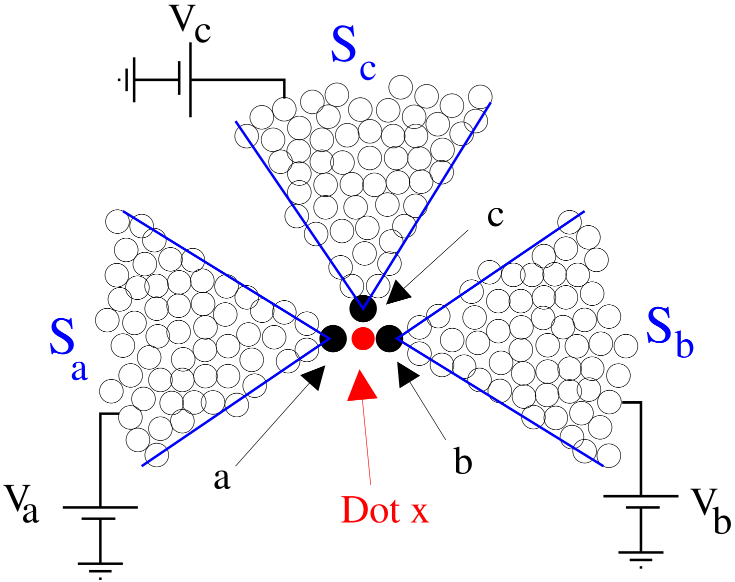
We want to understand mechanisms for the width of the resonances:

- Equilibration with quasiparticle semi-infinite continua
- Electron-phonon scattering

We want to determine connections between spectrum and DC-transport and noise:

- Same energy scales in spectrum and DC-transport ?
- Connection with DC-current
- Connection with noise (see talk by Yonathan Cohen)

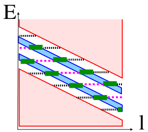
The Set-up for Numerical Experiments

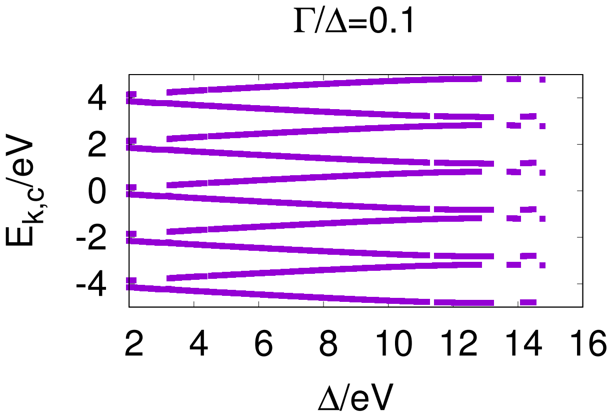


Floquet-Wannier-Stark-Andreev Resonances (1/2)

$\Gamma/\Delta = 0.1$

Inter-ladder tunneling for $\Delta/eV \simeq 14$
$\Rightarrow$ Landau-Zener-Stückelberg transitions

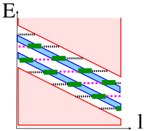


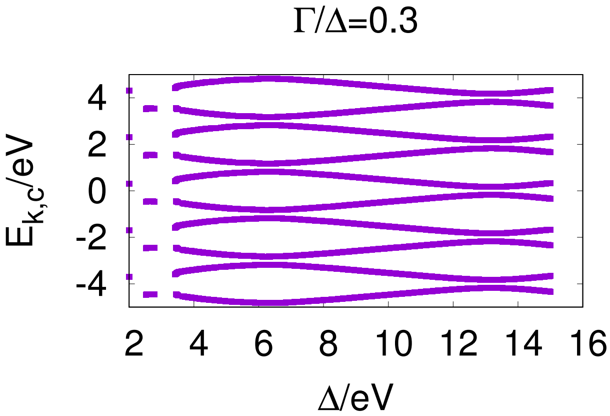


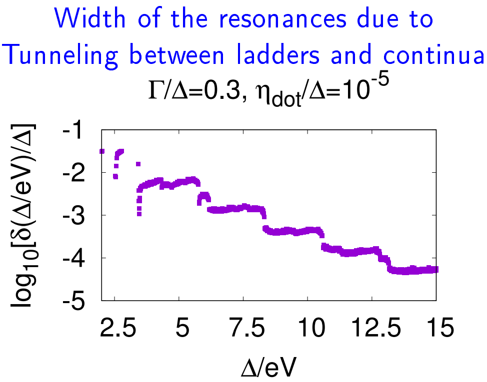
Floquet-Wannier-Stark-Andreev Resonances (2/2)

