Thermal decay of charge quantization in mesoscopic circuits

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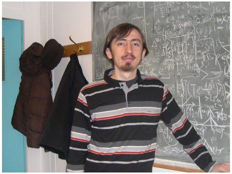
Collaboration



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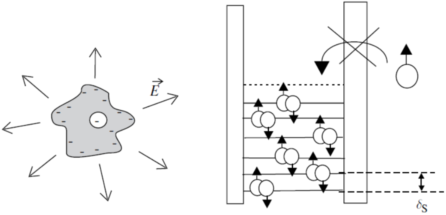
Outline

1. Introduction: charge quantization, energy scales, SET
2. Experiment in the group of Frederic Pierre: charge quantization with quantum fluctuations, *Nature 536, 5862 (2016)*
3. Theoretical explanation: perturbation, quantum and thermal regimes

Introduction: charge quantization



Isolated metallic island: $Q = eN$, where $e = 1.6 \times 10^{-19}$ C and N is an integer

Electrostatic energy: $E = \frac{Q^2}{2C} = E_C N$, where $E_C = \frac{e^2}{2C}$ is a charging energy

Mean level spacing: δ_S

For more details: *Y.V. Nazarov, Y.M. Blanter, Quantum Transport: Introduction to Nanoscience (2009)*

Introduction: energy scales

A cubic island of size L:

Number of atoms: $N_{at} \simeq (L/a)^3$, a is interatomic distance

Level spacing: $\delta_S \simeq E_F/N_{at}$; Charging energy: $E_C \simeq e^2/L$

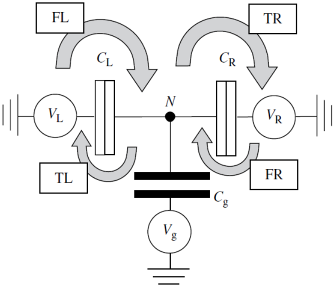
Fermi energy: $E_F \simeq e^2/a$; Ratio: $\delta_S/E_C \simeq 1/N_{at}^{2/3}$

Estimates: $L = 100nm$, $N_{at} = 10^9$, $E_F \simeq 10eV \Rightarrow \delta_S \simeq 10^{-8}eV$, $E_C \simeq 10^{-2}eV$

$$\delta_S/E_C \ll 1$$

For more details: *Y.V. Nazarov, Y.M. Blanter, Quantum Transport: Introduction to Nanoscience (2009)*

Introduction: single-electron transistor (SET)

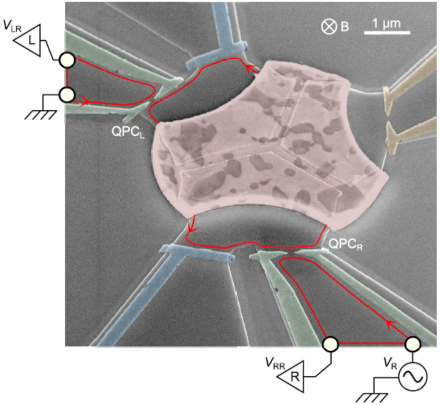


Coulomb blockade: $G_{L/R} \ll e^2/\hbar \simeq G_Q$ and $k_B T < E_C$

Typical capacitance: $C_{L/R} \sim 10^{-16}$ F and $C_g \ll C_{L/R}$

For more details: *Y.V. Nazarov, Y.M. Blanter, Quantum Transport: Introduction to Nanoscience (2009)*

Experiment: *S. Jezouin et al. Nature 536, 5862 (2016)*



Magnetic field: $B \simeq 4$ T; Level spacing: $\delta_S \simeq k_B \times 0.2\mu K \simeq 2 \times 10^{-11} eV$
Charging energy: $E_C \simeq k_B \times 0.3K \simeq 3 \times 10^{-5} eV$

Experiment: visibility

Degree of charge quantization:

$$V(T/E_C) = \frac{G_{max} - G_{min}}{G_{max} + G_{min}}$$

Linear conductance: $G = \frac{d\langle I \rangle}{d\Delta\mu}\Big|_{\Delta\mu=0}$