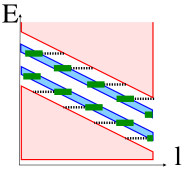
$\Gamma/\Delta = 0.3$

Inter-ladder tunneling for $\Delta/eV \simeq 6, 13$
$\Rightarrow$ Landau-Zener-Stückelberg transitions









- Envelope $\delta(\Delta/eV) \sim \exp(-\Delta/eV)$ because of tunneling through classically forbidden region of length $\sim \Delta/eV$

- Steps related to thresholds of multiple Andreev reflections coupling quantum dot to quasiparticle continua (discreteness of auxiliary variable l)

$\Rightarrow$ Requirement for another relaxation mechanism providing low-voltage cut-off (e.g. at large Δ/eV)

Emergence of exponentially small energy scales

$\Rightarrow$ Those should be compared to lots of things
$\Rightarrow$ Dynes parameter

Dynes parameter

Dynes, Narayanamurti, Garno, Phys. Rev. Lett. **41**, 1509 (1978)

Strong-coupling superconductor $Pb_{0.9}Bi_{0.1}$

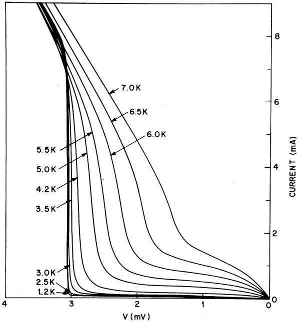


FIG. 1. I-V characteristic for a $Pb_{0.9}Bi_{0.1}$ tunnel junc-

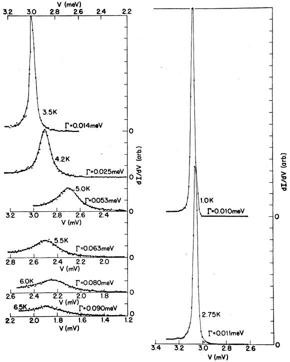


FIG. 2. dI/dV vs V determined from the data using Fig. 1. The solid curves are fits to the data using Eq. (2) with Γ an adjustable parameter.

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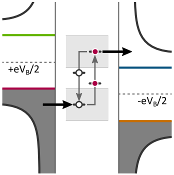
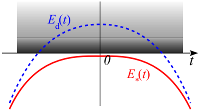
$$I = G_N \int_{-\infty}^{+\infty} \rho(E)\rho(E+V)\left[f(E)-f(E+V)\right] dE$$

$$\rho(E,\Gamma) = (E-i\Gamma)/\left[(E-i\Gamma)^2-\Delta^2\right]^{1/2}$$

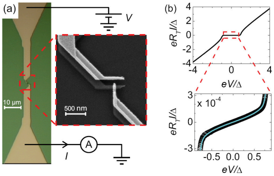
Improvement of performance of an electron turnstile



Experiment: van Zanten, Basko, Khaymovich, Pekola, Courtois, Winkelmann (PRL '16)



Theory: Basko ('16)



Dynes parameters can originate from electromagnetic environment: Pekola, Maisi, Kafanov, Chekurov, Kemppinen (PRL, '10)

Motivations for Introducing η_{dot}

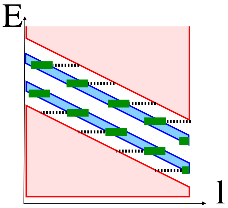
- Dynes parameter η_{dot} on the quantum dot
- Dynes parameter η_S in superconductors

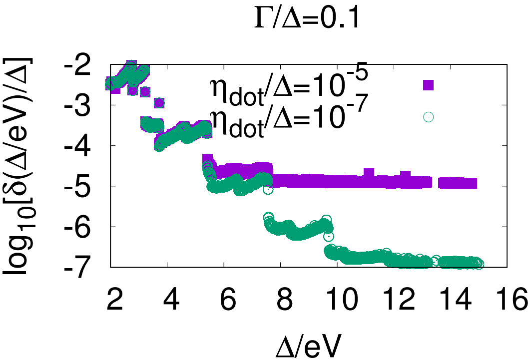
η_{dot} has much stronger influence on current than η_S

Considered scenario for η_{dot}: Electron-phonon scattering

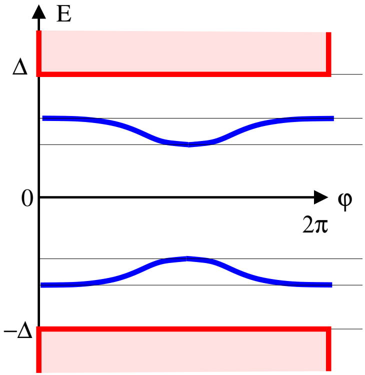
Width of Floquet-Wannier-Stark-Andreev Resonances (2/2)

- Tunneling between ladders and continua
- + Relaxation
  (Dynes parameter on quantum dot)



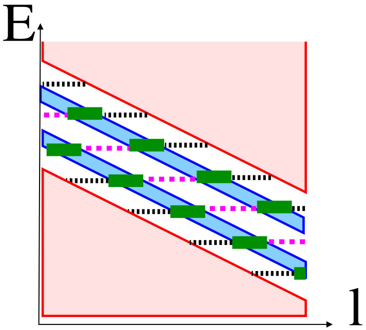


Equilibrium
Andreev bound states



$$I = -\frac{2e}{\hbar}\ \frac{d}{d\varphi} E(\varphi)$$

Nonequilibrium
Resonances



$$I = ???$$

Spectrum and Self-Induced Rabi Resonances

1) Two Floquet-Wannier-Stark-Andreev ladders:

$$\begin{cases} E_j = 2eVj + \langle E \rangle \\ E'_{j'} = 2eVj' - \langle E \rangle \end{cases}, \text{ with } \langle E \rangle = \frac{1}{2\pi} \int_0^{2\pi} E(\varphi) d\varphi.$$

2) Self-induced Rabi resonance whenever $E_j = E'_{j'} \Rightarrow$

$$2eVj + \langle E \rangle = 2eVj' - \langle E \rangle \Rightarrow eV_k = \frac{\langle E \rangle}{k}, \text{ with } k = j' - j$$

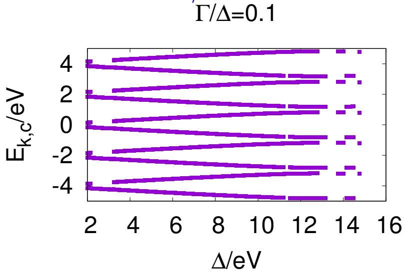
Spectrum ↔ Transport: Current $I_c(eV/\Delta)$

Spectrum

x-axis=Δ/eV

Avoided crossing

at $\Delta/eV \simeq 14$

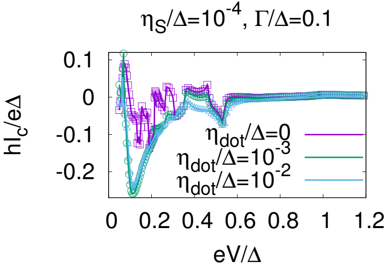


Transport, current I_c

x-axis=eV/Δ

Maximum in $|I_c|$

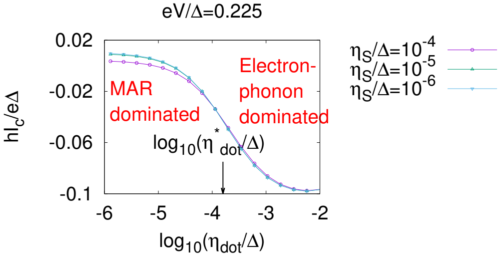
at $eV/\Delta \simeq 0.1$



- Possible explanations for difference between the two:
  - Transport couples also to Floquet wave-function and populations
  - Two cross-over values evaluated with different methods
- Ultra-sensitivity on tiny η_{dot}/Δ

  $\Rightarrow$ Additional energy scale η^*_{dot}/Δ

Energy Scale η^*_{dot}/Δ in Current (1/3)