Full charge quantization: $V = 1$

Absence of charge quantization: $V = 0$

Energy scales in the experiment:

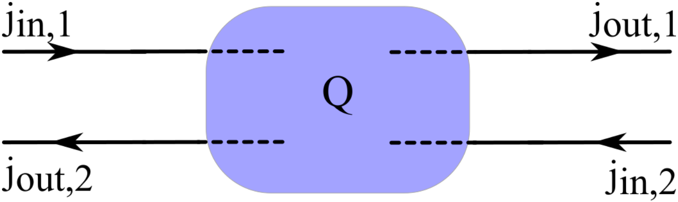
Level spacing: $\delta_S \simeq k_B \times 0.2\mu K \simeq 2 \times 10^{-11} eV$

Charging energy: $E_C \simeq k_B \times 0.3K \simeq 3 \times 10^{-5} eV$

Temperature: $T \simeq 15 \div 200 mK [(13 \div 200) \times 10^{-7} eV]$

Theory: low energy effective theory of QH edge states

Bosonisation: $\psi(x) \propto \exp[i\phi(x)]$, $(e = \hbar = k_B = 1)$



Edge states are collective fluctuations of the charge densities:

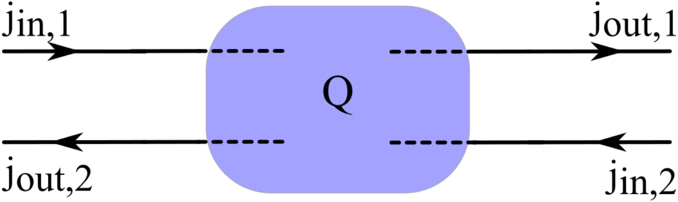
$$\rho_{\alpha j}(x) = (1/2\pi)\partial_x \phi_{\alpha j}(x)$$

Canonical commutation relations:

$$[\partial_x \phi_{\alpha j}(x), \phi_{\beta k}(y)] = (-1)^{\alpha} 2\pi i \delta_{\alpha\beta}\delta_{jk}\delta(x-y)$$

Take into account Coulomb interaction at the metallic node!!!

Theory: total Hamiltonian



Hamiltonian: $H = H_0 + H_{\text{int}}$

Kinetic term: $H_0 = \frac{v_F}{4\pi}\sum_{\alpha,j}\int dx(\partial_x\phi_{\alpha j}(x))^2$

Coulomb interaction at the metallic node: $H_{\text{int}} = \frac{(Q-Q_0)^2}{2C}$

$Q =$
$\frac{1}{2\pi}\int_0^\infty dx\,[\partial_x\phi_{\text{in}1}(x) + \partial_x\phi_{\text{out}2}(x)] + \frac{1}{2\pi}\int_{-\infty}^0 dx\,[\partial_x\phi_{\text{in}2}(x) + \partial_x\phi_{\text{out}1}(x)]$
$Q_0 = C_g V_g$

Theory: bare conductance

Quantum Langevin equations:

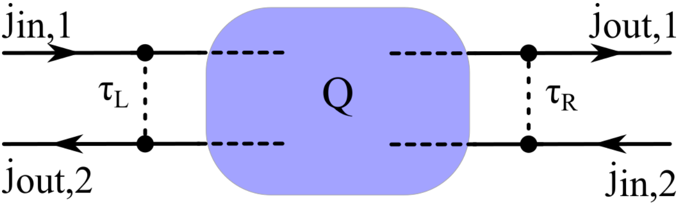
$$\frac{dQ(t)}{dt} = \sum_{\alpha=1,2} j_{\mathrm{in}\alpha}(t) - \sum_{\alpha=1,2} j_{\mathrm{out}\alpha}(t),$$

$$j_{\mathrm{out}\alpha}(t) = \frac{Q(t) - Q_0}{2\pi C} + j^s_\alpha(t).$$

Charge stored in the metallic grain: $\langle Q \rangle = Q_0 + \Delta\mu C/2$

Average current: $\langle I \rangle_0 = \Delta\mu/4\pi$

Bare conductance: $G_0 = 1/4\pi$



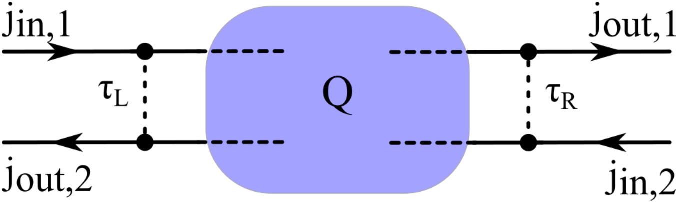
Tunneling Hamitonian:

$$H_T = A_L + A_R + \text{H.c.},$$

$$A_L = \frac{\tau_L}{a} e^{i\phi_{\text{in}1}(0) - i\phi_{\text{out}2}(0)}$$

$$A_R = \frac{\tau_R}{a} e^{i\phi_{\text{out}1}(0) - i\phi_{\text{in}2}(0)}$$

Theory: symmetric barriers



Symmetric barriers: left and right QPC's are almost fully open

Current: $\langle I \rangle = \langle I \rangle_0 + I_{inc} + I_{coh}$

$I_{inc} = I_{LL} + I_{RR}$ is an incoherent contribution

$I_{coh} = I_{LR} + I_{RL}$ is an coherent contribution

Perturbation in tunneling couplings $\tau_{L,R}$: $I_{ll'} = -\frac{1}{2} \int dt \langle \left[A_l^\dagger(t), A_{l'}(0) \right] \rangle_0$

Averaging: $\rho_0 \propto \exp\left[-(H_0 + H_{\text{int}})/T\right]$ is an equilibrium density matrix

Theory: symmetric barriers, conductance

Quantum regime: $T/E_C \ll 1$ $(T \gg \Gamma(Q_0))$

$$G = \frac{1}{4\pi}\left(1 - \frac{\Gamma(Q_0)}{T}\right)$$

$$\Gamma(Q_0) = \frac{e^{\gamma} E_C}{2\pi v_F^2}\left[|\tau_L|^2 + |\tau_R|^2 + 2|\tau_L||\tau_R|\cos(2\pi Q_0)\right]$$

A. Furusaki, K. A. Matveev, Phys. Rev. B 52, 16676 (1995)

Thermal regime: $T/E_C \gg 1$

$$G = \frac{1}{4\pi}\left[1 - \frac{|\tau_L|^2 + |\tau_R|^2 + 2|\tau_L||\tau_R| F(T)\cos(2\pi Q_0)}{2v_F^2}\right]$$

$$F(T) = 2\pi\sqrt{\frac{\pi T}{E_C}}\exp\left[-\frac{\pi^2 T}{E_C}\right]$$

Theory: symmetric barriers, visibility

Quantum regime: $T/E_C \ll 1$

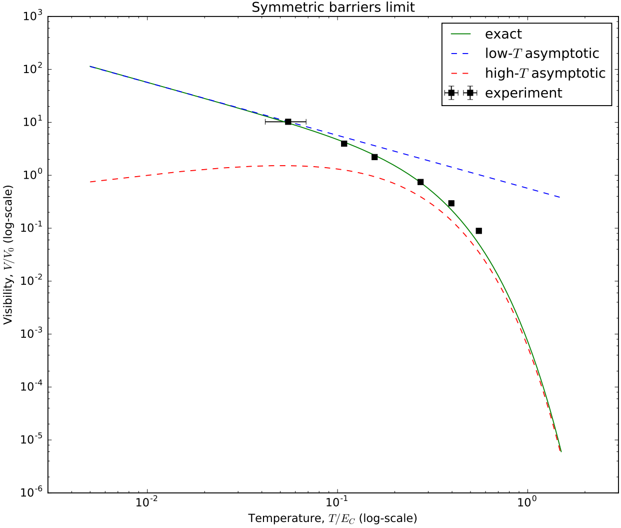
$$V/V_0 = \frac{2e^\gamma E_C}{T}$$

$V_0 = |\tau_L||\tau_R|/2\pi v_F^2$ and $\gamma \approx 0.5772$ is an Euler's constant