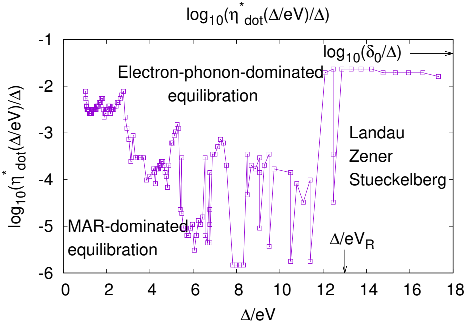


- $\log_{10}(\eta^*_{dot}/\Delta)$ defined as inflection point on those curves
- $\hbar/\eta^*_{dot}$ is intrinsic characteristic time
- Important effect of η_{dot}/Δ (change of sign in current I_c)

- Much stronger effect of η_{dot}/Δ than η_S/Δ
- Possible experimental relevance of new regime $\eta_{dot} \gg \eta^*_{dot}$ in which quantum dot degrees of freedom are nonequilibrated with quasiparticle continua

Energy Scale η^*_{dot}/Δ in Current (2/3)

$\log[\eta_{dot}/\Delta]$ as function of Δ/eV



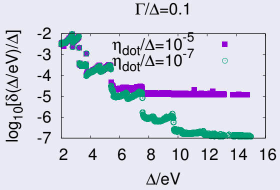
Exponentially small energy scales in current and in line-width broadening

Remarkably:

Spectrum ↔ current relation holds qualitatively (but not exactly).

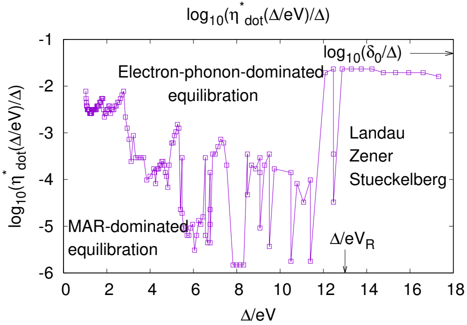
Namely:

log[line-width broadening]



Energy Scale η^*_{dot}/Δ in Current (3/3)

$\log[\eta_{dot}/\Delta]$ as function of Δ/eV



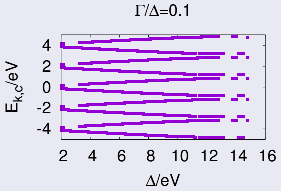
Compatible with Landau-Zener-Stückelberg resonance splitting δ_0

Remarkably:

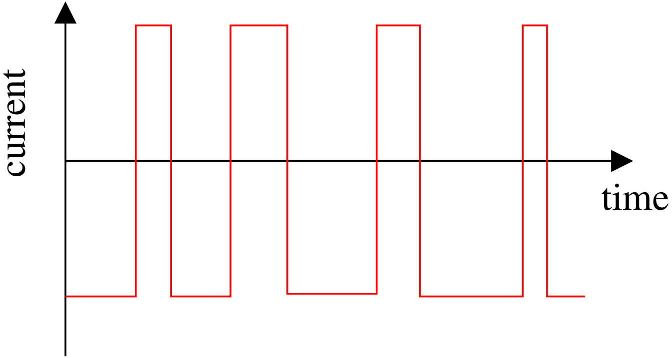
Spectrum $\leftrightarrow$ current relation holds qualitatively (but not exactly).

Namely:

FWS-Andreev spectrum

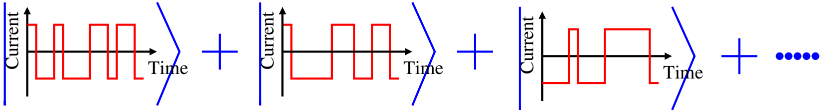


Thermal noise in a two-terminal point contact:



Possible Connection with Yonathan Cohen's Talk (2/3)

Quantum noise of quartet state:



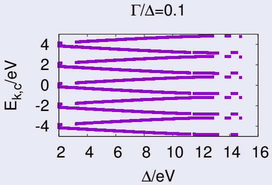
Possibility of fabricating Schrödinger cats of Cooper pairs.