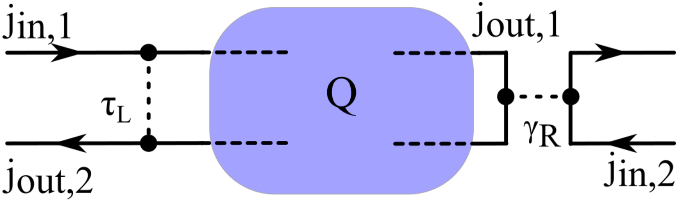
Thermal regime: $T/E_C \gg 1$

$$V/V_0 = 4\pi\sqrt{\pi}\sqrt{\frac{\pi^2 T}{E_C}} \exp\left[-\frac{\pi^2 T}{E_C}\right]$$

Theory: symmetric barriers, visibility



Theory: asymmetric barriers



Asymmetric barriers: the left QPC is almost fully open while right one is fully closed

Quantum Langevin equations: the right arm is free

Double perturbation theory, first in γ_R and corrections to it in τ_L:

$$G = G_{inc} + G_{coh}$$

Theory: asymmetric barriers, conductance

Quantum regime: $T/E_C \ll 1$

$$G = G_R \frac{2\pi^4 T^2}{3e^{2\gamma} E_C^2} \left[1 - \xi \frac{|\tau_L|}{v_F} \cos(2\pi Q_0) \right]$$

$G_R = |\gamma_R|^2/2\pi v_F^2$, *A. Furusaki, K. A. Matveev, Phys. Rev. B 52, 16676 (1995)*

Thermal regime: $T/E_C \gg 1$

$$G = G_R \left[1 - \frac{2|\tau_L|}{v_F} M(T) \cos(2\pi Q_0) \right]$$

$$M(T) = 2 \left(\frac{\pi T}{E_C} \right)^{3/2} \exp\left[-\frac{\pi^2 T}{E_C} \right]$$

Theory: asymmetric barriers, visibility

Quantum regime: $T/E_C \ll 1$

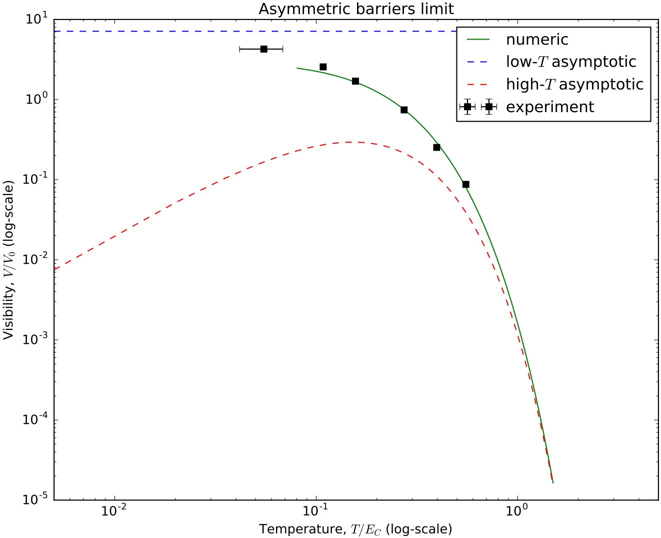
$$V/V_0 = \sqrt{2\pi}\xi$$

$V_0 = |\tau_L|/\sqrt{2\pi}v_F$

Thermal regime: $T/E_C \gg 1$

$$V/V_0 = 4\sqrt{\frac{2}{\pi}}\left(\frac{\pi^2 T}{E_C}\right)^{3/2}\exp\left[-\frac{\pi^2 T}{E_C}\right]$$

Theory: asymmetric barriers, visibility



Conclusion

1. Transport properties and charge quantization phenomenon in SET
2. Perturbation theory in tunneling coupling. Linear conductance
3. Two regimes: quantum, $T \ll E_C$ and thermal, $T \gg E_C$
4. Comparison with experiment
5. Non-perturbation. Non-linear Langevin equations

Acknowledgments

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