Identification of energy scales between this experiment and Floquet-Wannier-Stark-Andreev ladders:

Correlation time for sign of current=$\hbar/\delta_0$

δ_0 = Level splitting at avoided crossings between Floquet-Wannier-Stark-Andreev resonances

FWS-Andreev spectrum as function of Δ/eV



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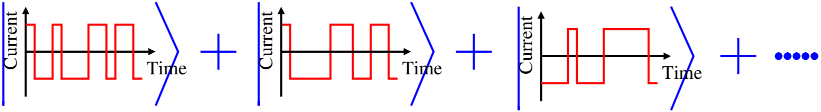
Quantum noise of quartet state:



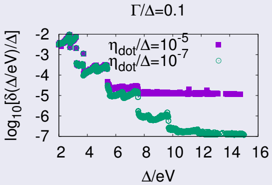
Possibility of fabricating Schrödinger cats of Cooper pairs.

Identification of energy scales between this experiment and Floquet-Wannier-Stark-Andreev ladders:

Correlation time for absolute value of current $= \hbar/\delta$

$\delta =$ width of Floquet-Wannier-Stark-Andreev resonances

log[line-width broadening] as function of Δ/eV

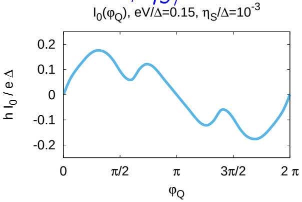


Connection with RF-irradiated Josephson junctions (1/2)

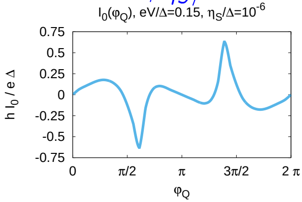
Role of Dynes parameter η_S/Δ, with $\eta_{dot}/\Delta = 0$

Double quantum dot: Four Floquet-Wannier-Stark-Andreev ladders

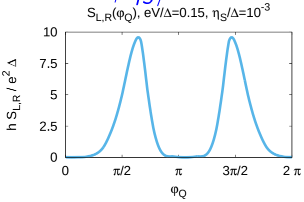
Current, $\eta_S/\Delta = 10^{-3}$



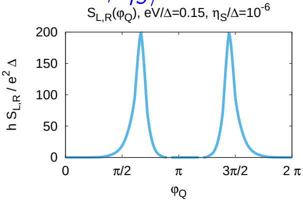
Current, $\eta_S/\Delta = 10^{-6}$



Noise, $\eta_S/\Delta = 10^{-3}$



Noise, $\eta_S/\Delta = 10^{-6}$



Interpretation: Avoided crossings between Floquet-Wannier-Stark-Andreev ladders tuned by quartet phase

Emergence of a tiny η_S^*/Δ

Connection With RF-Irradiated Josephson Junctions (2/2)

Bergeret, Virtanen, Ozaeta, Heikilä, Cuevas,
Phys. Rev. B **84**, 054504 (2011)

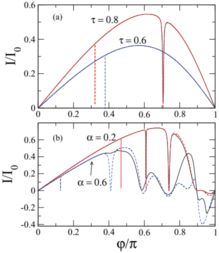


FIG. 5. (Color online) (a) The CPR for $\alpha = 0.1, \hbar\omega = 0.6\Delta$, and two values of the transmission coefficient, $\tau = 0.8$ and $\tau = 0.6$. (b) The CPR for $\hbar\omega = 0.3\Delta$, $\tau = 0.95$, and two values of α, 0.2 and 0.6. In both panels the solid lines correspond to the microscopic theory and the dashed lines to the two-level model.

Recall also following paper:
Voltage-induced Shapiro steps
in a superconducting multiterminal structure
J.C. Cuevas and H. Pothier, Phys. Rev. B **75**, 174513 (2007)

Conclusions

Three relevant low-energy scales:
1) Line-width broadening of Floquet-Wannier-Stark-Andreev resonances
2) Resonance level splitting at avoided crossings of Floquet-Wannier-Stark-Andreev resonances
3) Cross-over Dynes parameter η_S^*/Δ or η_{dot}^*/Δ

New predictions for spectroscopy experiments

Qualitative connection between spectrum and transport:
even in presence of strong effect of weak relaxation

Interesting perspective on quantum thermodynamics:
In infinite-gap limit, no entropy flows from dot to superconducting leads $\Rightarrow$ Interest of investigating heat transport, and, maybe, in connection with entanglement of quartet state

Interesting perspective on semi-classics:
Kang Yang and Benoît Douçot are now developing semi-classical theory on the basis of the Floquet-Wannier-Stark-Andreev viewpoint $\Rightarrow$ Analytical results